Prof: L. B. Stewart.

University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING
SCHOOL OF ENGINEERING RESEARCH

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Aeronautical Research Paper No. 8.
Sept. 1921.

THICK AEROFOILS

By J. H. PARKIN, *Assistant Professor of Mechanical Engineering,*
H. C. CRANE and J. S. E. MACALLISTER, *Research Assistants*

GENERAL SURVEY OF SITUATION

Introduction

The immediate problem in commercial aviation is that of improving the present type of aeroplane so as to increase its safety of operation. From the aerodynamic standpoint the principal defects in this respect are that the minimum flying speed is too high and the stability and controllability in the neighborhood of minimum speed are insufficient. In short, the difficulty is that of "putting the machine on the ground". To lessen this difficulty whilst retaining the principal advantage of aerial transportation, namely speed, requires in the first place an increase in the speed range, *i.e.*, a reduction in the minimum speed with the same or increased top speed.

An increase in speed range may be obtained either by increasing the maximum lift of the wing or by variation of the wing area. The methods employed for increasing the maximum lift are:

1. Development of aerofoil sections having high maximum lifts, that is so called high lift or thick sections.

2. Use of variable lift devices:

(*a*) Variable camber wings including flexible construction and wings with flaps.

(*b*) Slotted type wings.

Variation of the area of the supporting surface may be effected by:

(1) Increasing the chord.

(2) Increasing the span.

The method most frequently used and the one offering the simplest and most straightforward solution with fewest mechanical difficulties is that making use of high lift aerofoil sections. The other methods listed are in various stages of development and have not been applied practically to any great extent. The slotted wing, flexible variable camber and variable area devices have been applied only in experimental machines, if at all.

Classes of Aerofoil Sections

It is now the practice to divide wing sections into two principal classes, thin or high speed sections and thick or high lift sections, with sometimes an intermediate class known as the medium thick section. A thick section may be arbitrarily defined as one having a thickness to chord ratio, t/c in excess of 0.125, while a medium thick section ranges in t/c from 0.09 to 0.125, a thin section being one with a t/c of 0.09 or less.

In early machines[1] and until comparatively recently all wing sections were of the thin type. During the war the thick section was used with success in certain enemy machines (Fokker and Junker) while at the present time a large proportion of the machines designed and in use for commercial purposes employ thick section cantilever wings.

Practice is however divided between the older and conventional externally braced truss type construction generally of biplane form using thin aerofoils and the more recent internally braced thick section cantilever usually of monoplane form.[2] While the use of a thin wing does not necessitate biplane construction nor thick wings require monoplane, biplane construction with the thick aerofoil would appear to be inefficient due to the excessive interference effects.

Relative advantages of Thick and Thin Wing Sections

While there is some controversy regarding the relative merits of thick and thin wings which can only be finally settled by extended full scale tests of machines of the two types the relative advantages may be briefly stated as follows:

1. The thick wing has a much higher maximum lift coefficient, ranging from 0.75 to over 0.80 as compared with about 0.60 for the average thin section. This permits of higher wing loading and hence reduced wing area for the same landing speed or a lower landing speed with the same wing loading.

2. The thick section is well adapted to the internally braced cantilever wing construction which possesses desirable advantages from the aerodynamic and construction standpoints.

---

[1] An exception to this is the Antoinette monoplane of 1909 having wings, tapered in plan form and of deep section, the upper surface having a camber of about 0.155 and the lower one of about 0.06. The span was 46 ft. and average chord 8.2 ft. so that the wings were approximately one foot thick at the fuselage.

[2] This is exemplified by two recent de Havilland commercial aeroplanes the D.H. 29 being a cantilever thick section monoplane and the D.H. 32 a thin section biplane. The designer, Captain de Havilland evidently possessing an open mind as to the relative merits of the two types.

3. The efficiency of the thick section is less than that of thin wings and although the L/D is being constantly improved and increases at a greater rate with vl (see later paper) than for thin sections it is improbable that the L/D of the thick wing alone will ever approach that of the thin wing alone. When the efficiency of the complete machine is considered however that for the machine with the thick wing equals or exceeds that of the externally braced thin wing machine.

4. The lift of most thick section aerofoils is subject to an abrupt falling off at the critical angle due to discontinuity in the flow. This would render machines fitted with such wings easily stalled did it not appear that the stalling angle increases with vl.

5. The general lack of information and data regarding thick sections, particularly as to the scale effect on the characteristics and especially the critical angle is a serious disadvantage of these aerofoils at present.

Advantages of Internally Braced Cantilever Monoplane Construction

Some of the principal points of superiority which render the cantilever monoplane exceptionally well suited to commercial work are:

1. Improved aerodynamic performance with the result that the monoplane can carry the same load with less horse power or greater load with same horse power. This improved performance is due to:

(*a*) Higher overall L/D which results from the greater lift coupled with reduced drag. The latter results not only from the reduction in parasite resistance through the elimination of the interplane bracing but also because the greater lift requires less wing area for the same lift and the reduced wing area in turn means less weight permitting further reduction in wing area, all tending to reduce drag and engine power.

(*b*) Absence of interference such as occurs in superimposed or tandem wings and which renders the lift of the monoplane 25% more effective than that of the biplane.

2. Lends itself particularly well to all metal construction. As to whether all metal construction is wholly desirable can only be proved by experience. The smoother surface is an advantage aerodynamically but the structure is more rigid than the usual wood construction and the fine vibrations from the engine and propeller may prove detrimental through the induced stresses resulting in fatigue and crystallization.

3. Reduction in the number of parts making for more robust construction with decrease in depreciation, inspection and maintenance costs. Depreciation charges at present constitute the major part of the cost of aerial transportation.

4. Possibility of placing fuel, freight, engines, etc., in the deep wings.

5. Ability to make quick repairs by replacing the whole wing.

6. Greater comfort of passengers, including improved vision, accessibility of cabin, etc.

On the question of the relative weights of the two types of construction there is some difference of opinion. At first glance the internally braced cantilever construction would appear to be at a serious disadvantage. As a matter of fact this is less the case than at first appears for, while the cantilever wing will possess a greater weight per square foot for the same strength, it requires a much smaller area due to its greater lift. To derive full advantage from the internally braced cantilever construction the wings are generally tapered both in plan form and section from the centre to the tips. The aerodynamical advantages will appear from the tests described herein, the structural advantages are:

1. The spar depth is increased enabling more economical tapering spars to be used and giving widely separated points of attachment at the body thus reducing the stresses. Further the centre of lift of the wings is brought closer to the body thus decreasing the bending moment in the spars. Recent tests (see N.A.C.A. Report No. 150, p. 6-7) have shown a reduction of 20-30% in bending moment due to a 10-15% of semi span inward movement of the centre of lift.

2. A reasonably large aspect ratio is permitted without necessitating unduly heavy spars.

3. Greater space is available in the wings close to the fuselage for tanks, engines, etc.

While the use of rectangular plan form wings would possess certain advantages from the production standpoint since all of the ribs would be of the same section there are serious structural and aerodynamic drawbacks.

PART I. EFFECT OF TAPERING IN PLAN FORM ON CHARACTERISTICS OF THICK SECTION WINGS

Object of Investigation

The investigation here described is the first of a series on the general subject of thick high lift aerofoils. It was undertaken at the suggestion of the Canadian Air Board[1] for the purpose of determining the effect on the characteristics of tapering a thick section aerofoil from centre to tips while maintaining the same section parallel to the wind. It was desired to know the nature and magnitude of the effect on the lift and L/D and particularly on centre of pressure movement as it was feared that the greatly increased chord at the body might result in excessive centre of pressure movement.

---

[1] The models of the U.S.A. 27 were supplied by the Air Board and the results for this aerofoil are here published with the permission of the Board.

Basic Sections

Of the four basic sections originally selected for this research two were of British and two of American design. At the time this investigation was planned no British thick aerofoil sections had been published as such, with the exception of the RAF-19, which was unsuitable in many ways. British sections of this character were developed for propeller purposes and termed airscrew sections. Two of the latter, suitable for aerofoil purposes were therefore selected, the airscrew 4 and 5 (see R. and M. 322) designated here NPLAS-4 and 5. The former is a comparatively thin high lift section possessing good maximum L/D and maximum lift with low centre of pressure travel. The NPLAS-5 section is poor aerodynamically—but was chosen as one of the basic sections because of its great thickness in order to determine the effect of tapering on sections having a large thickness to chord ratio.

The American sections adopted were the USATS-5 and 10. The former is recognized as the best of the American thick sections suitable for heavy internally braced monoplanes (see A.S.I.C., Vol. II, No. 117). This section has a high L/D at low lifts and high maximum lift. The centre of pressure movement is rather large. The USATS-5 has the same upper curvature as the Durand 13 an American propeller section possessing a very high maximum lift coefficient. The under surface has a camber resembling an exaggerated Phillips entry to permit of the use of a deep forward spar. The latter point was another consideration leading to its adoption in this study.

The USATS-10 was developed at the same time as the latter section. It is intermediate in thickness between the NPLAS-4 and USATS-5, and has a flat undersurface. It possesses the greatest maximum lift of the basic sections with good L/D and fair C.P. movement.

Later, the characteristics of the USA 27 became available and because of their excellence this section was adopted as a fifth basic section. The USA 27 is a medium thick section having a t/c ratio of 0.119 and possesses a high maximum lift, exceptionally good maximum L/D with small C.P. travel. It is a section suitable for small internally braced monoplanes or large biplanes with interplane bracing and long bays.

The contours of the basic sections are tabulated in Table I. The characteristics of the sections as determined at Toronto under similar conditions are plotted in Plate 7. A comparison of the properties of the five basic sections (on a basis of the Toronto data[2]) with various other wing sections is made in Table 2 using the American system outlined in A.S.I.C., Vol. II, No. 117 and Vol. III, No. 267.

On a basis of the Toronto results, see Plate 7, at vl. 10, the USA-27 possesses the best L/D over the whole range of lifts, the maximum being 15.29. The efficiency of the NPLAS-4 is very close to that of the USA-27 for the higher lifts, but is somewhat less at the lower lifts. The USATS-5 and 10 are similar with respect to L/D on C_y, the former being somewhat the better of the two especially at low lifts. The NPLAS-5 as was to be expected has a very poor L/D on C_y curve.

The USATS-10 has the highest maximum lift coefficient, that of the NPLAS-4 being slightly less and the USA-27 next in order. The NPLAS-5 has a very low maximum lift. The USATS-5 although of high t/c ratio does not exhibit the abrupt drop in lift at the critical angle, usual with thick sections. This is possibly an effect of the Phillips entry under surface.

The USA-27 has the least drag of any of the sections, the NPLAS-5 the greatest. The drag curve of the USATS-5 differs in form from the remainder of the curves as it shows little tendency to rise at lifts below that of minimum drag.

The USA-27 and NPLAS-4 sections are practically identical, both with respect to travel and position of the centre of pressure, travel of the USA-27 being slightly the better. The C.P. of the NPLAS-5 is located generally farther back than in the previous two sections, while for the USATS-5 and 10, the location of the C.P. is still farther back and is practically the same for the two aerofoils. The travel for the latter three aerofoils is excessive.

Description of Models

For each basic section a series of five tapered models was tested, based on the standard uniform chord aerofoil. The following principles were observed in designing the tapered models:

1. Leading edge was maintained straight.

2. Section of model parallel to the air current was everywhere of basic section.

---

[2] Two sets of results which differ considerably have been published for USATS-5. The difference is due apparently to a difference in the method of aligning the model. In the original results, which agree with the Toronto figures, the base chord of the model was used in aligning, while in the later (revised) published results apparently the tangent to the undersurface was used. The angle between chord and tangent is 2.63 degrees.

3. The base chord of the model was maintained flat. Where the undersurface of the basic section was straight, the undersurfaces of all the tapered models were flat. In the case of USATS-5 the rear portion only of the undersurface, where it coincides with the base chord was flat. For the USA-27 the base chord is tangent to the curved undersurface and the tangent plane for the tapered models was flat.

4. The ends of the models were finished off perpendicular to the leading edge.

5. The mean chord of the models was in each case 3 inches, with a span of 18 inches.

6. The tapers used were expressed in even fractions of the average (3″) chord. These tapers were:

No. 1 Uniform 3 inch chord
" 2 1.1×0.9, *i.e.*, centre chord 3.3″ tip chord 2.7″
" 3 1.2×0.8 " " " 3.6 " " 2.4
" 4 1.3×0.7 " " " 3.9 " " 2.1
" 5 1.4×0.6 " " " 4.2 " " 1.8

The taper was carried no farther because of the difficulty of fixing the socket in the very thin tip section for chords less than 1.8″. It would have been interesting to have carried the taper to the limit, *i.e.*, tip chord zero or pointed but this would have necessitated the use of a cranked spindle with which C.P. determinations are unsatisfactory. As one of the chief objects of the study was the determination of the effect of taper on C.P. travel, the use of the cranked spindle was not permissible.

The taper was carried to the point where the tip chord was 1.8″ to permit of the study of the influence of the length of the tip chord on the maximum lift and nature of flow.

The models were well seasoned mahogany, carefully scraped to template by hand, shellacked and rubbed. In most cases it was found that the models held their shapes well. Subsequent tests, made one year after the original tests showed that in most cases no warping had occurred. However to eliminate any possible error from this cause the models were roughed out and allowed to stand for possibly a week, then finished and were invariably tested the same day as finished. The condition and changes in the atmosphere had a marked effect on the warping. Models finished on hot humid days proved to be much more liable to warp than those made on dry days.

Conditions of Test

Each model was tested for the usual characteristics, lift, drag, lift/drag and centre of pressure travel, at an air speed of 40 f.p.s. over a range of incidences from -6 to $+20$ degrees. The critical angle was also noted as accurately as possible but is subject to variations of possibly 1° depending on the way in which the model is brought up to the critical angle. The base chord (from which the ordinates are measured) was used in each case in aligning the model.

Results

The results of the different tests are tabulated in Tables 3 to 27 on Plates 2 to 6 and plotted on Plates 8 to 12.

The effects of taper on the characteristics are shown perhaps more effectively on Plate 13 where the principal characteristics are plotted on total taper in per cent. of mean chord.

The figures used are in most cases the averages of the results of two independent tests, in some cases three, which were in good agreement. In calculating the results end spindle corrections have been applied. A subsequent report (No. 12) will give details of the method of determining the end spindle corrections.

The centres of pressure have been calculated by means of a new graphical method possessing advantages in the way of simplicity and rapidity whilst sacrificing none of the accuracy of the usual method. This will be described in Report No. 15.

In tabulating values of C_y these have been given to four places. This does not mean that this degree of accuracy is claimed for the figures but is so given to permit the reader to fix the nearest third figure himself.

The centres of pressure for the tapered models are expressed in per cent. of the *mean chord* (3″) from the leading edge.

In plotting the results the usual angle of incidence base has been discarded. The angle is of no importance in selecting a wing, the important factor being the variation of the characteristics with the lift, *i.e.*, C_y. Hence the characteristics are plotted on a C_y base.

General Conclusions

The general conclusions to be drawn from this investigation of the effect of tapering in plan form on the characteristics of thick section wings may be summarized as follows. These conclusions should be generally applicable to all thick sections since the series was a fairly representative one in range of general form, thickness, efficiency and lift of the sections tested. The conclusions are only applicable however where the tapering is carried out according to the principles followed in this study.

Lift

Tapering the wing affects the lift but little, particularly at small lifts, although as a general rule the lift appears to be slightly increased.

The maximum lift coefficient increases very slightly with increase of taper until the 0.7×1.3 taper is reached beyond which the lift begins to decrease. This seems to indicate that for vl. 10 tapering may not be carried beyond the point where the tip chord is reduced to 2 inches. This is in agreement with the statement in the N.A.C.A. Report No. 75, p. 17, to the effect that reducing the chord below 2 inches introduces a break in the air flow with reduction in the maximum lift.

The critical angle for all the sections, with the exception of the USATS-10, is increased by tapering the wing. The data is not sufficiently definite to determine whether the critical angle is decreased when the tip chord becomes less than 2 inches but there is some indication that such is the case.

Drag

The drag at lifts above that of minimum drag is decreased by tapering, the uniform chord model showing the greatest drag, while below minimum drag, the reverse is the case, the uniform model having the least drag.

The minimum drag, as a general rule, decreases slightly as the taper is increased for the more moderate tapers, while for the greatest taper tested, there is a more pronounced reduction in the minimum drag. The minimum drag occurs at slightly greater incidences and lifts as the taper becomes greater.

Lift/Drag

An increase of from 1½ to 3% in maximum L/D is obtained by tapering a wing the maximum amount tested in this investigation (0.6×1.4). In the case of the USATS-5 tapering does not appear to influence in any way the maximum L/D. The effect on the L/D at lifts below that of maximum L/D is small and indefinite but for lifts above that of maximum L/D there is a slight though definite increase in efficiency. There appears to be no change in the angle of maximum L/D as a result of tapering.

Centre of Pressure

The centre of pressure travel, expressed as a fraction of the mean chord is increased by tapering. The graphs plotted on Plate 13 show the variation on C.P. travel between the position corresponding to 0.25 C_y max. and the maximum forward position. The upper set of graphs show that the C.P. travel (as a fraction of the mean chord) for the 0.6×1.4 taper is increased from 7½ to 35% as compared with that of the uniform chord model. The increase is less the thicker the section, the NPLAS-5 having an increase of 7.7%, the USATS-5 one of 10.9%, the USATS-10 13.7%, the NPLAS-4 33.3% and the USA-27 36.8%.

On the other hand, as shown in the lower set of graphs the C.P. travel expressed as a fraction of the maximum chord at midspan decreases with increase of taper. The thinner sections USA-27 and NPLAS-4 exhibiting the smaller reductions, namely 1.6 and 5.1% respectively, the USATS-10 showing a 12.8,% decrease and the USATS-5 and NPLAS-5 reductions of 23.6% and 28.0% respectively for the maximum taper as compared with the uniform chord model.

The above effects of taper on C.P. travel agree with the data given in the N.A.C.A. Report No. 150, pp. 19, 20, 21, derived from tests on the distribution of pressure over thick tapered aerofoils. For a flat bottomed aerofoil section the C.P. movement corresponding to a given change in incidence was found to increase more or less consistently with increase in taper.

Thickness of aerofoil section.

The results of the tests on the NPLAS-5 section agree with the results of tests given in N.A.C.A. Report No. 75, pp. 8 and 21, which indicated that 0.159 is a critical value of the t/c ratio at vl. 10, any greater thickness resulting in discontinuity of flow accompanied by a reduction in maximum lift. The results of the tests at Toronto on the NPLAS-5 were erratic and unsatisfactory apparently because of the discontinuous nature of the flow about the section. The aerofoil has a very low critical angle at vl. 10 and consequently a low maximum lift. Tapering appears to have no more marked effect on this thick section than on any of the other sections tested.

APPENDIX

PREPARATION OF TEMPLATES AND FORM OF SOCKETS FOR THE END SPINDLE

Templates

The templates were prepared in the following manner. A master drawing of the basic section was made with a 30″ chord, and reduced photographically to the desired chord. A readily secured accuracy of 0.01″ on the drawing thus became, on the average, 0.001″ on the photographic plate. It was found possible to produce the negative to within less than 0.003″ of the required chord. The negative was then printed on 3×1/16″ cold rolled steel and the template cut. The tool maker in cutting the template could readily split the line printed on the steel so that the accuracy of the template particularly with respect to ordinates is quite high.

The advantages in the way of accuracy of contour and cost of this method as compared with the usual method of laying out such a template are obvious. From the standpoint of this particular investigation for the production of the tapered models the method is particularly good. Thus nine templates of each basic section were made having chords of 1.8, 2.1, 2.4, 2.7, 3.0, 3.3, 3.6, 3.9 and 4.2 inches. By this method all nine were of exactly the same form whereas if separately laid out small irregularities in form would inevitably have resulted causing differences in the models.

In preparing the master drawing, the ordinate lines at each 1/10 of the chord were carried beyond the section contour and being printed on the steel remained after the template was cut. The model maker in using the templates first roughed out the centre and tip sections, and marked thereon these ordinates. The model was then finished so that corresponding ordinates at centre and tip were joined by straight lines, which was readily checked by means of a straight edge. An additional check was afforded by the use of templates at intermediate points between centre and tip.

In the case of the USATS-5 and USA-27 having curved undersurfaces the template was made in two parts (requiring two prints on steel), cut with stepped joints at the ends and held together in clamps.

Sockets

The end spindle was screwed into a brass socket in the end of the aerofoil. The most satisfactory form for this socket proved to be one of brass, 1½″ long, straight knurled, *i.e.*, ribbed, tapped with an 8×32 thread, with the outside diameter made as small as possible over this thread. A hole slightly smaller than the outside diameter of the knurled socket was drilled in the end of the model and the socket forced in and set with shellac. In most cases the socket was located the usual 1″ from the leading edge but for some of the larger tapers in order to take advantage of the greatest thickness of the section the sockets were located ½″ from the leading edge.

Aug 1921

PLATE No. I.

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THICK AEROFOILS

Table No. I CONTOURS OF BASIC SECTIONS.

Fraction of chord	NPLAS·4 Upper	NPLAS·4 Lower	NPLAS·5 Upper	NPLAS·5 Lower	USATS·5 Upper	USATS·5 Lower	USATS·10 Upper	USATS·10 Lower	USA·27 Upper	USA·27 Lower	Fraction of chord
0.00	2.00	2.00	2.50	2.50	2.00	2.00	1.50	1.50	1.77	1.77	0.00
1.25					4.40	0.00	4.00	0.10	3.80	0.50	1.25
2.50					5.50	−0.80	5.60	0.00	5.07	0.33	2.50
5.00	7.94	0.00	11.67	0.00	7.40	−1.80	8.30	0.00	6.93	0.17	5.00
7.50					8.80	−2.50	10.00	0.00	8.22	0.10	7.50
10.00	10.20	0.00	15.05	0.00	10.00	−3.00	11.40	0.00	9.17	0.00	10.00
15.00					11.80	−3.40	13.30	0.00	10.50	0.10	15.00
20.00	12.18	0.00	18.10	0.00	13.10	−3.50	14.50	0.00	11.33	0.35	20.00
30.00	12.70	0.00	18.80	0.00	14.70	−2.90	15.80	0.00	11.90	0.95	30.00
40.00	12.44	0.00	18.16	0.00	14.80	−1.50	15.70	0.00	11.57	1.17	40.00
50.00	11.51	0.00	16.65	0.00	13.90	−0.70	14.70	0.00	10.77	0.90	50.00
60.00	10.20	0.00	14.55	0.00	12.30	−0.30	13.10	0.00	9.52	0.25	60.00
70.00	8.60	0.00	12.10	0.00	10.30	−0.20	10.90	0.00	8.00	0.10	70.00
80.00	6.68	0.00	9.25	0.00	7.80	−0.10	8.00	0.00	6.03	0.05	80.00
90.00	4.23	0.00	5.87	0.00	4.90	0.00	5.00	0.00	3.65	0.15	90.00
95.00					3.30	0.00	3.40	0.00	2.00	0.47	95.00
100.00					1.00	1.00	1.00	1.00	0.67	0.67	100.00
Rad L.E	2.00		2.50		2.00		1.50		1.05		Rad. L.E
" T.E									0.21		" T.E
Reference	R&M. -	322	R&M -	322	NACA -	Rep. 93	NACA ·	Rep. 93	ASIC Vol	II. No. 198	Reference

Table No 2 COMPARISON OF CHARACTERISTICS OF VARIOUS SECTIONS.

		R.A.F.				Durand	U.S.A	Glenn Martin		Gottingen		NPLAS		USATS		USA	
		6	6a	15	19	13	33	2	4	227	244	4	5	5	10	27	
1	Landing Speed · maximum Cy	0.615	0.581	0.517	0.845	0.799	0.713	0.781	0.746	0.863	0.861	0.709	0.595	0.694	0.766	0.674	1
2	Cruising Speed · maximum L/D	16.00	17.10	17.30	12.00	13.26	12.80	14.00	13.60	12.40	12.00	14.95	11.75	12.80	12.68	15.29	2
3	High speed · pursuit L/D at $Cy_{max}/9.0$	4.50	4.80	8.10	2.45	3.10	3.90	4.40	4.80	2.80	1.45	3.35	1.80	4.00	2.90	5.00	3
4	" " · reconnoisance fighter L/D at $Cy_{max}/6.25$	6.40	7.35	10.70	3.70	5.10	5.80	6.50	6.80	5.80	2.55	6.30	2.75	5.75	4.50	7.45	4
5	" " · bomber · commercial L/D at $Cy_{max}/4.0$	10.00	11.90	14.80	6.10	8.60	9.00	11.40	10.80	9.40	5.50	10.40	5.15	8.85	7.80	11.30	5
6	Speed Range $\sqrt{Cy_{max}} \div \sqrt[3]{Cx_{min}}$	3.22	3.33	3.67	2.81	3.13	3.13	3.50	3.31	3.25	2.92	3.38	2.62	3.13	3.02	3.37	6
7	Ceiling and Climb · cons loading · max value $Cy^{3/2} \div Cx$	8.95	9.14	8.80	9.05	8.77	8.35	8.49	8.28	8.81	8.58	8.90	7.50	8.58	8.46	9.09	7
8	" " " · cons landing speed minimum value · $\sqrt{\frac{Cy_{max}}{Cy}} \div L/D$	0.880	0.0833	0.0820	0.1020	0.1020	0.1012	0.1040	0.1044	0.1080	0.106	0.0930	0.1013	0.970	0.1035	0.0905	8
9	Maximum forward position of C.P.	0.28*	0.29	0.276	0.339		0.266			0.307	0.373	0.303	0.318	0.308	0.313	0.300	9
10	C.P. travel between max. forward position and that at $Cy_{max}/6.25$	0.23*	0.265	0.114	0.211		0.224			0.693	0.372	0.366	0.492	0.512**	0.443	0.340	10
11	" " " " " $Cy_{max}/4.00$	0.16*	0.185	0.092	0.161		0.134			0.423	0.292	0.208	0.302	0.290	0.257	0.180	11
12	Spar Depths · 10% from leading edge	5.70	6.20	6.01	10.40	10.00	10.40	11.16	11.43	12.20	14.50	10.20	15.05	13.00	11.40	9.17	12
13	15% " " "	6.40	7.05	6.30	10.60	11.80	12.80	12.60	13.50	14.10	16.30	11.40	16.80	15.20	13.30	10.40	13
14	60% " " "	6.30	6.80	5.20	5.50	12.30	12.40	10.93	13.09	8.80	9.40	10.20	14.55	12.60	13.10	9.27	14
15	70% " " "	5.40	5.80	4.60	4.50	10.30	10.60	9.40	11.37	6.30	6.70	8.60	12.10	10.50	10.90	7.90	15
16	Model.	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	3×18	16
17	vl.	10	10	10	10	11	11	11	11	11	11	10	10	10	10	10	17
18	Reference	R&M 110 pt. II pp 97	R&M 195 Sec I pp 75	R&M 248 sec 5 p 87	R&M 415 p. 132	NACA 93 p 37	ASIC III 244	Journal · March	S.A.E. 1921	ASIC III No 267	ASIC III No 248		— This Report —				18

*For 12.5×2.1 Model see R&M 78. **Estimated.

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CHARACTERISTICS NPL AS-4. TAPERED AEROFOILS

TABLE No. 3 UNIFORM CHORD

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0794	0.0359	-2.21	77.7
-4	0.0383	0.0236	1.62	
-2	0.1313	0.0182	7.50	60.6
0	0.2040	0.0173	11.81	46.6
+2	0.2769	0.0190	14.60	40.5
4	0.3513	0.0237	14.81	36.7
6	0.4216	0.0303	13.91	34.3
8	0.4908	0.0384	12.78	32.8
10	0.5582	0.0481	11.60	31.9
12	0.6195	0.0588	10.52	31.0
14	0.6753	0.0703	9.60	30.6
16	0.7084	0.0827	8.56	30.3
18	0.4256	0.1369	3.11	38.0
20	0.4166	0.1564	2.66	39.4

TABLE No. 4 TAPERED - 0.9x1.1

Angle of Incidence (deg.)	Lift Coefficient (abs)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0993	0.0400	-2.48	66.2
-4	0.0249	0.0263	0.95	
-2	0.1279	0.0196	6.53	60.1
0	0.2055	0.0183	11.27	46.9
-2	0.2742	0.0196	14.16	39.4
4	0.3526	0.0237	14.89	36.2
6	0.4244	0.0302	14.02	33.6
8	0.4953	0.0385	12.88	32.0
10	0.5634	0.0478	11.79	30.7
12	0.6260	0.0584	10.73	30.2
14	0.6840	0.0700	9.76	29.2
16	0.7324	0.0825	8.85	29.2
18	0.4378	0.1379	3.17	36.7
20	0.4234	0.1574	2.68	36.9

TABLE No. 5 TAPERED - 0.8x1.2

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs)	L/D	Centre of Pressure % chord
-6	-0.0984	0.0401	0.246	2.2
-4	0.0263	0.0263	0.997	
-2	0.1286	0.0196	7.24	66.1
0	0.2058	0.0178	11.59	48.8
+2	0.2794	0.0192	14.51	41.0
4	0.3517	0.0235	14.93	37.1
6	0.4234	0.0301	14.09	34.8
8	0.4933	0.0382	12.92	32.8
10	0.5618	0.0475	11.81	31.9
12	0.6264	0.0584	10.70	30.8
14	0.6824	0.0697	9.79	30.0
16	0.7270	0.0819	8.87	29.6
18	0.4262	0.1377	3.09	37.6
20	0.4095	0.1559	2.64	38.5

TABLE No. 6 TAPERED - 0.7x1.3

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0846	0.0369	-2.30	0.3
-4	0.0341	0.0240	1.42	
-2	0.1304	0.0180	7.25	63.3
0	0.2047	0.0173	11.83	48.7
+2	0.2760	0.0189	14.60	40.0
4	0.3491	0.0232	15.01	35.6
6	0.4201	0.0298	14.12	32.8
8	0.4878	0.0377	12.91	32.4
10	0.5490	0.0465	11.81	31.1
12	0.6134	0.0568	10.80	29.6
14	0.6662	0.0676	9.86	28.8
16	0.7111	0.0791	9.00	28.3
18	0.4248	0.1344	3.16	36.0
20	0.4054	0.1516	2.68	36.0

TABLE No. 7 TAPERED - 0.6x1.4

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0934	0.0379	-2.46	
-4	0.0219	0.0251	0.87	
-2	0.1201	0.0189	6.34	71.9
0	0.1969	0.0173	11.40	52.8
+2	0.2688	0.0186	14.46	43.2
4	0.3410	0.0225	15.17	37.3
6	0.4091	0.0284	14.13	36.5
8	0.4775	0.0362	13.20	33.6
10	0.5440	0.0451	12.07	33.5
12	0.6076	0.0553	11.01	32.3
14	0.6648	0.0661	10.04	31.5
16	0.7070	0.0773	9.13	29.8
18	0.4329	0.1321	3.27	35.3
20	0.4123	0.1491	2.76	37.4

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CHARACTERISTICS NPL AS-5 TAPERED AEROFOILS

TABLE No. 8 UNIFORM CHORD

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure %chord
-6	0.0724	0.0376	1.92	90.9
-4	0.1415	0.0289	4.90	64.2
-2	0.2105	0.0255	8.25	49.6
0	0.2812	0.0270	10.41	45.8
+2	0.3525	0.0305	11.52	38.4
4	0.4215	0.0362	11.67	35.4
6	0.4872	0.0447	10.88	33.8
8	0.5475	0.0543	10.07	32.3
10	0.5947	0.0658	9.04	31.8
12	0.3435	0.1066	3.23	37.1
14	0.3383	0.1238	2.73	37.4
16	0.3478	0.1400	2.48	37.5
18	0.3622	0.1577	2.30	37.8
20	0.3797	0.1744	2.18	37.6

TABLE No. 9 TAPERED-0.9×1.1

Angle of Incidence (deg.)	Lift Coefficient (abs)	Drag Coefficient (abs.)	L/D	Centre of Pressure %chord
-6	0.0746	0.0412	1.81	81.6
-4	0.1393	0.0317	4.30	63.0
-2	0.2123	0.0274	7.76	50.5
0	0.2826	0.0278	10.17	43.3
+2	0.3564	0.0311	11.47	39.1
4	0.4258	0.0365	11.69	36.4
6	0.4922	0.0444	11.10	34.6
8	0.5558	0.0539	10.32	33.2
10	0.6138	0.0644	9.52	32.3
12	0.6630	0.0759	8.72	31.4
14	0.3411	0.1236	2.76	37.7
16	0.3469	0.1389	2.47	37.8
18	0.3544	0.1533	2.28	37.9
20	0.3644	0.1682	2.14	37.6

TABLE No. 10 TAPERED-0.8×1.2

Angle of Incidence (deg)	Lift Coefficient (abs.)	Drag Coefficient (abs)	L/D	Centre of Pressure %chord
-6	0.0720	0.0417	1.73	83.7
-4	0.1335	0.0319	4.18	64.1
-2	0.2077	0.0275	7.52	50.4
0	0.2787	0.0277	10.07	43.4
+2	0.3515	0.0306	11.48	39.1
4	0.4199	0.0355	11.81	36.3
6	0.4864	0.0433	11.22	34.6
8	0.5502	0.0529	10.38	33.1
10	0.6028	0.0633	9.52	32.5
12	0.3481	0.1057	3.30	37.0
14	0.3278	0.1211	2.71	36.3
16	0.3281	0.1357	2.42	37.4
18	0.3344	0.1499	2.24	37.3
20	0.3472	0.1625	2.13	37.0

TABLE No. 11 TAPERED-0.7×1.3

Angle of Incidence (deg.)	Lift Coefficient (abs)	Drag Coefficient (abs.)	L/D	Centre of Pressure %chord
-6	0.0703	0.0402	1.75	97.4
-4	0.1338	0.0310	4.32	65.9
-2	0.2066	0.0270	7.65	53.4
0	0.2779	0.0276	10.06	44.4
+2	0.3497	0.0306	11.41	40.7
4	0.4185	0.0357	11.73	37.6
6	0.4858	0.0433	11.21	35.7
8	0.5483	0.0526	10.42	34.4
10	0.6096	0.0628	9.72	33.1
12	0.6641	0.0733	9.05	32.0
14	0.3343	0.1213	2.76	37.8
16	0.3267	0.1352	2.42	37.6
18	0.3348	0.1487	2.25	37.9
20	0.3468	0.1656	2.09	37.9

TABLE No. 12 TAPERED-0.6×1.4

Angle of Incidence (deg)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure %chord
-6	0.0671	0.0394	1.70	100.0
-4	0.1336	0.0300	4.45	70.5
-2	0.2050	0.0259	7.90	54.1
0	0.2744	0.0262	10.48	46.6
+2	0.3458	0.0287	12.10	41.4
4	0.4141	0.0338	12.24	37.8
6	0.4792	0.0409	11.70	36.6
8	0.5415	0.0499	10.86	34.8
10	0.5966	0.0598	9.96	33.6
12	0.3307	0.1049	3.15	37.4
14	0.3199	0.1193	2.68	38.0
16	0.3213	0.1393	2.31	37.9
18	0.3306	0.1465	2.25	38.0
20	0.3430	0.1642	2.15	38.1

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TABLE No. 13 UNIFORM CHORD

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0276	0.0203	-1.36	
-4	0.0640	0.0189	3.39	
-2	0.1412	0.0191	7.32	68.1
0	0.2122	0.0206	10.18	52.7
+2	0.2864	0.0235	12.00	44.9
4	0.3600	0.0280	12.72	40.3
6	0.4336	0.0337	12.72	37.4
8	0.5038	0.0419	12.01	35.4
10	0.5704	0.0511	11.14	33.8
12	0.6315	0.0609	10.37	32.8
14	0.6757	0.0701	9.65	31.7
16	0.6909	0.0808	8.54	30.8
18	0.6606	0.0967	6.50	30.4
20	0.6388	0.1118	5.71	30.6

TABLE No. 14 TAPERED - 0.9×1.1

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0296	0.0192	-1.53	
-4	0.0555	0.0187	2.97	
-2	0.1331	0.0194	7.19	68.5
0	0.2053	0.0209	9.85	53.5
+2	0.2780	0.0237	11.70	45.3
4	0.3533	0.0279	12.64	40.3
6	0.4283	0.0337	12.70	37.3
8	0.4995	0.0410	12.19	35.2
10	0.5666	0.0499	11.37	33.4
12	0.6310	0.0597	10.58	32.2
14	0.6835	0.0691	9.90	31.2
16	0.7150	0.0782	8.99	30.2
18	0.6918	0.0911	7.60	29.4
20	0.6438	0.1069	6.05	29.7

TABLE No. 15 TAPERED - 0.8×1.2

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0199	0.0183	-1.09	
-4	0.0601	0.0178	3.37	
-2	0.1351	0.0180	7.52	70.9
0	0.2057	0.0203	10.13	54.3
+2	0.2791	0.0232	12.00	46.2
4	0.3539	0.0277	12.75	41.0
6	0.4280	0.0340	12.60	38.2
8	0.4996	0.0415	12.03	36.0
10	0.5701	0.0503	11.32	34.4
12	0.6344	0.0595	10.67	33.3
14	0.6844	0.0686	9.96	32.0
16	0.7040	0.0789	8.92	30.9
18	0.6740	0.0939	7.16	30.6
20	0.6409	0.1091	5.86	31.1

TABLE No. 16 TAPERED - 0.7×1.3

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0282	0.0175	-1.61	
-4	0.0556	0.0168	3.30	
-2	0.1317	0.0187	7.02	72.0
0	0.2047	0.0206	9.94	54.3
+2	0.2769	0.0235	11.78	47.0
4	0.3520	0.0277	12.70	41.6
6	0.4248	0.0334	12.70	38.3
8	0.4956	0.0409	12.13	36.1
10	0.5624	0.0494	11.39	34.2
12	0.6245	0.0587	10.62	32.5
14	0.6797	0.0674	10.08	30.6
16	0.6940	0.0758	9.15	29.7
18	0.6589	0.0909	7.25	29.4
20	0.6125	0.1058	5.78	30.3

TABLE No. 17 TAPERED - 0.6×1.4

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	-0.0289	0.0187	-1.54	
-4	0.0512	0.0178	2.87	
-2	0.1264	0.0188	6.71	75.3
0	0.2008	0.0206	9.74	56.0
+2	0.2738	0.0232	11.88	47.6
4	0.3489	0.0275	12.68	42.3
6	0.4200	0.0331	12.68	39.1
8	0.4927	0.0405	12.18	36.8
10	0.5593	0.0490	11.41	35.4
12	0.6208	0.0581	10.69	33.8
14	0.6745	0.0671	10.04	32.7
16	0.7018	0.0759	9.23	31.8
18	0.6923	0.0886	7.81	31.2
20	0.6620	0.1030	6.47	30.7

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CHARACTERISTICS U.S.A.T.S-10 TAPERED AEROFOILS

TABLE No. 18 UNIFORM CHORD

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0319	0.0375	0.85	
-4	0.1044	0.0286	3.65	85.3
-2	0.1760	0.0246	7.16	59.8
0	0.2448	0.0245	9.98	48.2
+2	0.3195	0.0266	12.00	42.1
4	0.3917	0.0309	12.68	38.9
6	0.4631	0.0381	12.16	36.5
8	0.5341	0.0466	11.47	34.7
10	0.6020	0.0547	10.72	33.6
12	0.6668	0.0666	10.00	32.6
14	0.7194	0.0780	9.22	31.9
16	0.7560	0.0899	8.40	31.3
18	0.4256	0.1378	3.09	37.1
20	0.4237	0.1555	2.72	37.4

TABLE No. 19 TAPERED - 0.9×1.1

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0302	0.0388	0.778	
-4	0.1030	0.0294	3.50	84.1
-2	0.1760	0.0247	7.13	57.7
0	0.2454	0.0246	9.98	46.9
+2	0.3201	0.0269	11.91	42.6
4	0.3933	0.0308	12.75	38.6
6	0.4667	0.0378	12.32	36.2
8	0.5406	0.0463	11.65	35.2
10	0.5924	0.0553	10.70	33.4
12	0.6690	0.0659	10.17	32.3
14	0.7243	0.0776	9.34	32.2
16	0.7620	0.0896	8.50	31.0
18	0.4279	0.1373	3.11	37.3
20	0.4186	0.1526	2.74	37.9

TABLE No. 20 TAPERED - 0.8×1.2

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0294	0.0391	0.75	
-4	0.1002	0.0299	3.34	85.4
-2	0.1744	0.0253	6.89	60.2
0	0.2453	0.0244	10.07	47.3
+2	0.3201	0.0263	12.17	41.7
4	0.3931	0.0304	12.90	38.2
6	0.4662	0.0371	12.57	35.7
8	0.5376	0.0453	11.88	34.8
10	0.6048	0.0549	11.02	32.8
12	0.6665	0.0652	10.22	32.2
14	0.7205	0.0770	9.36	31.6
16	0.7566	0.0892	8.46	30.8
18	0.4098	0.1355	3.02	37.8
20	0.4036	0.1511	2.66	36.4

TABLE No. 21 TAPERED - 0.7×1.3

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0260	0.0376	0.69	
-4	0.0997	0.0282	3.52	87.8
-2	0.1737	0.0240	7.23	62.4
0	0.2458	0.0239	10.23	48.4
+2	0.3194	0.0258	12.38	42.7
4	0.3928	0.0298	13.15	38.7
6	0.4649	0.0367	12.69	36.4
8	0.5363	0.0449	11.94	35.6
10	0.6040	0.0541	11.10	34.3
12	0.6651	0.0643	10.35	32.8
14	0.7168	0.0754	9.50	31.7
16	0.7556	0.0871	8.65	31.1
18	0.4169	0.1300	3.11	36.4
20	0.4020	0.1484	2.71	37.0

TABLE No. 22 TAPERED - 0.6×1.4

Angle of Incidence (deg.)	Lift Coefficient (abs.)	Drag Coefficient (abs.)	L/D	Centre of Pressure % chord
-6	0.0164	0.0403	0.41	
-4	0.0878	0.0302	2.91.	97.5
-2	0.1648	0.0248	6.94	64.3
0	0.2367	0.0243	9.74	50.8
+2	0.3100	0.0258	12.02	44.4
4	0.3817	0.0294	13.00	40.2
6	0.4523	0.0358	12.62	38.0
8	0.5201	0.0437	11.90	36.3
10	0.5852	0.0527	11.10	34.4
12	0.6469	0.0628	10.31	33.2
14	0.7020	0.0736	9.54	32.4
16	0.7420	0.0848	8.75	30.6
18	0.4246	0.1302	3.26	35.9
20	0.4006	0.1444	2.79	36.0

Aug·1921. Aeronautical Research Paper No.8. Plate ⑥

UNIVERSITY OF TORONTO

CHARACTERISTICS-U.S.A.-27 TAPERED AEROFOILS

TABLE No.23 UNIFORM CHORD

Angle of Incidence (deg)	Lift Coefficient (abs)	Drag Coefficient (abs)	L/D	Centre of Pressure %chord
-6	-0·0898	0·0331	-2·71	
-4	0·0275	0·0183	1·50	
-2	0·1208	0·0145	8·32	60·0
0	0·1930	0·0152	12·67	44·7
+2	0·2695	0·0178	15·10	40·4
4	0·3437	0·0230	14·97	36·5
6	0·4151	0·0297	13·98	34·2
8	0·4860	0·0376	12·91	32·9
10	0·5547	0·0467	11·88	31·6
12	0·6183	0·0565	10·95	31·3
14	0·6610	0·0662	9·99	30·5
16	0·6715	0·0777	8·65	30·0
18	0·5171	0·1343	3·89	35·2
20	0·4417	0·1607	2·78	38·6

TABLE No.24 TAPERED - 0·9x1·1

Angle of Incidence (deg)	Lift Coefficient (abs)	Drag Coefficient (abs)	L/D	Centre of Pressure %chord
-6	-0·0898	0.0346	-2·59	
-4	0·0257	0·0196	1·31	
-2	0·1258	0·0151	8·17	61·1
0	0·2040	0·0157	12·79	45·4
+2	0·2837	0·0181	15·40	39·3
4	0·3567	0·0232	15·18	35·2
6	0·4290	0·0300	14·09	33·2
8	0·4977	0.0377	13·00	31·4
10	0·5652	0·0467	11·92	30·8
12	0·6237	0·0572	10·91	30·5
14	0·6789	0·0676	10·02	28·9
16	0·7249	0·0783	9·25	28·2
18	0·7367	0·0883	8·34	27·7
20	0·6882	0·1075	6·39	27·9

TABLE No.25 TAPERED-0·8x1·2

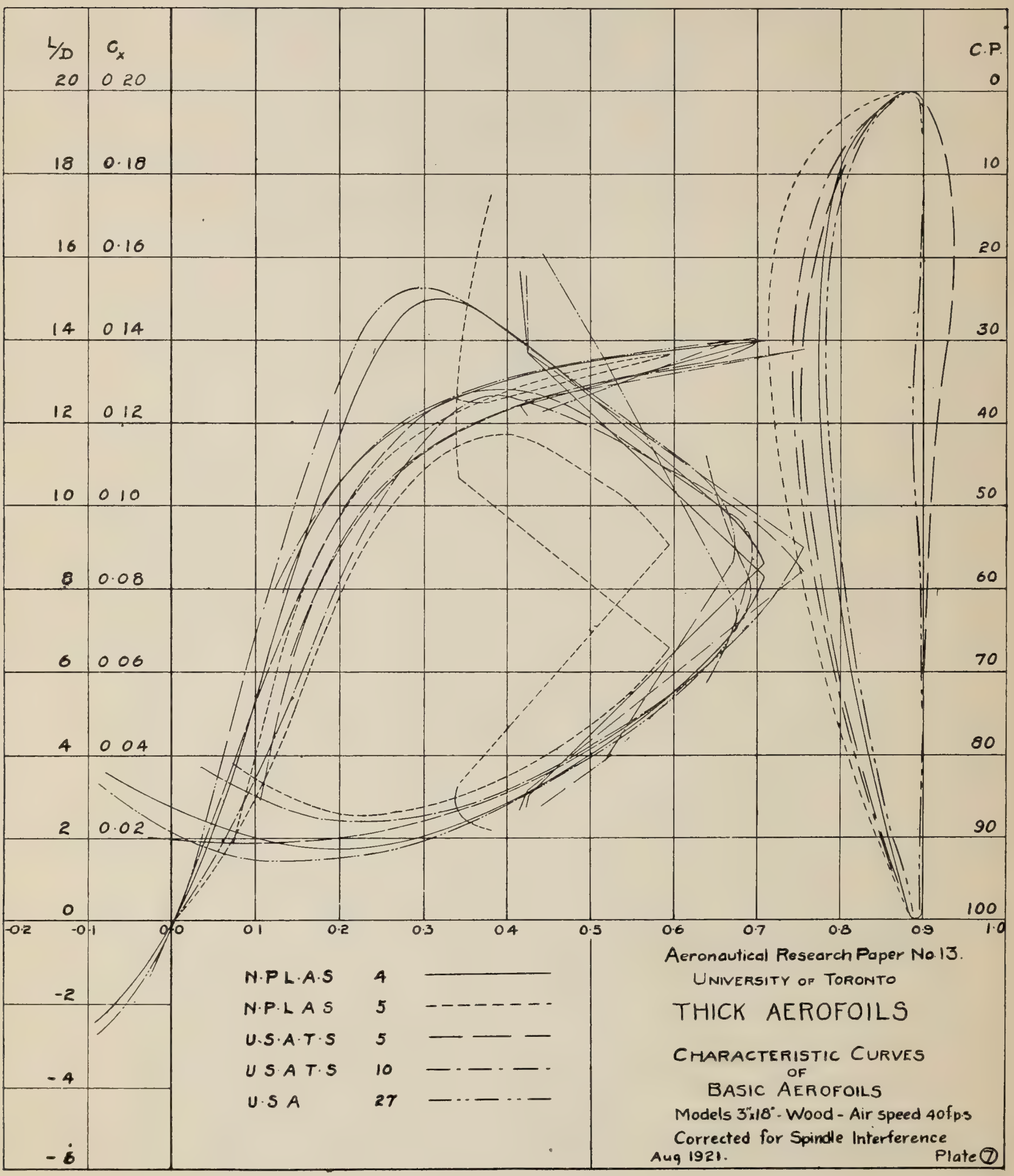
Angle of Incidence (deg)	Lift Coefficient (abs)	Drag Coefficient (abs)	L/D	Centre of Pressure %chord
-6	-0·0859	0·0334	-2·57	
-4	0·0263	0·0190	1·39	
-2	0·1221	0·0150	8·16	65·6
0	0·2004	0·0151	13·60	47·2
+2	0·2778	0·0178	15·62	40·3
4	0·3532	0·0228	15·30	35·9
6	0·4183	0·0297	14·09	33·6
8	0·4844	0·0367	13·07	31·4
10	0·5507	0·0459	12·00	30·6
12	0·6148	0·0560	10·99	29·6
14	0·6700	0·0659	10·18	28·4
16	0·7093	0·0755	9·38	27·6
18	0·6983	0·0881	7·92	27·4
20	0.6559	0·1093	5·99	28·8

TABLE No.26 TAPERED-0·7x1·3

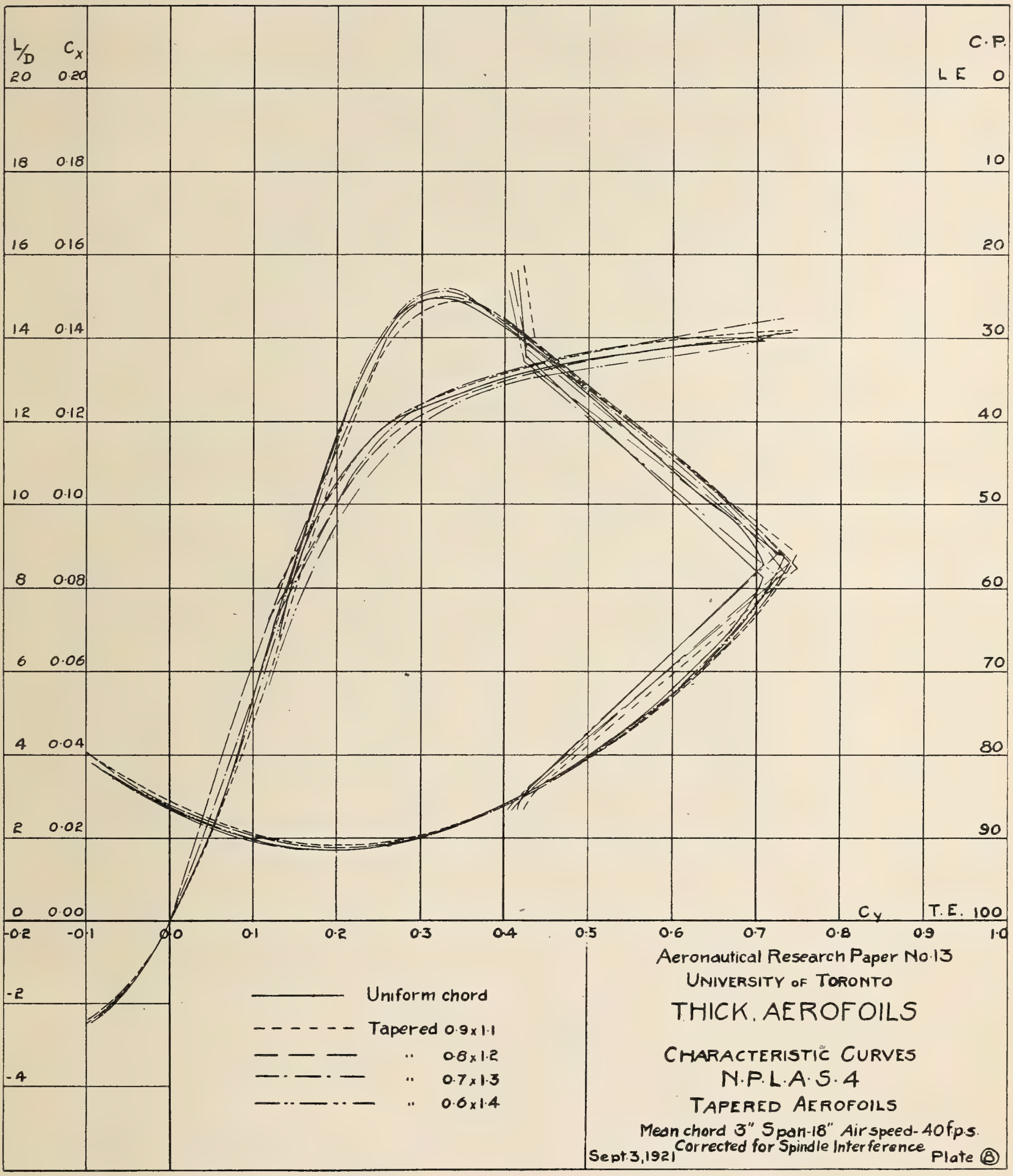
Angle of Incidence (deg)	Lift Coefficient (abs)	Drag Coefficient (abs)	L/D	Centre of Pressure % chord
-6	-0·1027	0·0353	-2·82	
-4	0·0211	0·0211	1·00	
-2	0·1176	0·0157	7·53	68·2
0	0·1885	0·0150	12·54	50·7
+2	0·2668	0·0174	15·30	41·2
4	0·3394	0·0223	15·23	37·1
6	0·4068	0·0284	14·36	34·3
8	0·4734	0·0357	13·22	32·2
10	0·5383	0·0443	12·12	31·3
12	0·6002	0·0537	11·16	30·2
14	0·6602	0·0639	10·33	29·1
16	0·7089	0·0739	9·59	28·0
18	0·7334	0·0830	7·48	27·3
20	0·7008	0·0979	7·15	28·2

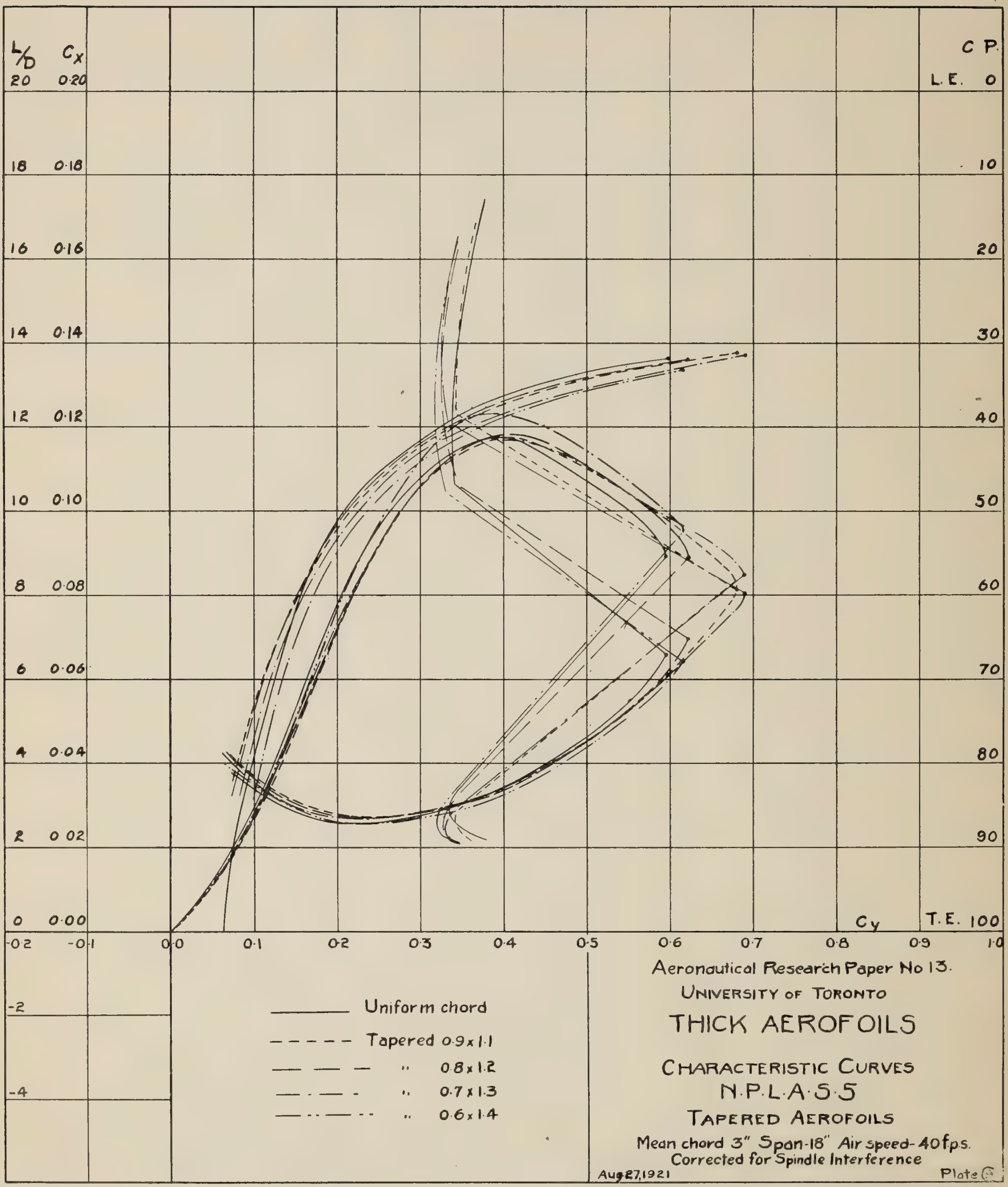
TABLE No.27 TAPERED-0·6x1·4

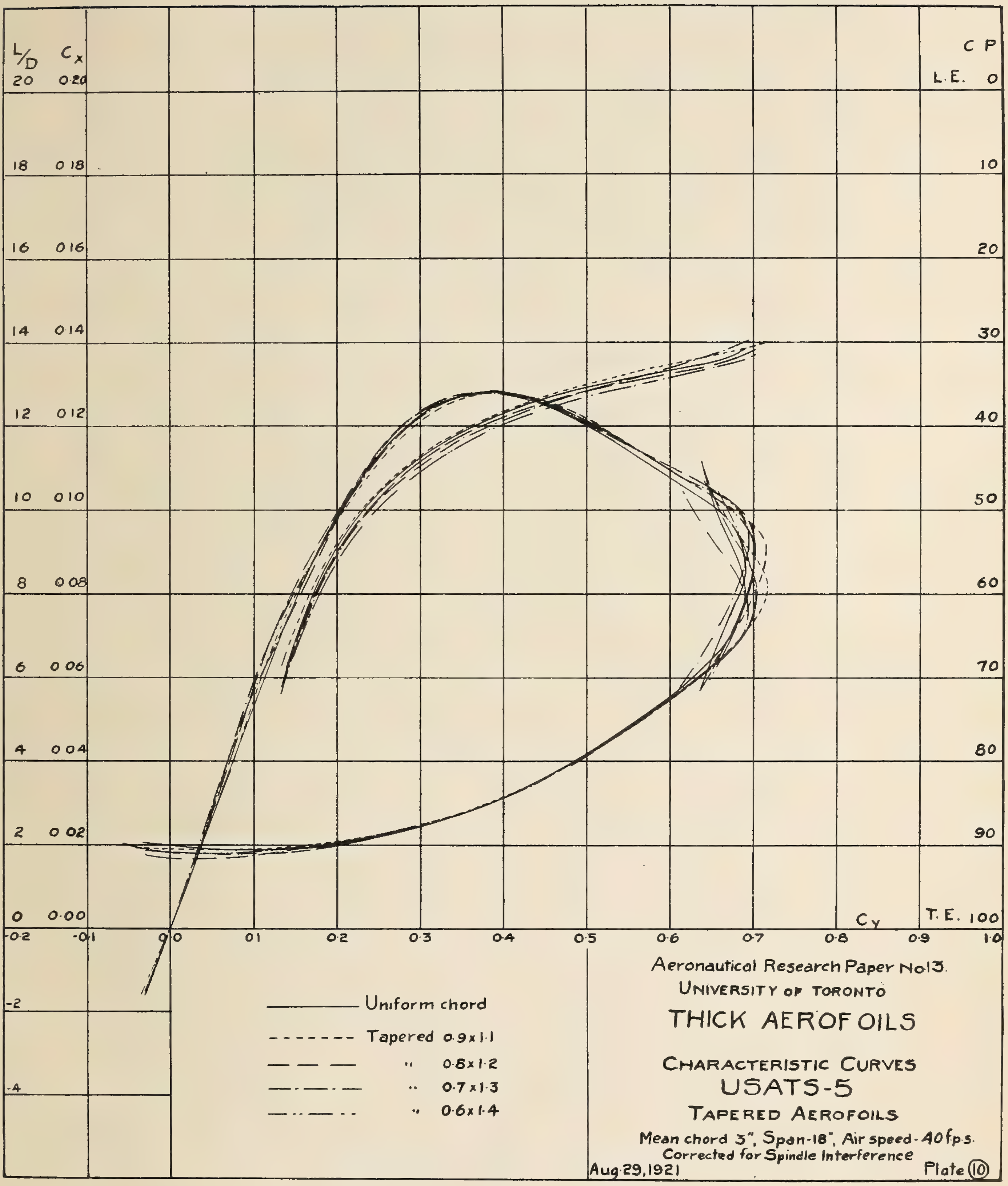
Angle of Incidence (deg)	Lift Coefficient (abs)	Drag Coefficient (abs)	L/D	Centre of Pressure %chord
-6	-0·0979	0·0353	-2·77	01·0
-4	0·0119	0·0222	0·53	
-2	0·1079	0·0164	6·56	69·8
0	0·1880	0·0152	12·29	51·1
+2	0·2614	0·0172	15·20	43·6
4	0·3351	0·0213	15·76	41·9
6	0·4043	0·0277	14·62	35·4
8	0·4693	0·0348	13·44	33·9
10	0·5332	0·0432	12·33	32·5
12	0·5962	0·0526	11·32	31·6
14	0·6552	0·0630	10·40	30·9
16	0·7010	0·0728	9·62	30·1
18	0·7286	0·0821	8·87	28·8
20	0·5312	0·1337	4·97	29·4

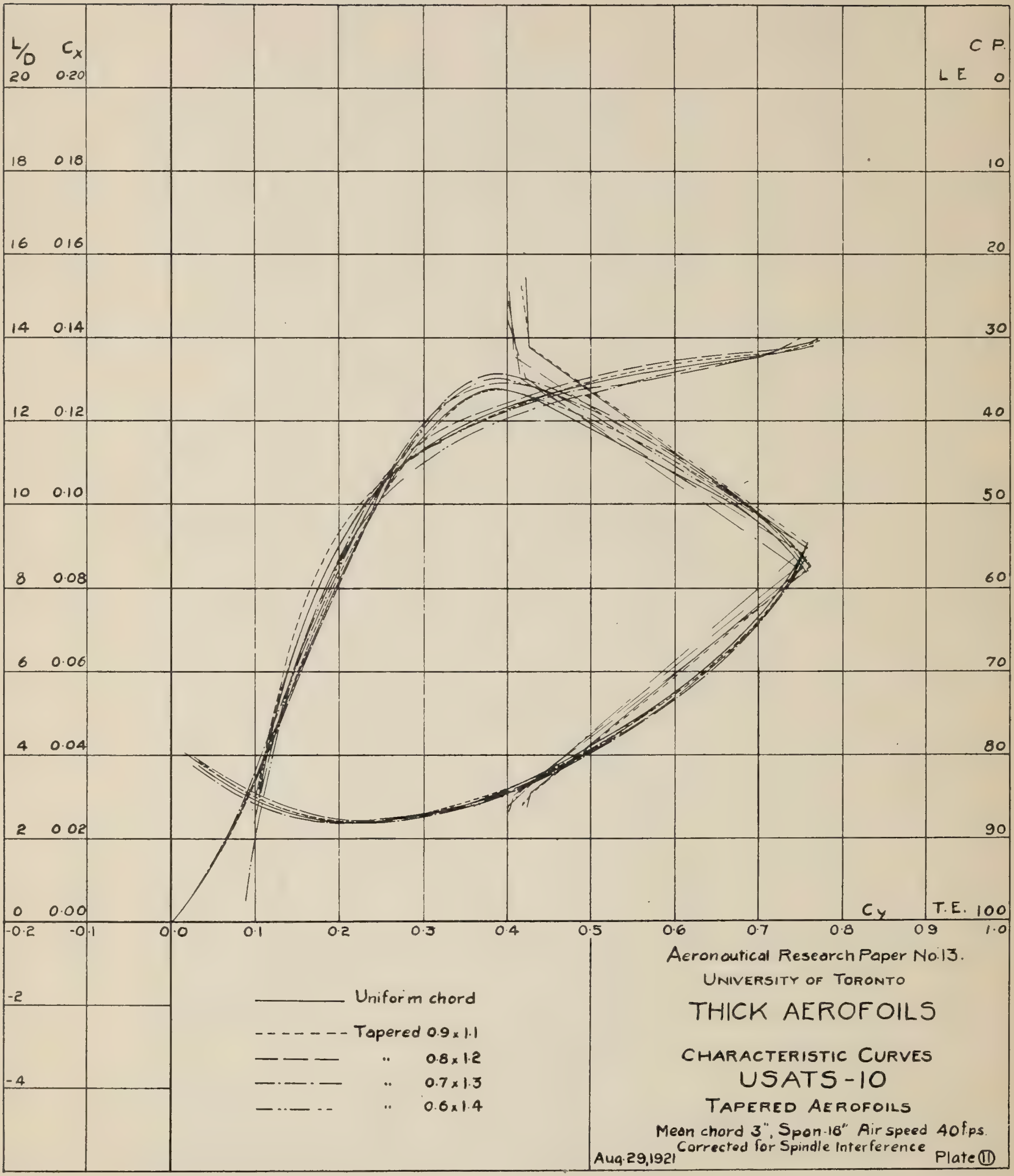


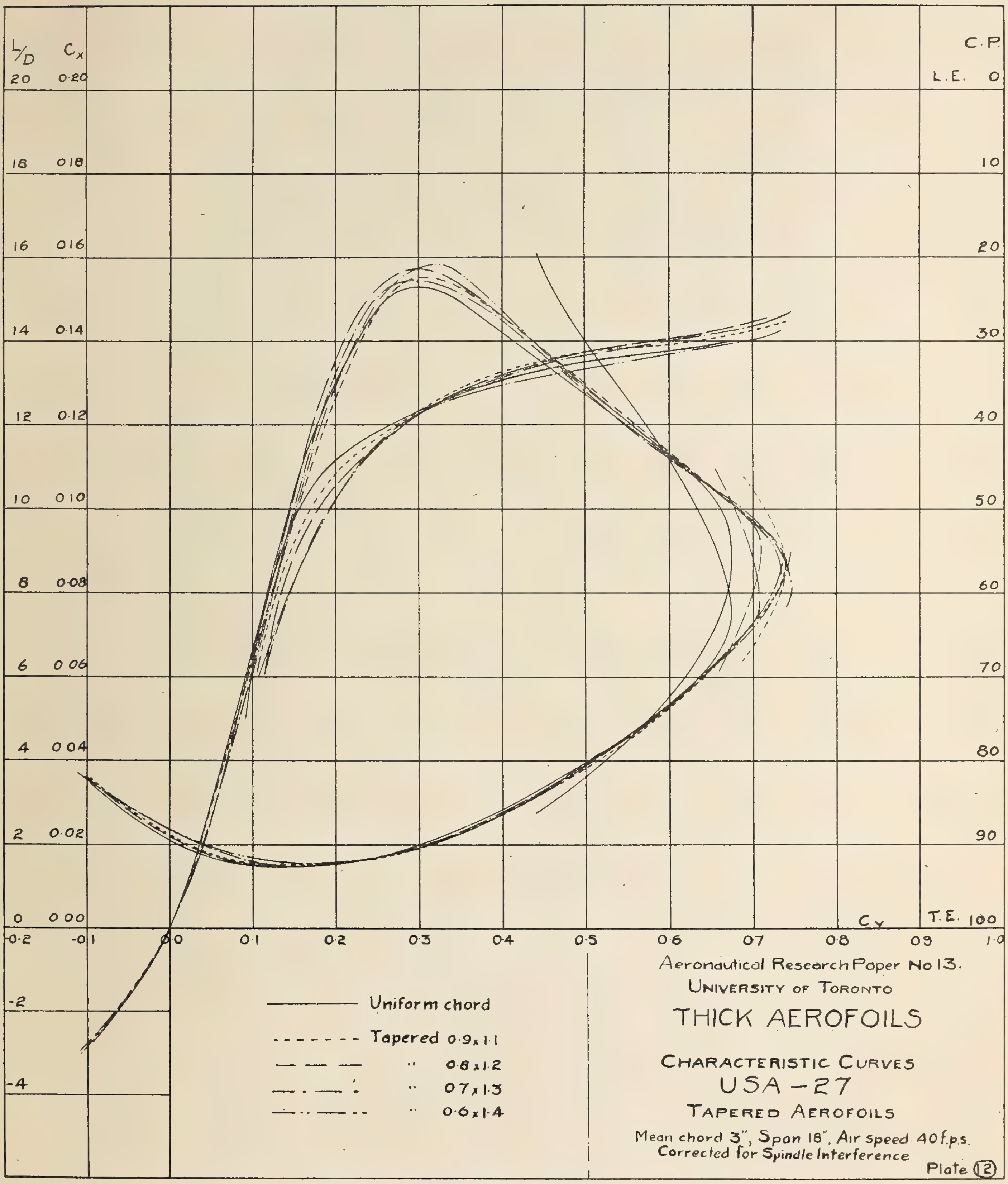
Aeronautical Research Paper No. 13.
UNIVERSITY OF TORONTO
THICK AEROFOILS
CHARACTERISTIC CURVES
OF
BASIC AEROFOILS
Models 3"x18" - Wood - Air speed 40f.ps
Corrected for Spindle Interference
Aug 1921. Plate ⑦

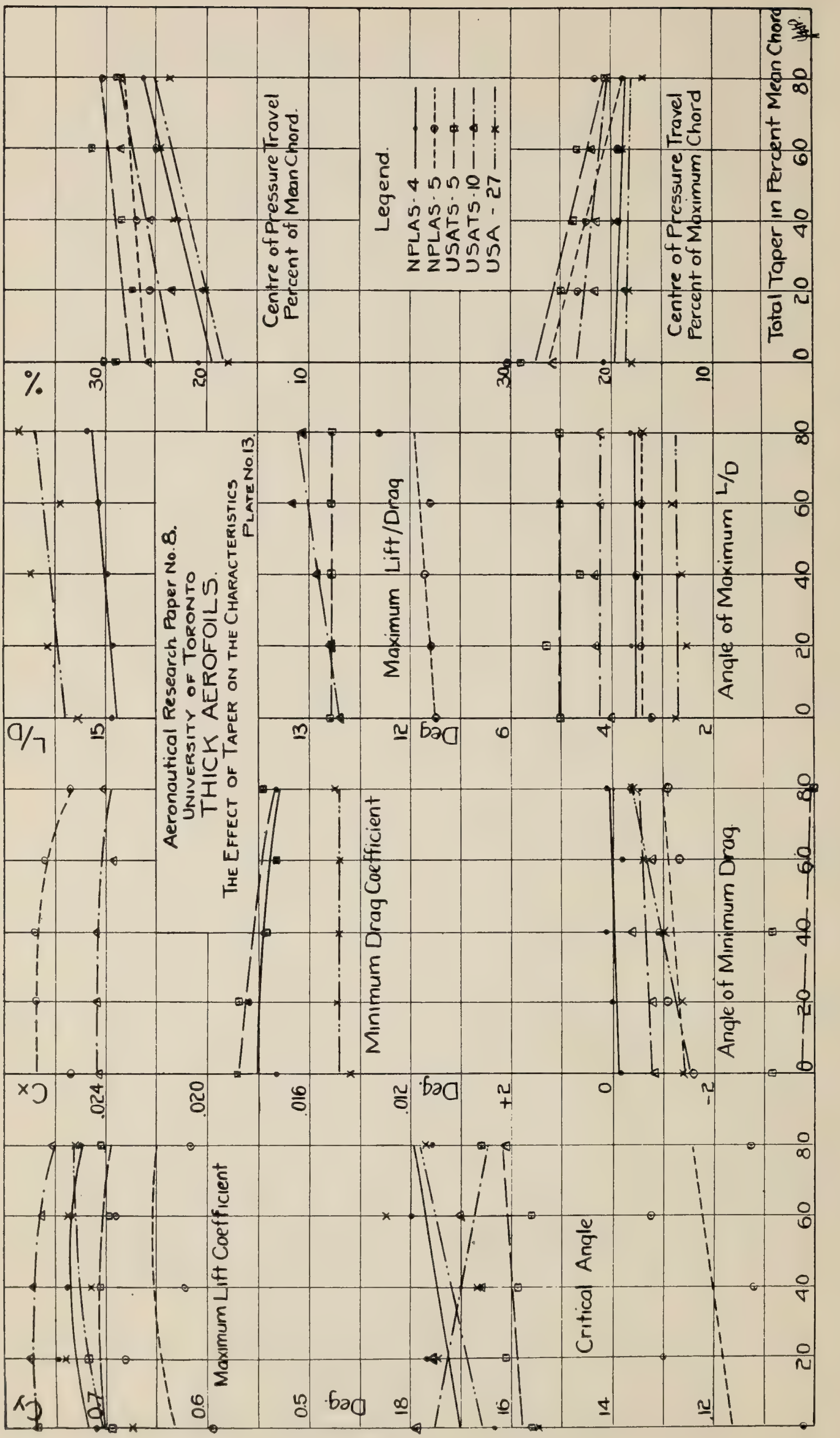

L/D
Cx
C·P.
LE 0
20 0·20
18 0·18
16 0·16
14 0·14
12 0·12
10 0·10
8 0·08
6 0·06
4 0·04
2 0·02
0 0·00
10
20
30
40
50
60
70
80
90
T.E. 100
-0·2
-0·1
0·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Cy
-2
-4
Uniform chord
Tapered 0·9x1·1
" 0·8x1·2
" 0·7x1·3
" 0·6x1·4
Aeronautical Research Paper No·13
University of Toronto
THICK. AEROFOILS
Characteristic Curves
N·P·L·A·S·4
Tapered Aerofoils
Mean chord 3" Span-18" Airspeed-40f.p.s.
Corrected for Spindle Interference
Sept·3,1921
Plate ⑧


L/D
Cx
C P.
20 0·20
L.E. 0
18 0·18
10
16 0·16
20
14 0·14
30
12 0·12
40
10 0·10
50
8 0·08
60
6 0·06
70
4 0·04
80
2 0 02
90
0 0·00
Cy
T.E. 100
-0·2
-0·1
0·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
-2
-4
Uniform chord
Tapered 0·9 x 1·1
" 0·8 x 1·2
" 0·7 x 1·3
" 0·6 x 1·4
Aeronautical Research Paper No 13.
UNIVERSITY OF TORONTO
THICK AEROFOILS
CHARACTERISTIC CURVES
N·P·L·A·S·5
TAPERED AEROFOILS
Mean chord 3" Span-18" Air speed-40 fps.
Corrected for Spindle Interference
Aug 27, 1921
Plate 6


L/D
Cx
CP
L.E. 0
20 0.20
18 0.18
16 0.16
14 0.14
12 0.12
10 0.10
8 0.08
6 0.06
4 0.04
2 0.02
0 0.00
10
20
30
40
50
60
70
80
90
T.E. 100
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cy
-2
-4
Uniform chord
Tapered 0.9x1.1
" 0.8x1.2
" 0.7x1.3
" 0.6x1.4
Aeronautical Research Paper No13.
UNIVERSITY OF TORONTO
THICK AEROFOILS
CHARACTERISTIC CURVES
USATS-5
TAPERED AEROFOILS
Mean chord 3", Span-18", Air speed-40 f.p.s.
Corrected for Spindle Interference
Aug.29,1921
Plate ⑩


L/D
Cx
C.P.
L.E.
20 0.20 0
18 0.18 10
16 0.16 20
14 0.14 30
12 0.12 40
10 0.10 50
8 0.08 60
6 0.06 70
4 0.04 80
2 0.02 90
0 0.00 100
T.E.
Cy
-0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-2
-4
Uniform chord
Tapered 0.9 x 1.1
" 0.8 x 1.2
" 0.7 x 1.3
" 0.6 x 1.4
Aeronautical Research Paper No. 13.
University of Toronto
THICK AEROFOILS
Characteristic Curves
USATS-10
Tapered Aerofoils
Mean chord 3", Span 18" Air speed 40 f.p.s.
Corrected for Spindle Interference
Aug. 29, 1921
Plate ⑪


L/D
Cx
C.P.
L.E. 0
20 0.20
18 0.18
16 0.16
14 0.14
12 0.12
10 0.10
8 0.08
6 0.06
4 0.04
2 0.02
0 0.00
10
20
30
40
50
60
70
80
90
T.E. 100
Cy
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
-2
-4
Uniform chord
Tapered 0.9x1.1
" 0.8x1.2
" 0.7x1.3
" 0.6x1.4
Aeronautical Research Paper No 13.
University of Toronto
THICK AEROFOILS
Characteristic Curves
USA - 27
Tapered Aerofoils
Mean chord 3", Span 18", Air speed 40 f.p.s.
Corrected for Spindle Interference
Plate ⑫


Aeronautical Research Paper No. 8.
University of Toronto
THICK AEROFOILS.
The Effect of Taper on the Characteristics
Plate No. 13.
Legend.
NPLAS-4
NPLAS-5
USATS-5
USATS-10
USA-27
Maximum Lift Coefficient
Critical Angle
Minimum Drag Coefficient
Angle of Minimum Drag
Maximum Lift/Drag
Angle of Maximum L/D
Centre of Pressure Travel Percent of Mean Chord.
Centre of Pressure Travel Percent of Maximum Chord
Total Taper in Percent Mean Chord
C_y
0.7
0.6
0.5
Deg.
18
16
14
12
C_x
.024
.020
.016
.012
Deg.
+2
0
-2
L/D
15
13
12
Deg
6
4
2
%
30
20
10
30
20
10
0
20
40
60
80

Aeronautical Research Paper No. 10.
September 1922.

DOWNWASH

By J. H. PARKIN, *Assistant Professor of Mechanical Engineering,*
H. C. CRANE, J. S. E. MACALLISTER, and E. V. AHARA.
Research Assistants

SUMMARY

(*a*) *Introduction.—Reasons for inquiry.*—The inquiry was instituted because of the increased popularity of thick cantilever tapered sections, the importance of a knowledge of the velocity and direction of the air in the region occupied by the tail, and the lack of data concerning downwash for thick or tapered wings.

(*b*) *Methods.*—Three methods for measuring downwash have been used: (1) yawmeter, (2) exploring plane, (3) silk streamers. The yawmeter was used in this research.

(*c*) *Arrangement of Apparatus.*—The study was carried out in the four foot N.P.L. type wind channel with apparatus specially designed to facilitate operation.

(*d*) *Extent of Investigation.*—Five sections ranging in thickness to chord ratio from 12 to 19 were selected. For each section, for each of five models (uniform chord and four tapers) the downwash was determined at 18 points in the rear of the wing for various incidences.

(*e*) *Results and conclusions.*—The change in velocity behind the wing is negligible compared to the angular deflection of the stream. The angle of downwash is found to conform closely to the formula

$$\epsilon=(Cy+c)k10^{nx}(1+st)$$

in which c, k, n and s are constants whose values are given. The factor $(1+st)$ is the taper correction, 10^{nx} the correction for distance in the rear and $k(C\gamma+c)$ the value of ϵ at the trailing edge at the given values of y and z. ϵ is seen to be a linear function of Cy but not proportional to it as has been usually assumed. Because of the extent of the investigation the results should be applicable to any thick section and probably to medium and thin sections as well.

INTRODUCTION

An aerofoil moving through the air has exerted upon it an upward lift which is accompanied by a downward deflection of the air passing the aerofoil so that the latter is trailed by a wash having a certain downward motion. The downward momentum thus imparted to the air equals the magnitude of the upward force or reaction on the aerofoil. The reaction therefore depends upon the mass of air affected and the downward velocity imparted to it or the angle of deflection. The downward velocity of the air is thus directly related to the lift of the aerofoil.

The angle through which the trailing air stream is deflected with regard to the undisturbed wind is termed the "angle of downwash."

Since the tail unit of the aeroplane is situated in the downwash from the main planes, to properly design and set the tail planes for longitudinal control and stability a knowledge of the magnitude of the angle of downwash is essential.

Considerable data have been published on the downwash behind thin aerofoils generally in biplane or triplane combinations (see references at end of this paper) but very little information regarding the downwash in the rear of thick sections has been published and as far as the writers know none for thick tapered aerofoils. Owing to the marked tendency toward the construction of cantilever thick section monoplanes for commercial work such information is urgently required. The present investigation has therefore been undertaken for the purpose of furnishing data concerning the downwash in the neighbourhood of the tail unit for machines fitted with thick tapered wings.

METHODS OF MEASUREMENT.

There are three methods that have been developed for the measurement of the direction and magnitude of the downwash in wind channels.

1. *By the Use of the Direction and Velocity Meter or So-called "Yawmeter."*—This instrument was developed at the N.P.L. for this particular purpose and is essentially a very small five-point Pitot tube, four of the tubes being used as direction finders and the fifth, placed in the rear of a small cone for the determination of velocity. The instrument has been fully described in various reports listed at the end of this paper, and has a precision of 0.10 degree for direction and 0.50 per cent. for velocity. It is convenient to read and few subsequent calculations are necessary.

2. *By the Use of an Exploring Plane.*—The exploring plane may be of ordinary tail plane section or a thin flat plate. The former type is generally of a size corresponding to the tail plane to scale in order to give the average downwash direction of the tail plane region instead of the direction at a point. It is placed in the downwash and adjusted to the position of no lift, thus indicating the mean air stream deflection. The thin flat plate is usually quite small, ½″×2½″, and is carried in a specially constructed, light, sensitive balance. From the measured forces upon it when placed at small incidences to the downwash the

velocity and direction of the latter may be calculated. The method is sensitive to 0.05 degrees but is tedious and inconvenient.

3. *By the Use of Silk Streamers.*—Fine silk strands are attached to wires stretched across the downwash region and take up the general direction of the air stream. The streamers are photographed from two directions at right angles, the streamers being so coloured as to be distinguishable on the plates. The angles of downwash are then measured from the plates.

Since the Direction and Velocity Meter possesses advantages with regard to simplicity of apparatus, convenience and rapidity of reading, and gives ample precision for the investigation, this instrument was selected for use in these tests.

ARRANGEMENT OF APPARATUS.

The apparatus for the study was arranged in the 4 ft. N.P.L. type wind channel at Toronto, as shown in Plate 1.

The yawmeter was clamped on the chuck of the balance, the pressure connections between it and the gauges being made through fine rubber tubing. The instrument was connected to two Chattock micromanometers. Since the investigation was not primarily to determine the direction of flow in the rear of the aerofoil in the plane of the span only one pair of the direction openings of the yawmeter was employed and connected to one Chattock gauge. Having determined the downwash direction with the meter the velocity of the wash was measured by means of the velocity opening and the second Chattock gauge which had been previously calibrated. The normal wind speed in the channel was indicated by a Direct Lift Gauge connected to the side plate (see Aero. Res. Paper No. 1).

The support of the model was arranged as shown to facilitate rapidity of adjustment with sufficient flexibility to permit of downwash determinations over a considerable region. Two carefully machined rails were screwed to the channel floor and on these a cross rail was arranged to slide. The ends of the cross rail were shaped to prevent lifting off the rails or cross binding. The cross rail could be moved longitudinally along the channel by means of rack and pinion manipulated from outside the channel. The cross rail carried a small slide which could be clamped in any position on the cross rail, and carried a standard fitted with a protractor and a chuck similar to that of the balance. It will be seen that this permitted all the necessary adjustments. Positions of meter in the rear of the aerofoil were adjusted from outside the channel, positions above and below the aerofoil by moving the small slide, and positions along the span by using spindles of different lengths. Incidence of the aerofoil was set using the protractor on the standard. The apparatus was found satisfactory in practice and to be as convenient and rapid as was possible considering the number of adjustments necessary. The rails and cross rail were all made thin in the direction of air flow in order to introduce as little resistance and disturbance as possible.

Extent of Investigation.

The downwash was measured in direction and velocity in the rear of tapered wings of five different sections, namely,

N.P.L.A.S.-4
N.P.L.A.S.-5
U.S.A.T.S.-5
U.S.A.T.S.-10
U.S.A.-27.

For each aerofoil section, five models of 18″ span and 3″ average chord having the tapers listed were tested. The taper is expressed in fractions of average chord at mid-span and at the tip:—

Uniform chord, designated $t=0$
1.1×0.9 taper " $t=.1$
1.2×0.8 taper " $t=.2$
1.3×0.7 taper " $t=.3$
1.4×0.6 taper " $t=.4$

The models were those used in the previous thick tapered section research (see Aero. Res. Paper No. 8) and were tapered in such a way that the section parallel to the wind was always of the basic section.

Measurements of downwash were made at positions shown in Fig. 1, Plate 2, namely, in the longitudinal plane containing the centre of the span, at 1, 2 and 3 chords behind the trailing edge in the plane of the aerofoil, *i.e.*, the plane of the centre of the spindle located 1″ (1/3 chord) from the leading edge and midway between the upper and lower surfaces (see Fig. 1, Plate 2), and one chord above and below the aerofoil; and a similar set of nine readings in the longitudinal plane one chord from mid-span. The latter were made in order to investigate the effect of taper on the downwash. It will be evident that these eighteen points fully cover the region ordinarily occupied by the tail unit.

For each model, the eighteen downwash determinations were made for incidences from −4°, by increments of 4° up to the critical angle and in certain cases for uniform chord models every 2°.

The velocity of the undisturbed air was in all tests 40 f.p.s.

Discussion of Results.

The observations confirm the results of previous experimenters in so far as change in velocity is concerned. The change is very small, in fact negligible compared to the angular deflection in tail plane calculations.

All the downwash observations for each aerofoil section have been plotted on two plates, the one containing plots of downwash at mid-span on lift coefficient, the other at one chord to the side. As may be seen by referring to plates No. 3 to No. 12 each sheet contains three horizontal rows of five squares each (also see Fig. 2, Plate 2). Contained in the five squares of each row are the results for the five tapers, the top row being for one chord above the aërofoil ($y=+1$), the middle row for the same level as the spindle ($y=0$) and the lower row for one chord below ($y=-1$).

In each square are three sets of points being observations of downwash and lift coefficient for one, two and three chords behind the trailing edge. ($x=1$, $x=2$ and $x=3$)

Roughly, variations in downwash due to changes in x, y, z and t (taper) may be seen by inspection of the curves: Variation due to x by the three curves in each square, variation due to y by the change in the curves in any vertical row, variation due to z by the difference between curves in similar positions on the two sheets, and variation due to taper by the change in the curves in any horizontal row.

Since the linear relation between downwash ϵ and lift coefficient Cy found by previous experimenters appeared to be confirmed the best straight lines were drawn in (in pencil) in all the plots. These lines did not pass through the origin nor were the intercepts small enough to be due to experimental errors. It was found that in any square the three lines for $x=1$, 2 and 3 appeared to cross on the Cy axis (at $Cy=-c$) and that the value of c was independent of taper except for the axis through the aerofoil in the line of flight ($y=0$, $z=0$). The values of c for each value of y and z were determined. These are given in Table No. 26. The curves were then readjusted to pass through the points ($\epsilon=0$, $Cy=-c$). The slopes of the curves were then determined and the variation of this slope ($d\epsilon/dCy$) with x was investigated. Plotting log $d\epsilon/dCy$ on log $(x+1)$ and on x both gave straight lines within the range of x investigated. The resulting equations are:

$$d\epsilon/dCy=k'(x+1)^{n}$$

and

$$d\epsilon/dCy=k10^{nx}$$

For several reasons the latter form was adopted; chiefly, however, because the exponential form does not require the addition of a constant (unity in this case) to give a finite value when $x=0$. It was found that the value of n was nearly independent of taper. Consequently an average value of n for each of the six points in the $y.z$ plane was determined. Then using this value of n the value of k for each of the tapers at each of the points in the *yz* plane was worked out. For each point the values of *k* were linear with respect to taper, and denoting the value when the taper was zero (uniform chord) by *k* the effect of taper was to multiply *k* by a factor $(1+st)$ where *t* was the fractional increase in chord at mid-span (or which is the same thing the fractional decrease in chord at the tip, see Fig. 1, Plate 2) and *s* a constant depending on the position in the *y.z* planes. The equation then assumes the form

$$\epsilon=(Cy+c).k.10^{nx}(1+st)$$

in which *c*, *k*, *n* and *s* are variables depending on the position in the *yz* plane and having the values given in Table No. 26. It may be observed that $k\ (Cy+c)$ is the value of ϵ at the trailing edge for a wing of uniform chord, terms involving *x* and *t* becoming unity. The value of 10^{nx} is easily found, it being the number whose logarithm is *nx*.

Conclusions.—The formula here adopted gives a convenient method of calculating the angle of incidence of a tail plane. The precision with which it represents the experimental data may be concluded by noting that the lines drawn in on the ϵ - *Cy* plots are the calculated ones, those from which the values of *c* and the slopes were taken having been erased. At $Cy=.5$ the mean deviation of calculated results from the original pencil lines is .18°. Bearing in mind that five sections of radically different shape were used and that five models of each section of varying taper were tested it appears that the above formula may be applied to any medium or thick section.

Heretofore it has been assumed that ϵ is proportional to *Cy*, while these results show that ϵ is proportional to *Cy* plus a constant. The explanation is that although the average downwash in any *yz* plane (normal to the flight axis (*x*)) is, according to theory, proportional to *Cy*, this does not require that at each point in the plane the downwash shall be proportional to *Cy*. In fact, when *Cy* is zero, although the average downwash is zero, there is obviously positive downwash above the wing and negative below it. This can be seen nicely in any plot of the flow about a wing.

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Aeronautical Research Paper No.
University of Toronto.

Flow in the Rear of Thick Tapered Aerofoils.

TABLE No. 1 NPLAS-4 UNIFORM CHORD

Incidence (deg.)	Elevation (chord)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel	Angle	Vel.	Angle	Vel.	Angle	Vel.
-6	+1	-0·55	40·0	-0·65	40·2	-0·70	40·1						
	0	-0·95	39·8	-1·15	40·8	-1·20	41·3						
	-1	-0·95	40·3	-0·85	40·1	-0·90	39·9						
-4	+1	0·40	40·3	0·05	40·0	0·15	40·2	0·40	40·4	0·25	40·3	0·15	40·3
	0	-0·15	37·4	-0·10	37·5	0·25	37·7	0·30	36·5	0·05	37·1	0·25	37·5
	-1	0·00	40·2	0·15	40·2	0·35	40·3	0·05	40·0	0·20	40·3	0·25	40·4
-2	+1	2·00	40·6	1·55	40·5	1·35	40·4						
	0	2·15	39·4	2·05	39·6	1·90	39·9						
	-1	1·65	40·2	1·45	40·4	1·70	40·5						
0	+1	1·85	40·4	1·65	40·4	1·40	40·5	1·70	40·4	1·45	40·4	1·25	40·4
	0	2·10	39·8	1·65	40·0	1·50	40·4	2·25	39·7	1·85	40·0	1·70	40·4
	-1	1·25	39·8	1·40	40·3	1·40	40·2	1·45	40·2	1·40	40·4	1·40	40·3
2	+1	1·90	40·4	1·65	40·2	1·50	40·3						
	0	2·65	39·9	2·25	40·0	1·90	40·0						
	-1	1·95	40·1	1·85	40·2	1·70	40·5						
4	+1	2·80	40·4	2·50	40·2	2·20	40·2	2·70	40·6	2·50	40·7	2·20	40·7
	0	3·60	39·8	3·00	40·0	2·80	40·3	3·90	40·3	3·20	40·4	2·95	40·8
	-1	3·00	39·8	2·80	40·0	2·60	40·2	2·90	40·1	2·70	40·6	2·55	40·8
6	+1	3·40	40·6	3·05	40·2	2·80	40·3						
	0	4·40	39·9	3·65	40·4	3·40	40·4						
	-1	3·45	39·8	3·30	40·3	3·15	40·3						
8	+1	3·85	40·5	3·35	40·4	3·05	40·6	3·85	40·8	3·30	40·7	2·95	40·8
	0	5·00	40·3	4·35	40·2	3·95	40·4	5·25	40·4	4·40	40·6	4·00	40·9
	-1	4·10	39·9	3·80	40·0	3·65	40·3	4·05	40·2	3·75	40·5	3·60	40·6
10	+1	4·35	40·6	3·85	40·7	3·50	40·5						
	0	5·55	49·3	4·80	40·3	4·40	40·6						
	-1	4·65	39·8	4·30	40·0	4·15	40·3						
12	+1	4·55	40·6	4·10	40·6	3·80	40·6	4·45	40·8	4·00	40·8	3·70	41·0
	0	6·15	40·0	5·15	40·4	4·60	40·6	6·15	40·5	5·15	40·6	4·85	40·8
	-1	5·05	39·6	4·65	40·4	4·45	40·5	5·05	40·2	4·70	40·8	4·45	40·8
14	+1	4·95	40·7	4·35	40·7	4·05	40·6						
	0	6·40	40·3	5·35	40·3	4·85	40·7						
	-1	5·30	39·6	5·00	40·0	4·70	40·5						
16	+1	2·40	40·2	1·25	40·2	0·75	40·3						
	0	5·90	40·3	4·75	40·4	4·35	40·8						
	-1	5·05	39·9	4·75	40·4	4·40	40·5						
TABLE No. 2		NPLAS-4						TAPERED-0·9×1·1					
-4	+1	0·75	40·4	0·85	40·4	0·75	40·4	0·75	40·2	0·65	40·2	0·65	40·2
	0	0·95	38·0	1·10	37·4	1·40	36·8	0·95	36·4	0·80	37·0	0·60	37·2
	-1	0·65	40·5	0·55	40·4	0·50	40·5	0·50	40·0	0·45	39·9	0·45	40·0
0	+1	2·10	40·5	1·75	40·6	1·55	40·6	2·35	40·5	2·05	40·3	1·85	40·2
	0	3·00	39·9	2·55	39·7	2·35	40·4	3·00	39·8	2·55	39·9	2·30	40·3
	-1	2·10	40·3	2·00	40·5	1·85	40·6	2·20	40·2	2·05	40·3	1·95	40·6
4	+1	3·00	40·6	3·30	40·7	3·65	41·0	3·30	40·5	2·85	40·4	2·70	40·4
	0	4·05	41·0	4·30	40·8	4·35	40·4	4·45	40·1	3·85	40·5	3·45	40·3
	-1	2·70	40·4	3·00	40·4	3·45	40·1	3·35	40·0	3·25	40·4	3·05	40·4
8	+1	4·35	41·0	3·85	40·7	3·60	40·9	4·35	40·8	3·95	40·7	3·55	40·9
	0	6·15	40·3	5·10	40·8	4·65	41·0	6·00	40·2	5·05	40·7	4·65	40·6
	-1	4·70	40·2	4·40	40·6	4·25	41·7	4·60	39·9	4·45	40·5	4·25	40·7
12	+1	5·45	41·1	4·70	40·8	4·45	40·9	5·40	40·6	4·70	40·7	4·25	40·8
	0	7·45	40·5	6·35	40·8	5·65	41·0	7·00	40·3	5·85	40·7	5·45	40·8
	-1	6·00	40·3	5·70	40·6	5·40	41·0	5·35	40·1	5·30	40·7	5·25	40·8
16	+1	6·05	41·2	5·40	41·3	5·10	41·1						
	0	8·35	40·7	7·00	41·0	6·10	40·9						
	-1	6·55	39·9	6·30	40·4	6·05	40·7						

Oct. 28th 1921

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UNIVERSITY OF TORONTO

FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 3 NPLAS-4 TAPERED-0·8×1·2

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1·		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	0·65	40·4	0·65	40·6	0·65	40·2	0·55	40·3	0·45	40·2	0·40	40·1
	0	1·65	35·1	1·40	36·2	1·10	37·2	0·90	35·8	0·65	36·8	0·55	36·9
	-1	0·25	40·3	0·45	40·5	0·50	40·6	0·25	40·2	0·35	40·3	0·45	40·3
0	+1	2·45	40·7	2·10	40·5	2·00	40·7	2·25	40·7	2·10	40·7	2·05	40·5
	0	3·65	39·5	3·00	40·4	2·80	40·4	3·20	39·6	2·60	40·2	2·35	40·3
	-1	2·15	40·2	2·10	40·3	2·05	40·4	2·10	40·3	2·00	40·7	1·95	40·6
4	+1	3·65	40·9	3·35	40·7	3·95	40·9	3·25	40·7	2·80	40·6	2·70	40·6
	0	5·35	40·5	4·55	40·8	3·95	40·7	4·55	40·2	4·10	40·4	3·75	40·7
	-1	3·60	39·9	3·55	40·6	3·35	40·8	3·40	40·0	3·10	40·4	3·00	40·5
8	+1	4·50	40·8	4·05	40·8	3·65	41·0	4·30	40·6	3·80	40·6	3·55	40·7
	0	6·70	40·6	5·55	40·7	4·90	41·1	6·35	40·3	5·55	40·5	5·15	40·5
	-1	4·90	40·0	4·55	40·5	4·45	40·6	4·55	40·2	4·40	40·4	4·20	40·5
12	+1	5·65	41·4	4·75	41·2	4·35	41·1	5·40	40·9	4·90	40·6	4·55	40·7
	0	7·90	40·6	6·45	40·8	5·80	40·9	7·55	40·4	6·55	40·6	5·05	40·7
	-1	6·05	39·9	5·75	40·6	5·80	40·8	5·85	40·0	5·50	40·5	5·25	40·8
16	+1	6·20	41·2	5·35	41·0	4·85	40·9						
	0	8·90	40·4	7·35	40·7	6·40	41·0						
	-1	7·00	40·1	6·75	40·5	6·70	40·8						
TABLE No. 4 NPLAS-4								TAPERED-0·7×1·3					
-4	+1	0·75	40·3	0·60	40·4	0·50	40·4	0·55	40·3	0·45	40·3	0·35	40·1
	0	1·60	34·9	1·20	36·8	1·05	37·7	0·60	40·5	0·40	36·4	0·40	37·2
	-1	0·25	40·4	0·60	40·6	0·45	40·4	0·15	40·3	0·25	40·2	0·40	40·4
0	+1	2·35	40·3	1·90	40·4	1·65	40·6	1·95	40·3	1·70	40·4	1·60	40·6
	0	3·90	39·4	3·05	40·0	2·65	39·8	3·10	39·6	2·75	40·0	2·15	40·4
	-1	2·10	39·8	2·00	40·4	2·00	40·3	1·90	39·9	1·75	40·4	1·65	40·4
4	+1	3·45	40·8	2·95	40·8	2·65	40·8	3·15	40·7	2·50	40·4	2·45	40·6
	0	5·45	40·2	4·45	40·6	4·00	40·8	4·75	40·3	4·15	40·4	3·70	40·7
	-1	3·50	40·0	3·45	40·5	3·25	40·7	3·25	40·0	2·95	40·3	2·95	40·6
8	+1	4·65	41·3	4·05	41·0	3·55	41·1	4·15	40·7	3·55	40·5	3·40	40·6
	0	7·15	40·4	5·95	40·9	5·20	41·1	6·40	40·3	5·35	40·4	4·65	40·8
	-1	5·05	40·0	4·95	40·7	4·85	40·7	4·45	39·9	4·25	40·5	4·20	40·8
12	+1	5·45	41·0	4·70	40·9	4·25	40·9	5·35	41·0	4·70	40·4	4·05	40·9
	0	8·70	40·5	6·85	40·8	6·10	41·1	7·75	40·4	6·60	40·8	6·00	40·9
	-1	6·50	40·2	6·50	40·8	6·50	40·9	5·70	39·8	5·45	40·6	5·20	40·7
16	+1	3·75	41·5	3·70	50·0	2·55	40·3						
	0	—	—	9·25	24·0	5·05	29·0						
	-1	1·70	41·2	2·45	40·0	3·10	39·0						
TABLE No. 5 NPLAS-4								TAPERED-0·6×1·4					
-4	+1	0·60	40·3	0·60	40·3	0·30	39·9	0·45	40·4	0·25	40·3	0·20	40·3
	0	1·85	34·9	1·35	36·8	1·15	37·9	0·25	35·1	0·05	36·3	0·10	37·2
	-1	0·00	40·3	0·35	40·3	0·25	40·2	0·10	40·2	0·10	40·4	0·10	40·6
0	+1	2·05	40·7	1·80	40·7	1·55	40·6	1·95	40·3	1·75	40·4	1·45	40·5
	0	4·05	39·6	2·95	40·4	2·55	40·6	2·55	39·4	2·20	39·9	1·70	40·2
	-1	1·70	40·2	1·70	40·3	1·75	40·4	0·45	40·0	1·35	40·5	1·40	40·8
4	+1	3·15	40·5	2·65	40·5	2·30	40·5	3·20	40·5	2·80	40·7	2·60	40·6
	0	5·45	40·0	4·30	40·7	3·80	40·5	4·80	40·3	3·90	40·4	3·60	40·5
	-1	3·20	39·8	3·25	40·4	3·10	40·4	2·90	40·0	2·75	40·5	2·60	40·6
8	+1	4·30	40·9	3·70	40·7	3·40	40·8	4·05	40·7	3·50	40·8	3·10	40·7
	0	7·10	40·1	5·70	40·4	5·05	40·6	6·15	40·0	5·25	40·6	4·75	40·8
	-1	1·85	39·9	4·80	40·5	4·70	40·7	4·15	40·0	4·00	40·4	3·85	40·8
12	+1	5·50	41·2	4·70	41·1	4·30	41·0	5·35	40·9	4·65	41·1	4·35	40·9
	0	8·85	40·2	7·05	40·8	5·35	40·6	7·85	40·2	6·45	40·9	5·95	41·0
	-1	6·35	39·8	6·55	40·5	6·80	40·6	5·50	40·0	5·20	40·7	5·05	40·8
16	+1	6·30	41·2	5·50	40·3	5·00	41·2						
	0	10·10	40·4	8·05	40·7	7·15	41·0						
	-1	8·00	39·7	8·15	40·5	8·40	39·3						

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UNIVERSITY OF TORONTO

FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 6 NPLAS-5 UNIFORM CHORD

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-6	+1	1·05	40·3	0·75	40·4	0·75	40·5						
	0	1·20	37·7	0·85	38·2	0·35	38·4						
	-1	0·25	40·4	0·35	40·3	0·40	40·5						
-4	+1	1·65	40·4	1·30	40·5	1·10	40·4	1·60	40·6	1·35	40·3	1·15	40·8
	0	1·80	39·2	1·50	39·5	1·30	39·5	1·90	38·8	1·85	39·2	1·50	39·8
	-1	1·05	40·0	1·10	40·2	1·05	40·5	1·20	40·5	1·10	40·6	1·10	40·6
-2	+1	1·95	40·6	1·50	40·5	1·50	40·5						
	0	1·70	39·8	2·05	39·8	2·00	40·1						
	-1	1·75	40·0	1·60	40·4	1·50	40·3						
0	+1	2·50	40·7	2·15	40·6	1·80	40·5	2·70	40·5	2·35	40·7	2·10	40·7
	0	3·20	40·3	2·80	40·4	2·30	40·2	3·60	40·2	3·00	40·4	2·70	40·6
	-1	2·20	40·1	2·20	40·4	2·00	40·6	2·50	40·5	2·25	40·5	2·25	40·9
2	+1	3·15	40·8	2·50	40·4	2·45	40·6						
	0	4·00	40·2	3·20	40·4	3·00	40·6						
	-1	3·05	40·2	2·80	40·2	2·60	40·8						
4	+1	3·65	41·0	3·10	40·4	2·75	40·8	3·55	40·8	3·00	40·9	2·80	40·8
	0	4·70	40·2	3·85	40·6	3·70	40·7	4·75	40·4	4·10	40·6	3·80	40·8
	-1	3·60	40·5	3·35	40·4	3·15	40·6	3·65	40·2	3·50	40·6	3·50	40·7
8	+1	4·45	40·9	3·80	41·0	3·60	41·0	4·30	41·0	3·95	40·9	3·65	40·9
	0	5·55	40·1	4·65	40·4	4·45	40·8	6·05	40·2	5·05	40·6	4·80	40·8
	-1	4·45	40·1	4·35	40·6	4·05	40·6	4·75	40·0	4·35	40·9	4·30	40·9
12	+1	2·35	40·4	1·40	40·0	0·95	40·3	2·55	40·5	1·65	40·3	1·45	40·4
	0	3·10	25·0	1·40	30·6	0·70	32·7	3·55	24·0	0·00	30·8	-0·70	33·4
	-1	0·20	40·2	0·90	40·2	1·05	40·4	0·65	40·3	1·45	40·5	1·40	40·4

TABLE No. 7 NPLAS-5 TAPERED-0·9×1·1

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	1·65	40·4	1·45	40·4	1·35	40·5	1·50	40·3	1·20	40·1	1·05	40·4
	0	2·20	37·6	1·90	38·5	1·60	39·0	1·85	38·0	1·60	38·5	1·50	39·4
	-1	1·25	40·4	1·25	40·3	1·25	40·3	1·00	40·2	0·95	40·0	0·95	40·0
0	+1	2·85	40·6	2·55	40·8	2·20	40·5	2·60	40·6	2·15	40·4	2·05	40·6
	0	4·05	39·5	3·30	40·5	2·95	40·6	3·75	39·8	3·10	40·3	2·70	40·5
	-1	2·35	40·2	2·55	40·6	2·40	40·6	2·35	40·2	2·20	40·4	2·10	40·7
4	+1	4·05	40·9	3·60	40·8	3·05	40·7	3·75	40·6	3·05	40·5	2·90	40·9
	0	5·55	40·1	4·60	40·5	4·00	40·7	5·10	40·0	4·10	40·5	3·90	40·8
	-1	3·90	40·1	3·80	40·4	3·50	40·8	3·50	40·0	3·35	40·3	3·10	40·6
8	+1	4·95	40·9	4·25	40·7	3·85	41·1	4·50	40·6	4·35	40·5	4·05	40·7
	0	6·90	40·3	5·80	40·8	5·20	40·6	6·60	39·9	5·30	40·4	4·70	40·6
	-1	5·00	40·0	5·05	40·5	4·80	41·0	4·70	39·9	4·45	40·2	4·45	40·6
12	+1	3·50	40·4	2·20	40·4	1·75	40·1	5·35	40·8	4·70	40·8	4·50	41·0
	0	8·00	40·4	6·35	40·8	5·80	40·9	-2·35	21·0	-1·80	30·1	-2·05	32·4
	-1	6·15	40·0	5·95	40·7	5·75	40·8	5·65	40·2	5·50	40·5	5·20	40·8

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FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 8 NPLAS-5 TAPERED-0·8×1·2

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	1·80	40·2	1·50	40·1	1·25	40·3	1·60	40·4	1·40	40·2	1·15	40·6
	0	2·75	36·7	2·10	38·2	1·85	38·8	2·20	24·0	1·80	38·7	1·50	39·4
	-1	1·10	39·9	1·30	40·4	1·25	40·4	1·05	40·0	1·10	40·4	1·10	40·4
0	+1	2·90	40·5	2·30	40·4	2·15	40·5	2·85	40·3	2·25	40·2	2·05	40·2
	0	4·45	39·6	3·60	40·4	3·30	40·6	3·85	39·6	3·20	40·1	3·00	40·4
	-1	2·65	40·0	2·70	40·3	2·50	40·3	2·55	39·9	2·45	40·4	2·50	40·5
4	+1	4·05	40·6	3·40	40·6	3·25	40·5	3·65	40·6	3·10	40·5	3·90	40·5
	0	5·90	40·1	4·85	40·5	4·50	40·4	5·40	40·0	4·50	40·7	3·10	40·8
	-1	4·10	40·3	3·90	40·4	3·90	40·8	3·65	40·1	3·75	40·5	3·25	40·7
8	+1	4·95	40·9	4·30	40·9	4·00	40·5	4·70	40·9	4·05	40·6	3·70	40·7
	0	7·35	40·3	6·00	40·7	5·30	40·7	6·70	40·2	5·50	40·5	5·00	40·7
	-1	5·40	40·2	5·30	40·6	5·15	40·7	5·00	40·2	4·80	40·7	4·50	40·8
12	+1	4·00	40·5	2·75	40·0	2·40	40·4	2·55	40·1	1·35	40·3	1·10	40·2
	0												
	-1	1·10	40·0	1·80	39·9	2·25	40·4	0·60	40·0	1·05	40·1	1·15	40·3
TABLE No. 9		NPLAS-5						TAPERED-0·7×1·3					
-4	+1	1·85	40·5	2·25	40·3	1·15	40·4	1·45	40·3	1·05	39·9	1·00	40·3
	0	3·10	36·2	2·35	38·1	2·10	39·3	1·80	37·5	1·35	38·2	1·20	38·9
	-1	1·25	40·1	1·20	40·4	1·20	40·4	0·75	40·0	0·95	40·3	0·90	40·3
0	+1	3·05	40·5	2·45	40·8	2·25	40·7	2·50	40·4	2·10	40·6	2·05	40·4
	0	4·90	39·6	4·00	40·4	3·55	40·7	3·75	39·7	3·20	40·2	2·80	40·2
	-1	2·60	40·0	2·60	40·4	2·60	40·5	2·30	40·3	2·35	40·3	2·20	40·7
4	+1	4·15	40·7	3·60	40·8	3·20	40·7	3·75	40·6	3·10	40·4	2·90	40·6
	0	6·35	40·3	5·25	40·8	4·70	40·8	5·35	40·0	4·60	40·2	4·30	40·4
	-1	4·00	39·8	4·05	40·5	3·95	40·6	3·55	40·0	3·40	40·4	3·35	40·7
8	+1	5·05	41·0	4·45	40·8	4·00	41·0	4·85	40·5	4·20	41·0	4·00	40·9
	0	7·80	40·0	6·45	40·6	5·70	41·0	6·90	40·1	5·90	40·5	5·35	40·9
	-1	5·50	40·2	5·45	40·5	5·50	40·8	4·85	40·1	4·85	40·4	4·30	41·1
12	+1	3·35	40·2	1·70	40·0	0·40	39·4	5·75	41·1	5·00	40·9	4·75	40·8
	0												
	-1	6·95	40·0	6·90	39·3	6·80	40·7	5·90	40·2	5·70	40·6	5·70	40·8
TABLE No. 10		NPLAS-5						TAPERED-0·6×1·4					
-4	+1	2·00	40·1	1·45	40·2	1·35	40·5	1·65	40·4	1·20	40·4	1·10	40·2
	0	3·65	33·0	2·75	36·8	2·25	38·3	1·95	40·4	1·45	37·2	1·30	38·0
	-1	1·00	40·1	1·20	40·0	1·35	40·3	0·90	40·1	0·90	40·2	1·00	40·3
0	+1	3·20	40·5	2·65	40·4	2·30	40·5	2·90	40·6	2·30	40·3	2·20	40·4
	0	5·50	38·6	4·40	40·1	3·95	40·7	4·20	39·2	3·55	40·0	3·25	40·3
	-1	2·80	39·9	2·70	40·2	2·70	40·5	2·45	40·0	2·55	40·4	2·30	40·5
4	+1	4·30	40·4	3·80	40·7	3·15	40·4	3·90	40·5	3·35	40·6	3·05	40·6
	0	7·20	39·5	5·75	40·4	5·10	40·5	6·05	39·9	5·05	40·6	4·55	40·8
	-1	4·20	39·6	4·20	40·1	4·20	40·6	3·80	40·3	3·75	40·3	3·65	40·7
8	+1	5·40	40·9	4·55	40·9	4·25	40·8	4·90	40·8	4·15	40·7	3·90	40·8
	0	8·85	39·9	6·95	40·3	6·10	40·7	7·40	40·1	6·10	40·8	5·60	40·8
	-1	5·60	39·6	5·85	40·5	5·75	40·7	5·00	40·0	4·85	40·5	4·70	40·8
12	+1	3·50	40·5	2·20	39·9	1·20	39·8	3·15	40·8	2·20	40·4	1·80	40·3
	0												
	-1	0·90	40·6	1·40	40·3	1·85	40·3	0·90	40·3	1·10	40·2	1·60	40·4

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TABLE No. 11 USATS-5 UNIFORM CHORD.

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-6	+1	-0·10	39·8	-0·10	40·4	-0·15	40·0						
	0	-0·45	36·2	-0·60	36·8	-0·60	37·1						
	-1	-0·50	39·9	-0·30	40·2	-0·35	40·5						
-4	+1	0·95	40·3	0·55	40·2	0·50	40·3	0·55	40·3	0·35	40·2	0·25	40·3
	0	0·85	37·7	0·85	38·4	0·50	38·7	0·35	37·2	0·20	37·8	0·10	38·3
	-1	0·60	40·3	0·55	40·3	0·35	40·4	0·20	40·1	0·25	40·1	0·25	40·2
-2	+1	1·50	40·6	1·30	40·3	1·10	40·3						
	0	1·95	38·7	1·55	39·4	1·35	39·6						
	-1	1·25	40·2	1·25	40·0	1·10	40·5						
0	+1	1·80	40·6	1·55	40·4	1·40	40·3	1·80	40·2	1·45	40·3	1·20	40·2
	0	2·35	39·6	2·00	39·9	1·85	40·3	2·40	39·3	1·95	39·9	1·70	40·2
	-1	1·65	40·0	1·50	40·4	1·40	40·4	1·65	40·3	1·55	40·5	1·35	40·4
2	+1	2·90	40·8	2·25	40·4	2·10	40·5						
	0	3·50	39·7	3·00	40·5	2·70	40·6						
	-1	2·55	40·0	2·50	40·4	2·30	40·7						
4	+1	3·50	40·6	3·05	40·5	2·80	40·8	2·80	40·5	2·30	40·5	2·20	40·6
	0	4·90	40·2	3·90	40·4	3·55	40·6	4·05	40·2	3·20	40·4	3·05	40·7
	-1	3·50	40·1	3·30	40·4	3·20	40·6	2·90	35·0	2·30	40·3	2·55	40·6
8	+1	4·55	40·6	3·95	40·3	3·65	41·1	3·65	40·6	3·25	40·7	2·95	40·6
	0	5·95	40·4	5·05	40·6	5·50	40·7	5·30	40·3	4·55	40·6	4·00	40·6
	-1	4·95	39·7	4·70	40·4	4·45	40·8	4·10	40·0	3·95	40·4	3·90	40·7
12	+1	5·00	40·7	4·20	40·5	4·00	40·8	4·55	40·9	4·05	40·8	3·65	40·7
	0	6·25	40·0	5·15	40·5	4·90	40·7	6·30	40·3	5·35	40·4	4·95	40·6
	-1	5·10	40·0	4·95	40·8	4·55	40·8	5·15	45·2	4·90	40·5	4·65	40·7
16	+1	5·05	40·8	4·25	41·0	4·00	40·8						
	0	6·05	40·0	5·55	40·2	5·05	40·6						
	-1	5·00	40·0	4·60	40·5	4·25	40·8						

TABLE No 12 USATS-5 TAPERED -0·9×1·1

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	0·90	40·1	0·70	40·1	0·70	40·4	1·00	40·3	0·60	40·3	0·65	40·1
	0	0·70	38·7	0·80	38·1	0·95	37·7	1·00	37·3	0·80	37·8	0·70	38·8
	-1	0·55	40·2	0·60	40·1	0·60	40·1	0·53	40·3	0·55	40·2	0·55	40·4
0	+1	2·10	40·4	1·85	40·5	1·65	40·4	2·10	40·4	1·75	40·2	1·50	40·1
	0	3·10	39·2	2·55	40·3	2·25	40·5	2·90	39·7	2·35	40·0	2·05	40·3
	-1	2·00	40·0	1·90	40·1	1·70	40·2	2·85	40·0	2·85	40·2	2·75	40·4
4	+1	3·25	40·7	2·85	40·5	2·55	40·6	3·20	40·4	2·70	40·6	2·60	40·4
	0	4·60	39·8	3·90	40·3	3·35	40·7	4·50	40·3	3·85	40·3	3·40	40·6
	-1	3·20	39·9	3·00	40·6	3·00	40·6	3·30	40·4	3·25	40·6	3·00	40·5
8	+1	4·35	41·0	3·85	40·8	3·55	40·8	4·10	40·8	3·65	40·6	3·40	40·6
	0	6·05	40·3	5·05	40·6	4·50	40·9	5·95	40·7	5·00	40·6	4·55	40·7
	-1	4·75	39·8	4·60	40·8	4·40	40·8	4·60	39·6	4·40	40·4	4·25	40·9
12	+1	5·15	40·9	4·60	40·9	4·15	40·9	5·00	40·7	4·40	40·8	4·00	40·9
	0	7·25	40·4	5·95	40·6	5·40	40·3	7·25	40·8	6·00	40·4	5·50	40·8
	-1	5·90	39·6	5·50	40·3	5·40	40·6	5·15	40·2	5·40	40·6	5·40	40·7
16	+1	5·55	40·6	4·90	41·1	4·20	40·8						
	0	8·15	40·2	6·70	40·6	5·90	40·6						
	-1	6·10	39·9	6·10	40·4	6·10	40·6						

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TABLE No. 13 USATS-5 TAPERED-0·8×1·2

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	1·05	40·3	0·75	40·3	0·65	40·2	0·75	40·3	0·55	40·1	0·35	40·0
	0	1·70	37·3	1·25	38·2	1·10	38·9	1·00	37·2	0·70	37·8	0·55	38·5
	-1	0·45	39·8	0·65	40·0	0·65	40·1	0·25	40·2	0·45	40·0	0·40	40·4
0	+1	2·25	40·5	1·80	40·7	1·80	40·7	1·85	40·6	1·55	40·5	1·40	40·6
	0	3·50	39·7	2·80	40·1	2·50	40·4	2·65	39·1	2·30	40·0	1·95	40·2
	-1	1·95	40·2	1·90	40·4	1·75	40·6	1·70	39·9	1·70	40·3	1·55	40·6
4	+1	3·50	40·8	2·95	40·5	2·65	40·8	3·00	40·7	2·55	40·5	2·40	40·4
	0	5·20	40·1	4·30	40·4	3·90	40·6	4·70	40·1	3·95	40·4	3·60	40·5
	-1	3·55	40·0	3·35	40·3	3·25	40·7	3·05	40·1	2·80	40·3	2·65	40·6
8	+1	4·55	40·9	4·00	40·8	3·65	40·8	3·95	40·6	3·60	40·9	3·20	40·6
	0	6·90	40·4	5·55	40·9	5·05	40·8	6·15	40·4	5·25	40·5	4·80	40·7
	-1	4·95	40·1	4·80	40·3	4·80	40·6	4·45	40·1	4·10	40·6	4·00	40·7
12	+1	5·50	41·0	4·75	40·9	4·20	40·8	5·30	41·1	4·70	40·7	4·20	40·6
	0	8·10	40·4	6·70	40·9	5·85	40·9	7·30	40·3	6·20	40·7	5·90	40·9
	-1	6·30	40·1	6·15	40·5	6·10	40·5	5·50	40·2	5·20	40·6	5·00	41·1
16	+1	5·15	40·7	4·50	40·7	4·20	41·0						
	0	9·99	40·1	8·25	40·6	7·20	40·4						
	-1	5·80	39·6	5·80	40·9	5·70	40·6						
TABLE No. 14				USATS-5				TAPERED-0·7×1·3					
-4	+1	1·10	40·2	0·80	40·4	0·65	40·4	0·55	40·2	0·35	40·1	0·25	40·2
	0	1·70	36·1	1·35	37·6	1·30	38·3	0·30	35·8	0·25	37·0	0·15	37·4
	-1	0·60	40·0	0·60	40·2	0·65	40·4	0·15	39·9	0·30	40·2	0·30	40·2
0	+1	2·30	40·3	1·90	40·5	1·60	40·4	1·85	40·3	1·55	40·4	1·30	40·3
	0	4·00	39·4	3·15	39·9	2·90	40·5	2·55	38·8	2·05	39·4	2·05	39·8
	-1	2·05	40·1	2·10	40·4	2·00	40·4	1·60	39·9	1·55	40·2	1·50	40·1
4	+1	3·55	40·5	3·05	40·6	2·85	40·7	3·05	40·6	2·50	40·5	2·20	40·5
	0	6·05	40·0	5·00	40·5	4·50	40·6	4·70	40·1	4·00	40·3	3·60	40·3
	-1	3·70	40·2	3·70	40·5	3·50	40·8	2·85	40·0	2·80	40·2	2·75	40·6
8	+1	4·65	40·8	4·15	40·7	3·80	40·8	4·15	40·9	3·70	41·0	3·20	40·6
	0	7·45	40·2	6·15	40·6	5·35	40·8	6·25	40·2	5·25	40·5	4·90	40·6
	-1	5·25	39·8	5·20	40·5	5·20	40·7	4·25	40·2	4·10	40·6	4·05	40·6
12	+1	5·95	41·0	4·95	40·7	4·65	41·1	5·40	41·2	4·65	40·9	4·50	40·7
	0	8·85	40·5	7·30	40·9	6·55	41·1	7·55	40·4	6·40	40·7	5·95	41·0
	-1	6·85	39·9	6·85	40·5	6·85	40·6	5·50	40·3	5·25	40·6	5·10	40·7
16	+1	6·30	41·2	5·55	40·9	4·95	41·0						
	0	9·55	40·4	7·70	40·8	6·90	40·8						
	-1	7·50	39·9	7·50	40·7	7·70	40·0						
TABLE No. 15				USATS-5				TAPERED-0·6×1·4					
-4	+1	1·30	40·6	0·90	40·2	0·80	40·3	1·05	40·4	0·80	40·2	0·65	40·3
	0	1·80	34·4	1·55	36·9	1·45	38·1	1·00	36·0	0·85	37·3	0·85	38·0
	-1	0·25	40·0	0·65	40·7	0·60	40·4	0·45	40·0	0·55	39·9	0·65	40·4
0	+1	2·55	40·7	2·00	29·0	1·90	40·8	2·05	40·3	1·75	40·5	1·55	40·6
	0	4·55	39·0	3·75	40·5	3·15	40·8	2·90	38·8	2·40	39·6	2·20	39·9
	-1	2·10	40·0	2·05	40·5	2·15	40·7	1·70	40·2	1·95	40·6	1·75	40·5
4	+1	3·90	40·8	3·10	40·5	2·75	40·5	3·55	40·6	3·15	40·6	2·95	40·5
	0	6·40	40·1	5·20	40·8	4·55	40·9	5·35	39·8	4·45	40·2	4·15	40·5
	-1	3·65	40·0	3·80	40·5	3·80	40·8	3·30	40·2	3·10	40·7	3·00	40·5
8	+1	5·00	41·1	4·25	41·1	3·90	40·9	5·30	41·0	4·75	40·9	4·25	40·9
	0	8·10	40·4	6·55	40·7	5·70	40·9	8·50	40·1	7·05	40·6	6·65	40·9
	-1	5·35	39·9	5·35	40·6	5·45	41·1	6·10	39·7	6·05	40·7	5·70	40·7
12	+1	6·00	41·2	5·10	40·7	4·60	41·0	5·90	40·9	5·05	40·9	4·80	41·0
	0	9·70	40·5	7·60	40·8	6·80	40·9	7·30	39·6	5·70	40·1	5·40	40·3
	-1	7·05	40·0	7·30	40·7	7·45	40·8	5·60	40·1	5·30	40·6	4·60	40·3
16	+1	6·35	41·2	5·55	41·0	4·95	41·3						
	0	10·75	40·5	8·35	40·6	7·35	40·8						
	-1	8·00	39·8	8·45	40·5	9·15	37·8						

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Aeronautical Research Paper No.
UNIVERSITY OF TORONTO

FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 16 USATS-10 UNIFORM CHORD.

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-6	+1	0·40	38·7	0·40	40·4	0·20	40·0						
	0	0·60	37·0	0·25	37·5	0·10	37·7						
	-1	0·00	40·3	0·05	40·4	0·10	40·5						
-4	+1	1·10	40·6	0·80	40·5	0·85	40·4	1·00	40·3	0·85	40·5	0·60	40·4
	0	1·25	38·8	1·00	38·8	0·95	39·0	1·75	38·1	0·95	38·7	0·80	39·1
	-1	0·80	40·1	0·75	40·2	0·55	40·6	0·50	40·5	0·50	40·2	0·65	40·5
-2	+1	1·65	40·4	1·50	40·7	1·15	40·4						
	0	2·05	39·2	1·60	39·7	1·60	39·9						
	-1	1·10	40·0	1·20	40·5	1·30	40·3						
0	+1	2·35	40·7	2·05	40·4	1·90	40·4	1·95	40·7	1·60	40·5	1·55	40·7
	0	2·90	40·0	2·20	40·2	2·00	40·4	2·75	39·8	2·35	40·4	2·05	40·3
	-1	1·95	39·9	1·90	40·5	1·80	40·6	1·90	40·1	1·80	40·6	1·70	40·6
2	+1	2·75	40·5	2·20	40·6	2·20	40·8						
	0	3·70	40·0	3·05	40·4	2·65	40·6						
	-1	2·80	40·2	2·60	40·8	2·30	40·5						
4	+1	3·25	40·8	2·80	40·7	2·55	40·6	3·00	40·9	2·70	40·8	2·45	40·6
	0	4·30	40·3	3·65	40·4	3·15	40·7	4·20	40·3	3·55	40·6	3·20	40·7
	-1	3·30	39·9	3·05	40·1	2·90	40·6	3·25	40·6	3·00	40·6	2·80	40·8
8	+1	4·10	40·9	3·65	40·9	3·40	40·8	4·10	41·0	3·60	40·9	3·15	40·9
	0	5·40	40·2	4·55	40·7	4·20	41·1	5·60	40·7	4·75	40·9	4·50	40·8
	-1	4·55	40·0	4·25	40·7	4·00	40·8	4·55	40·2	4·20	40·7	4·00	40·8
12	+1	5·00	41·0	4·50	41·0	4·10	41·1	4·85	41·0	4·50	40·8	4·05	41·0
	0	6·60	40·5	5·60	40·8	5·05	40·8	6·75	40·4	5·80	40·6	5·20	40·6
	-1	5·55	40·1	5·15	40·6	5·05	40·7	5·55	39·9	5·20	40·4	5·20	41·0
16	+1	5·60	41·2	5·00	41·1	4·50	41·1						
	0	7·20	40·6	6·00	40·7	5·35	40·9						
	-1	6·05	40·2	5·70	40·4	5·35	40·9						

TABLE No. 17 USATS-10 TAPERED -0·9×1·1

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	1·10	40·3	0·80	40·2	0·55	40·2	1·00	40·2	1·00	40·4	0·85	40·3
	0	1·35	37·0	0·95	38·0	0·85	38·7	1·40	37·1	0·95	38·0	0·85	38·4
	-1	0·55	40·4	0·65	40·5	0·50	40·1	0·55	40·2	0·75	40·0	0·70	40·3
0	+1	2·15	40·7	1·85	40·7	1·70	40·4	2·15	40·3	1·95	40·6	1·80	40·5
	0	3·15	39·4	2·60	40·4	2·35	40·4	3·05	39·1	2·65	39·9	2·25	40·5
	-1	2·00	40·0	1·90	40·3	1·85	40·8	2·10	40·2	2·00	40·4	1·90	40·6
4	+1	3·45	40·7	2·90	40·9	2·65	40·8	3·20	40·7	2·80	40·6	2·55	40·4
	0	4·90	40·1	3·90	40·5	3·60	40·6	4·60	40·1	4·00	40·4	3·55	40·5
	-1	3·30	40·1	3·25	40·7	3·05	40·6	3·35	40·2	3·25	40·7	3·05	40·7
8	+1	4·65	41·0	3·95	40·6	3·55	41·0	4·15	40·8	3·90	40·9	3·30	40·8
	0	6·20	40·4	5·15	40·7	4·65	40·8	6·00	40·3	5·05	40·6	4·60	40·8
	-1	4·70	39·9	4·60	40·4	4·35	40·8	4·75	40·0	4·40	40·4	4·20	40·4
12	+1	5·40	40·7	4·75	40·7	4·15	41·0	5·20	40·8	5·35	40·6	5·80	40·3
	0	7·35	40·3	6·00	40·6	5·45	41·0	5·60	40·6	6·10	40·6	7·10	40·4
	-1	5·90	39·8	5·70	40·7	5·45	40·8	4·10	40·8	4·60	40·6	5·10	40·8
16	+1	5·95	41·2	5·10	41·0	4·65	40·7						
	0	7·70	40·4	6·45	41·0	5·75	41·0						
	-1	6·40	39·8	6·20	40·6	6·05	40·9						

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FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 18 USATS-10 TAPERED-0·8×1·2

Incidence (deg.)	Elevation (chord.)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
-4	+1	1·25	40·5	1·00	40·4	0·90	40·6	1·05	40·6	0·95	40·6	0·70	40·6
	0	2·00	36·7	1·50	37·8	1·25	38·4	1·50	37·4	1·35	38·1	1·00	38·9
	-1	0·75	39·9	0·75	40·4	0·80	40·2	0·65	40·3	0·80	40·6	0·80	40·7
0	+1	2·55	40·7	2·10	40·4	1·80	40·6	2·25	40·7	2·00	40·7	1·95	40·6
	0	3·90	39·2	3·05	40·2	2·90	40·3	3·20	39·6	2·65	40·3	2·35	40·4
	-1	3·30	39·9	2·10	40·5	2·05	40·7	2·30	40·4	2·10	40·6	2·10	40·8
4	+1	3·50	41·0	3·05	40·8	2·80	40·7	3·35	40·7	2·80	40·7	2·65	40·8
	0	5·25	40·5	4·40	40·7	4·45	40·8	4·80	40·2	4·10	40·6	3·85	40·9
	-1	3·60	40·4	3·50	40·7	3·30	40·8	3·35	40·3	3·15	40·6	3·15	40·9
8	+1	4·75	41·3	4·10	40·7	3·85	41·2	4·45	41·0	3·80	40·8	3·45	41·0
	0	6·95	40·5	5·80	40·9	5·20	41·1	6·40	40·5	5·60	41·0	5·05	40·8
	-1	5·05	39·9	5·00	40·7	4·95	41·2	4·65	40·4	4·65	40·7	4·40	41·0
12	+1	5·65	41·2	5·00	41·1	4·40	41·2	5·50	41·2	4·70	41·2	4·30	41·1
	0	8·10	40·7	6·60	41·1	6·05	41·1	7·65	40·7	6·40	40·7	6·00	41·0
	-1	6·25	40·5	6·15	40·6	6·10	40·6	6·00	40·6	4·04	48·6	5·30	41·0
16	+1	4·90	41·4	5·30	41·3	6·15	41·4						
	0	9·00	40·6	7·15	41·3	6·55	41·3						
	-1	7·05	40·4	6·95	40·8	7·00	40·7						
TABLE No. 19		USATS-10						TAPERED-0·7×1·3					
-4	+1	1·20	40·3	0·90	40·2	0·80	40·3	0·95	40·3	0·80	40·4	0·50	40·2
	0	2·10	36·8	1·40	38·0	1·25	38·8	1·15	36·2	1·00	37·4	0·80	38·2
	-1	0·60	40·0	0·75	40·3	0·90	40·5	0·45	39·9	0·50	40·3	0·50	40·2
0	+1	2·55	40·6	2·05	40·7	1·80	40·4	2·10	40·4	1·65	40·2	1·55	40·5
	0	4·00	39·6	3·30	40·2	2·85	40·6	3·00	39·0	2·60	39·9	2·80	40·3
	-1	2·20	40·0	2·15	40·3	2·10	40·3	2·00	40·1	1·95	40·5	1·80	40·4
4	+1	3·65	40·7	3·05	40·5	2·80	40·6	3·20	40·6	2·85	40·7	2·70	40·7
	0	5·60	40·2	4·65	40·5	4·10	40·6	5·05	40·3	4·45	40·4	4·05	40·5
	-1	3·85	39·9	3·75	40·7	3·55	40·6	3·20	39·9	3·10	40·8	3·05	40·5
8	+1	4·70	40·9	4·00	40·6	3·55	40·7	4·30	40·5	3·90	40·6	3·80	40·9
	0	7·20	40·4	5·85	40·8	5·35	40·9	6·75	40·3	5·85	40·8	5·15	40·9
	-1	5·20	39·8	5·20	40·4	5·05	40·7	4·70	39·9	4·50	40·5	4·40	41·0
12	+1	5·60	41·0	4·95	41·0	4·35	41·1	5·60	40·8	5·00	40·6	4·25	40·9
	0	8·30	40·5	6·80	40·8	6·20	40·8	7·85	40·0	6·75	40·7	6·20	40·7
	-1	6·75	40·0	6·50	40·6	6·60	40·7	5·95	40·3	5·55	40·7	5·45	41·0
16	+1	6·30	41·0	5·50	41·0	5·05	41·2						
	0	9·40	40·3	7·70	40·9	6·90	41·0						
	-1	7·65	39·9	7·85	40·6	7·95	39·3						
TABLE No. 20		USATS-10						TAPERED-0·6×1·4					
-4	+1	1·50	40·4	1·20	40·2	0·90	40·3	1·30	40·2	1·00	40·6	0·85	40·4
	0	2·90	33·2	1·95	36·5	1·60	37·7	1·55	35·0	1·10	36·6	1·05	37·7
	-1	0·75	40·0	0·95	40·2	1·00	40·4	0·60	40·4	0·90	40·2	0·70	40·5
0	+1	2·70	40·3	2·10	40·6	1·90	40·4	2·35	40·4	2·00	40·4	1·65	40·5
	0	4·85	38·7	3·75	40·3	3·10	40·3	3·20	38·6	2·70	39·5	2·90	40·4
	-1	2·30	40·0	2·40	40·4	2·20	40·5	2·05	40·1	2·05	40·4	2·05	40·4
4	+1	4·05	40·6	3·35	40·6	3·00	40·5	3·50	40·7	3·05	40·6	2·85	40·8
	0	6·60	39·8	5·25	40·8	4·55	40·6	5·55	40·0	4·60	40·4	4·15	40·7
	-1	3·90	39·7	4·00	40·0	3·80	40·4	3·25	39·9	3·30	40·5	3·15	40·6
8	+1	5·25	40·8	4·40	41·1	4·00	41·1	3·60	40·9	4·00	40·8	4·65	40·8
	0	8·25	40·3	6·45	40·6	5·75	40·9	6·95	40·0	6·00	40·5	5·40	40·8
	-1	5·55	39·7	5·60	40·6	5·75	40·5	4·85	40·0	4·80	40·7	4·55	40·8
12	+1	6·05	41·1	5·30	41·4	4·90	41·2	5·75	41·0	5·00	41·0	4·80	40·8
	0	9·30	40·4	7·80	40·8	6·95	41·0	8·25	40·3	7·30	40·7	6·65	40·8
	-1	7·35	39·9	7·25	40·6	7·30	40·8	6·05	39·9	5·90	40·6	5·75	40·8
16	+1	6·80	41·3	5·95	41·2	2·00	40·4						
	0	17·70	14·0	-1·00	25·0	-0·70	29·0						
	-1	2·20	40·4	2·60	39·8	2·90	40·0						

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FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 21 U·S·A–27 UNIFORM CHORD

Incidence (deg.)	Elevation (chord)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
	+1	1·0	47·2	-0·5	47·3	0·1	47·1						
-6	0	0·1	46·1	0·1	46·1	0·1	46·4						
	-1	0·1	47·3	0·1	47·3	0·3	47·3						
	+1	0·6	47·4	0·6	47·4	0·2	47·5	0·7	47·5	0·5	47·9	0·5	47·7
-4	0	0·4	46·6	0·3	46·4	0·3	46·9	0·6	46·4	0·6	46·9	0·4	47·1
	-1	0·4	47·4	0·1	47·5	0·4	47·5	0·6	47·6	0·6	47·9	0·4	47·9
	+1	1·2	47·3	1·2	47·6	1·0	47·4						
-2	0	1·4	46·9	1·4	47·2	1·4	47·1						
	-1	0·8	47·1	1·4	47·4	1·2	47·5						
	+1	2·0	47·6	1·3	47·7	2·0	47·5	2·2	47·7	1·8	47·6	1·5	47·5
0	0	2·6	47·2	2·1	47·4	2·1	47·6	2·4	47·1	2·3	47·5	2·1	47·6
	-1	1·9	47·3	1·9	47·5	1·3	47·6	1·6	47·7	1·7	47·6	1·7	47·8
	+1	2·7	47·6	2·3	47·8	2·3	47·7						
2	0	3·2	47·4	3·6	47·6	2·3	47·3						
	-1	2·8	47·3	2·4	47·6	2·4	47·6						
	+1	3·1	47·9	2·7	47·7	2·7	47·6	3·3	48·2	3·0	48·2	2·6	48·0
4	0	4·1	47·2	3·5	47·6	3·2	47·4	4·2	47·4	3·8	47·9	3·3	48·2
	-1	3·1	47·3	3·1	47·3	3·1	47·4	3·0	47·6	3·0	47·7	3·0	47·9
	+1	4·3	47·9	3·7	47·9	3·4	47·7	4·6	48·3	4·0	48·2	3·7	48·2
8	0	5·5	47·4	4·5	47·8	4·4	47·6	6·0	47·3	4·9	48·0	4·7	47·9
	-1	4·3	46·9	4·0	47·7	4·0	47·8	4·3	47·6	4·1	47·7	3·8	47·9
	+1	5·0	48·2	4·4	48·2	4·0	48·2	5·3	48·6	4·8	48·3	4·5	48·2
12	0	6·7	47·9	5·6	47·7	5·0	47·8	7·3	47·8	6·1	47·9	5·7	47·6
	-1	5·4	47·2	5·0	47·5	4·9	47·7	5·3	48·5	4·9	48·2	4·5	48·2
	+1	5·0	48·2	4·2	48·2	4·0	48·2						
16	0	6·0	47·3	5·2	47·5	4·6	47·6						
	-1	5·0	47·2	4·5	47·6	4·4	47·7						
TABLE No. 22 U·S·A·-27· TAPERED 0·9×1·1													
	+1	0·6	47·7	0·4	47·3	0·3	47·4	0·4	47·4	0·3	47·3	0·3	47·6
-4	0	0·7	46·3	0·7	46·5	0·6	46·9	0·1	46·4	0·1	46·3	0·0	46·8
	-1	0·2	47·4	0·4	47·4	0·4	47·4	0·2	47·4	0·1	47·5	0·3	47·5
	+1	1·8	47·7	1·7	47·5	1·3	47·6	1·9	47·6	1·5	47·8	1·3	47·6
0	0	0·5	46·9	2·1	47·3	1·9	47·3	2·5	47·0	2·2	47·2	1·9	47·4
	-1	1·5	47·1	1·7	47·3	1·5	47·6	1·6	47·3	1·6	47·6	1·4	47·5
	+1	3·2	47·9	2·9	47·7	2·5	47·8	3·0	47·7	2·5	48·0	2·3	47·8
4	0	4·5	47·2	3·6	47·5	3·3	47·8	4·1	47·4	3·5	47·4	3·2	47·4
	-1	3·2	47·4	3·0	47·6	2·8	47·6	2·7	47·4	2·8	47·7	2·6	47·8
	+1	4·1	47·9	3·7	47·8	3·4	47·9	4·0	47·7	3·5	47·9	3·2	48·0
8	0	5·9	47·4	4·9	47·5	4·4	47·7	5·7	47·3	4·7	47·8	4·4	47·9
	-1	3·3	47·3	4·0	47·5	4·0	47·8	4·2	47·3	3·9	47·7	3·9	48·0
	+1	5·0	48·2	4·5	47·8	4·1	48·1	4·5	47·9	3·9	47·9	3·8	48·0
12	0	7·0	47·4	5·9	47·8	5·3	47·8	6·6	47·6	5·6	47·8	5·2	47·9
	-1	5·6	47·1	5·3	47·5	5·1	47·7	5·1	47·2	4·6	47·9	4·5	47·8
	+1	5·8	48·3	5·1	48·0	4·7	48·2						
16	0	7·7	47·4	6·5	47·7	5·8	47·9						
	-1	6·2	47·2	5·9	47·5	5·8	47·7						

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FLOW IN THE REAR OF THICK TAPERED AEROFOILS

TABLE No. 23 U.S.A.-27. TAPERED 0·8×1·2

Incidence (deg)	Elevation (chord)	At mid span						One chord to the side					
		Distance in Rear of Aerofoil						Distance in Rear of Aerofoil					
		1		2		3		1		2		3	
		Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.	Angle	Vel.
	1	0·3	47·4	0·2	47·5	0·2	47·5	0·6	47·7	0·3	47·5	0·2	47·3
-4	0	0·5	46·2	0·4	46·3	0·3	46·6	0·3	46·3	0·3	46·6	0·1	46·7
	-1	0·1	47·3	0·2	47·4	0·2	47·3	0·2	47·5	0·5	47·3	0·2	47·3
	+1	1·9	47·5	1·6	47·6	1·3	47·7	2·1	47·7	1·8	47·6	1·4	42·8
0	0	3·0	46·9	2·5	47·1	2·4	47·2	3·0	47·1	2·4	47·4	2·2	47·4
	-1	1·7	47·2	2·8	47·4	1·5	47·3	1·6	46·6	1·8	47·6	1·6	47·5
	+1	3·2	47·8	2·8	47·6	2·5	47·6	3·2	47·9	2·8	47·8	2·6	47·8
4	0	5·0	47·1	4·0	47·3	3·5	47·3	4·4	47·3	3·9	47·6	3·6	47·7
	-1	3·0	47·2	3·0	47·3	3·0	47·3	3·2	47·2	3·0	47·4	3·0	47·5
	+1	4·4	48·0	3·6	47·8	3·3	47·8	4·2	48·2	3·7	47·9	3·3	47·6
8	0	6·2	47·3	5·1	47·6	4·6	47·5	6·0	47·3	5·1	47·6	4·6	47·7
	-1	4·4	47·1	4·4	47·5	4·3	47·7	4·6	47·4	4·3	47·8	4·3	47·8
	+1	5·0	48·2	4·5	47·4	4·2	47·8	5·2	48·1	4·4	47·7	4·2	47·9
12	0	7·7	47·2	6·3	47·7	5·6	47·7	7·2	47·5	6·1	47·7	5·7	47·9
	-1	5·9	46·8	5·7	47·6	5·6	47·6	5·7	47·3	5·4	47·7	5·1	47·9
	+1	5·6	48·5	4·9	48·0	4·5	47·8						
16	0	8·5	47·5	6·7	47·6	5·9	47·8						
	-1	6·2	46·8	6·3	—	6·0	47·3						

TABLE No. 24 U.S.A.-27. TAPERED 0·7×1·3

Incidence (deg)	Elevation (chord)	At mid span 1 Angle	Vel.	2 Angle	Vel.	3 Angle	Vel.	One chord to the side 1 Angle	Vel.	2 Angle	Vel.	3 Angle	Vel.
	+1	0·8	47·6	0·5	47·2	0·5	47·4	0·4	47·3	0·2	47·3	0·0	47·4
-4	0	1·2	45·7	0·8	46·1	0·9	46·6	0·4	46·0	0·2	46·3	0·1	46·5
	-1	0·1	47·3	0·4	47·3	0·2	47·3	0·0	47·2	0·0	47·4	0·0	47·2
	+1	2·3	47·6	2·0	47·4	2·0	47·4	2·0	47·5	1·4	47·5	1·4	47·5
0	0	3·2	46·7	3·0	47·2	2·8	47·2	2·9	46·8	2·5	47·1	2·1	47·2
	-1	1·8	47·3	1·7	47·5	1·8	47·4	1·6	47·1	1·7	47·3	1·6	47·6
	+1	3·5	47·9	3·0	47·7	3·6	47·9	3·0	47·6	2·5	47·4	2·2	47·5
4	0	5·8	47·2	4·5	47·7	5·8	47·3	4·8	47·1	4·1	47·2	3·4	47·5
	-1	3·4	47·1	3·4	47·4	3·2	47·7	2·7	46·9	2·9	47·3	2·5	47·5
	+1	4·5	48·3	3·9	48·0	3·7	47·8	4·0	47·7	3·5	47·6	3·0	47·6
8	0	7·1	47·3	5·7	47·6	5·0	47·8	6·0	47·4	5·4	47·5	4·8	47·8
	-1	4·7	47·1	4·6	47·3	4·6	47·7	4·0	47·1	4·0	47·6	3·8	47·7
	+1	5·4	48·2	4·8	47·9	4·3	48·3	4·8	48·0	4·3	47·6	4·0	47·6
12	0	8·6	47·4	6·8	47·7	6·1	48·1	7·8	47·5	6·7	47·7	6·0	47·8
	-1	6·1	46·9	6·1	47·5	6·1	47·7	5·0	47·2	5·0	47·8	4·9	47·6
	+1	6·3	48·3	5·6	48·5	5·0	48·3						
16	0	9·8	47·6	7·8	47·9	7·0	48·0						
	-1	7·4	46·9	7·4	47·5	7·2	47·8						

TABLE No. 25 U.S.A.-27. TAPERED 0·6×1·4

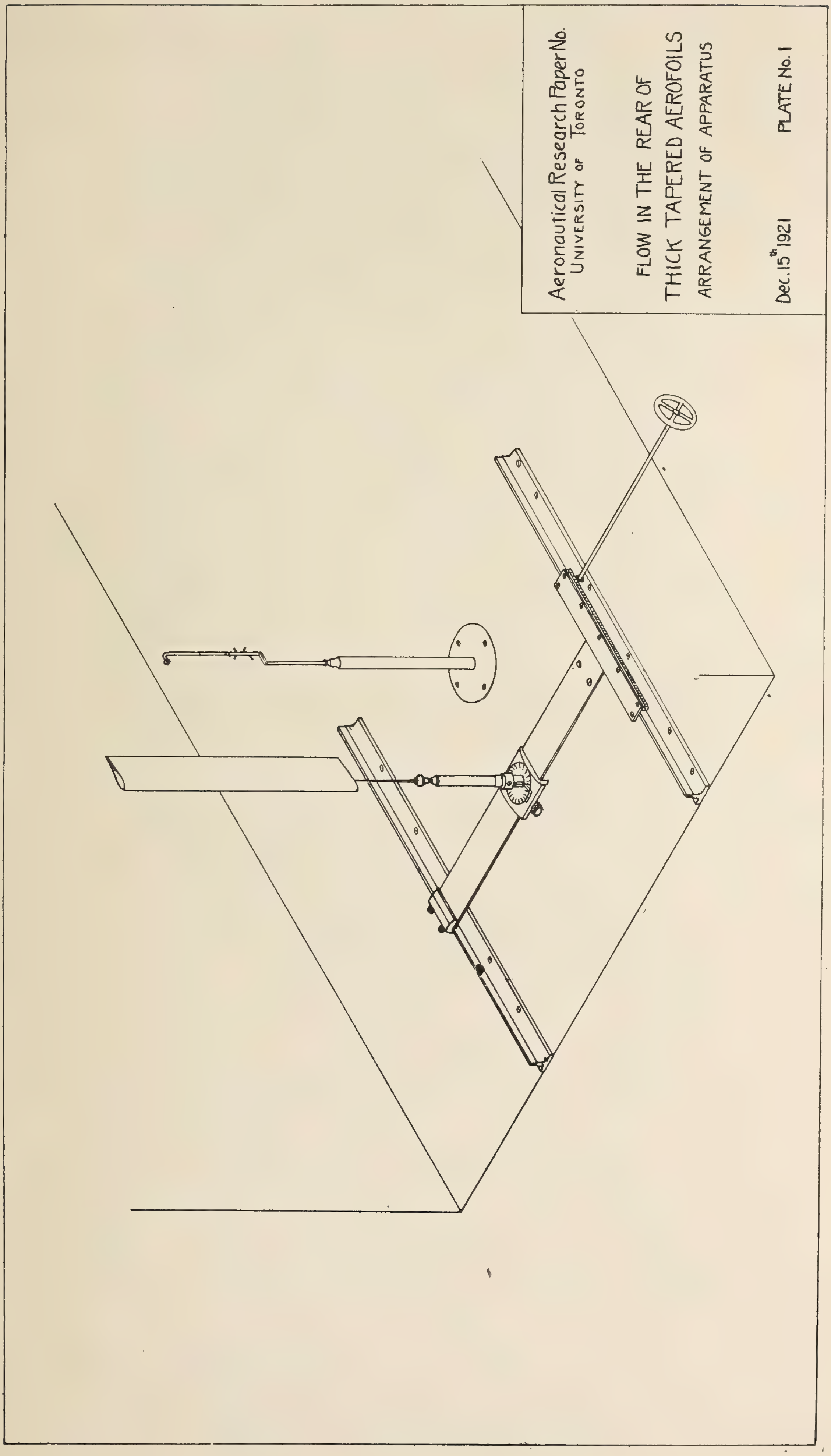
Incidence (deg)	Elevation (chord)	At mid span 1 Angle	Vel.	2 Angle	Vel.	3 Angle	Vel.	One chord to the side 1 Angle	Vel.	2 Angle	Vel.	3 Angle	Vel.
	+1	0·9	47·6	0·6	47·4	0·6	47·7	0·3	47·2	0·1	47·3	0·0	47·2
-4	0	1·6	56·6	1·4	46·3	1·3	46·6	0·0	45·9	-0·1	46·2	0·0	46·3
	-1	0·2	47·5	0·4	47·5	0·5	47·6	-0·1	47·2	0·2	47·4	-0·1	47·2
	+1	2·4	47·8	1·8	47·7	1·8	47·6	2·1	47·7	1·5	47·1	1·6	47·4
0	0	3·5	46·5	3·1	47·2	2·8	47·3	2·7	46·6	2·3	46·9	2·2	47·2
	-1	1·8	47·1	1·9	47·3	2·0	47·4	1·6	47·1	1·6	47·5	1·6	47·3
	+1	3·7	47·9	3·3	47·8	2·7	47·8	2·0	47·7	1·8	47·5	1·4	47·4
4	0	6·0	47·1	4·8	47·7	4·2	47·8	2·6	46·6	2·4	47·1	2·1	47·3
	-1	3·2	47·2	3·5	47·4	3·5	47·6	1·4	47·2	1·4	47·7	1·5	47·4
	+1	4·8	48·2	4·2	47·9	3·6	47·9	1·6	47·5	1·3	47·6	1·1	47·6
8	0	7·5	47·5	6·0	47·9	5·4	47·9	2·2	46·6	2·0	47·1	1·9	47·1
	-1	4·8	47·2	4·9	47·6	4·9	47·7	1·4	47·2	1·3	47·3	1·2	47·4
	+1	5·6	48·5	4·8	48·3	4·4	48·2	1·0	47·4	0·8	47·3	0·9	47·4
12	0	8·8	47·7	7·2	47·9	6·3	48·2	1·3	46·4	1·2	46·6	0·9	46·9
	-1	6·0	47·2	5·9	48·5	5·9	47·9	0·7	47·2	0·6	47·4	0·7	47·6
	+1	6·4	48·6	5·5	48·5	5·1	48·3						
16	0	10·2	47·8	7·9	48·1	7·0	48·2						
	-1	7·1	47·2	7·1	47·7	7·2	47·7						

TABLE No 26

VALUES OF TERMS IN THE EQUATION

$$\epsilon = (C_Y + C) \cdot K \cdot 10^{nx} (1 + St)$$

Z = 0				
Y	C	K	η	S
1	055	8 55	−·055	·351
0	·04+·235t	11 69	− 068	·516
−1	− 01	8 44	− 012	·924
Z = 1				
Y	C	K	η	S
1	·040	8·13	−·045	·332
0	025	11·77	−·055	·628
−1	− 010	8 98	−·020	·223



Aeronautical Research Paper No.
UNIVERSITY OF TORONTO

FLOW IN THE REAR OF
THICK TAPERED AEROFOILS
ARRANGEMENT OF APPARATUS

Dec. 15th 1921
PLATE No. 1

PLATE N° 2

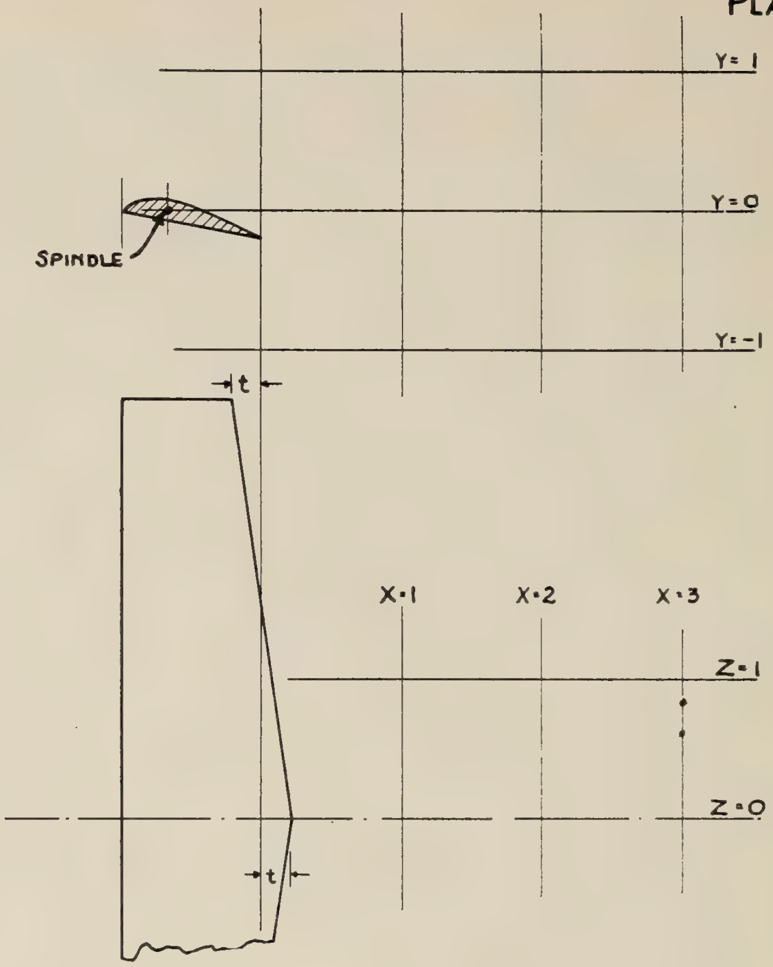


FIG. 1.— LOCATION OF POINTS OF OBSERVATION.

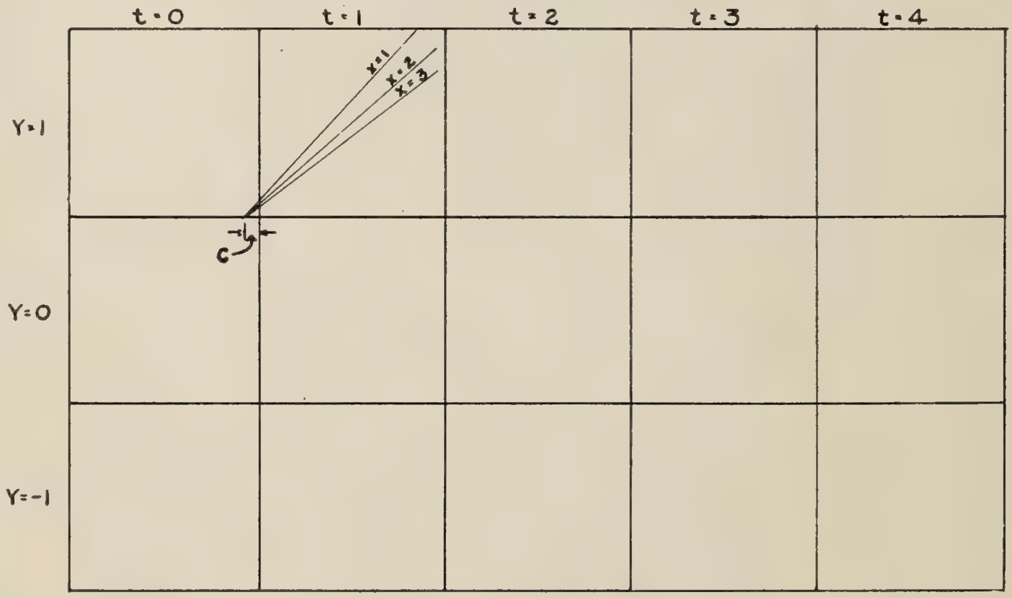


FIG. 2.— DIAGRAM SHOWING METHOD OF PLOTTING RESULTS.

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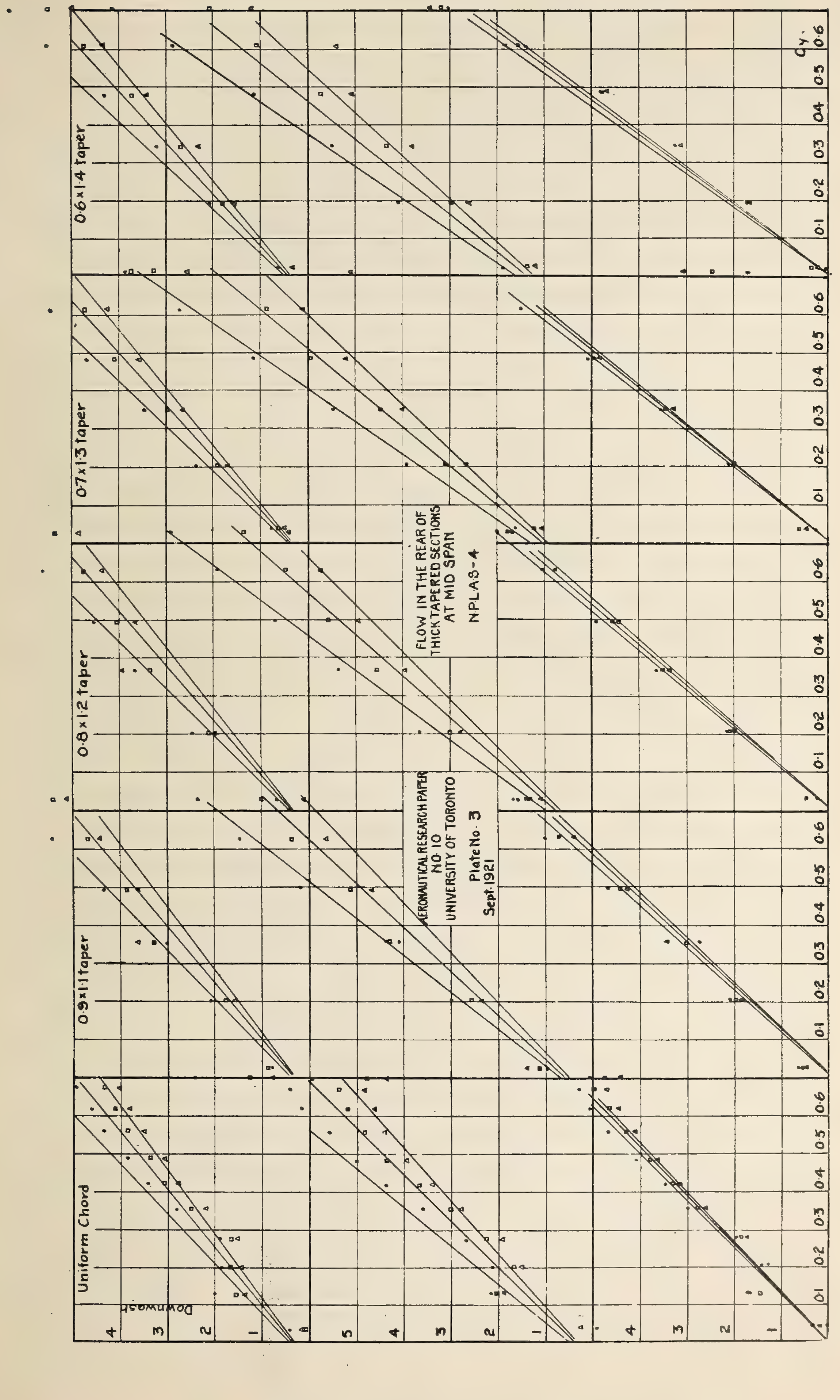

FLOW IN THE REAR OF THICK TAPERED SECTIONS AT MID SPAN

N.P.L. A.S-4

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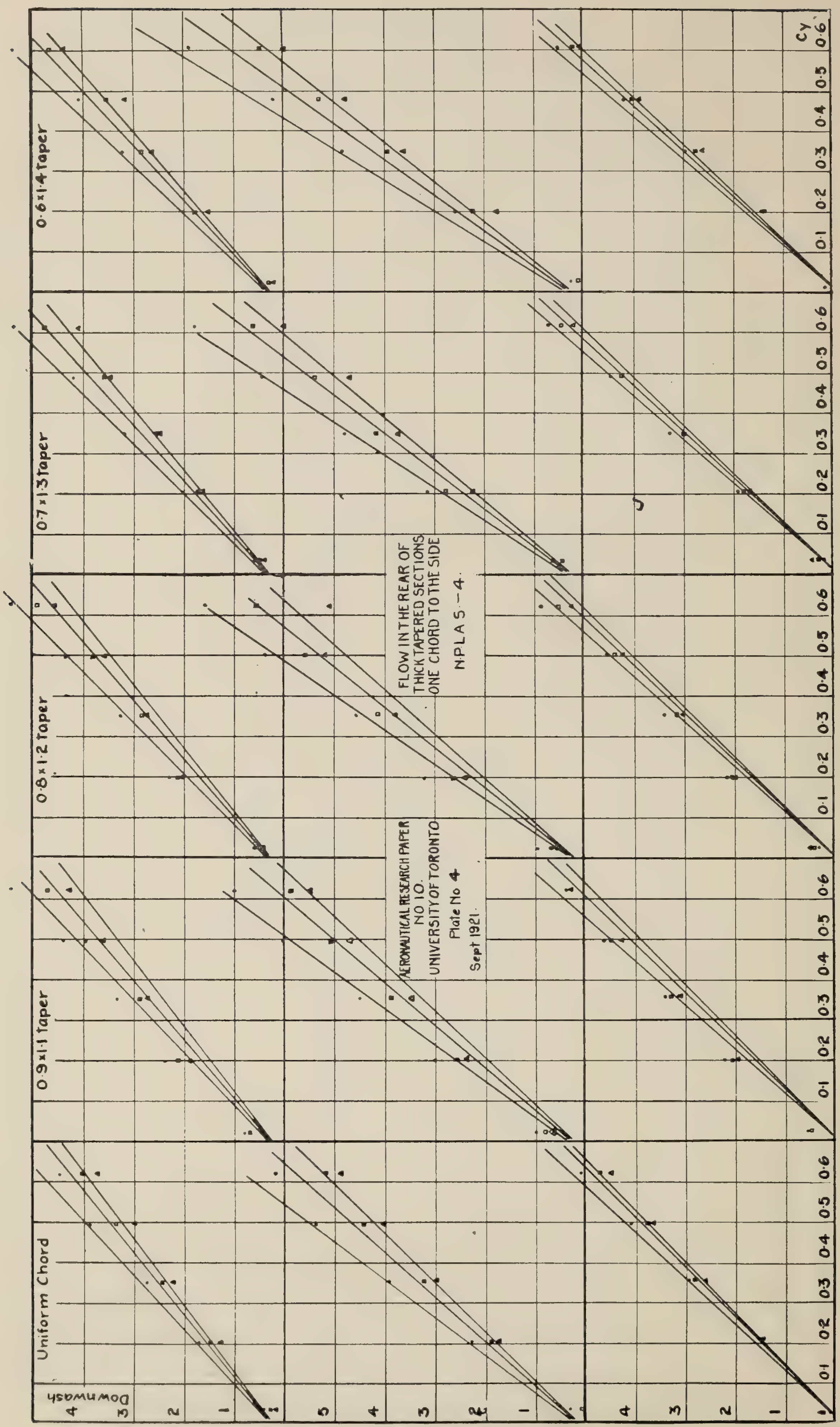
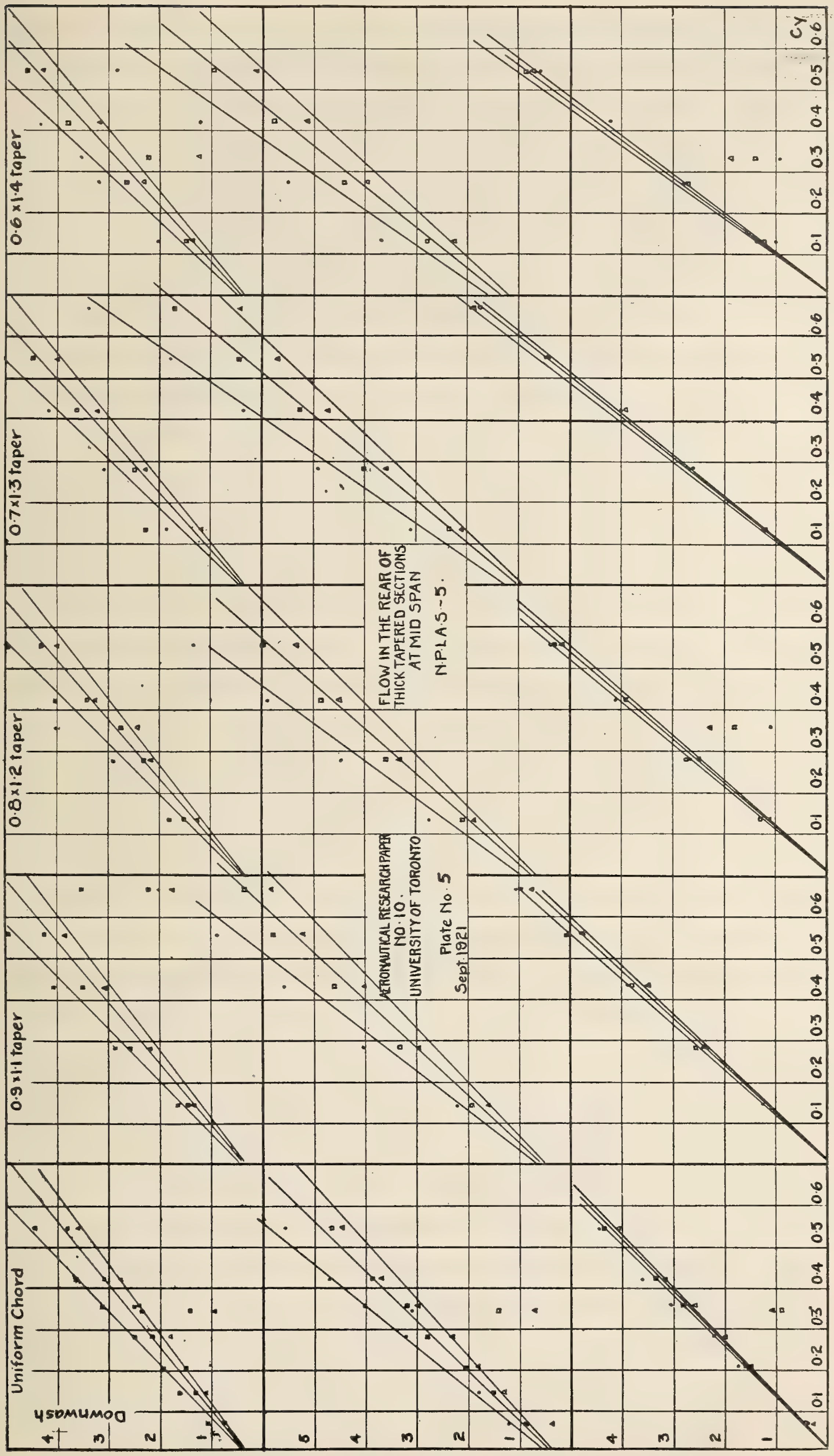
Plate No. 3

Sept. 1921


Downwash
4
3
2
1
5
4
3
2
1
4
3
2
1
Cy
0·1
0·2
0·3
0·4
0·5
0·6
Uniform Chord
0·9×1·1 taper
0·8×1·2 taper
0·7×1·3 taper
0·6×1·4 taper
FLOW IN THE REAR OF THICK TAPERED SECTIONS ONE CHORD TO THE SIDE
N.P.L A 5-4
AERONAUTICAL RESEARCH PAPER NO 10.
UNIVERSITY OF TORONTO
Plate No 4
Sept 1921



FLOW IN THE REAR OF THICK TAPERED SECTIONS AT MID SPAN

N.P.L.A.5.-5.

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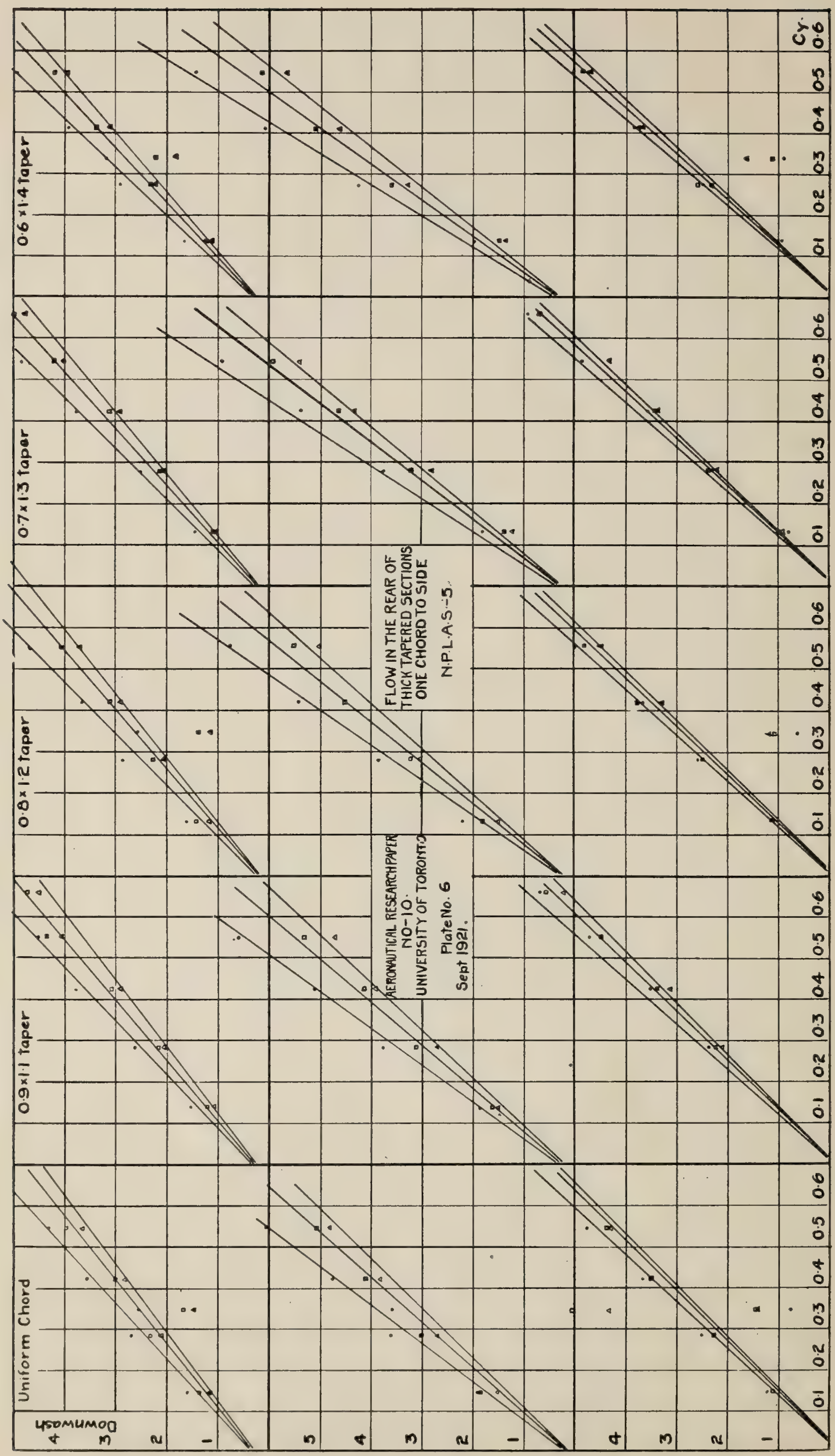
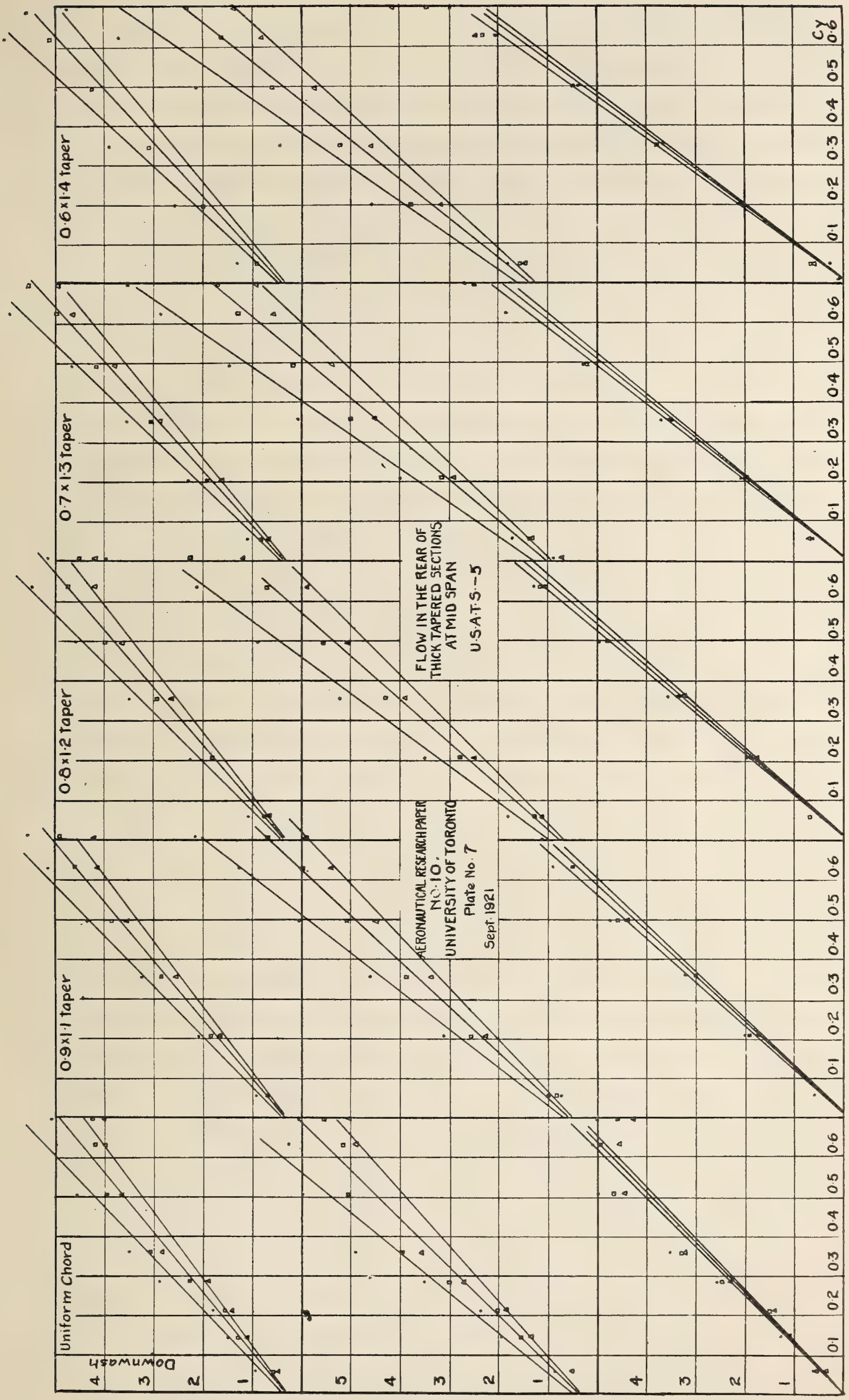
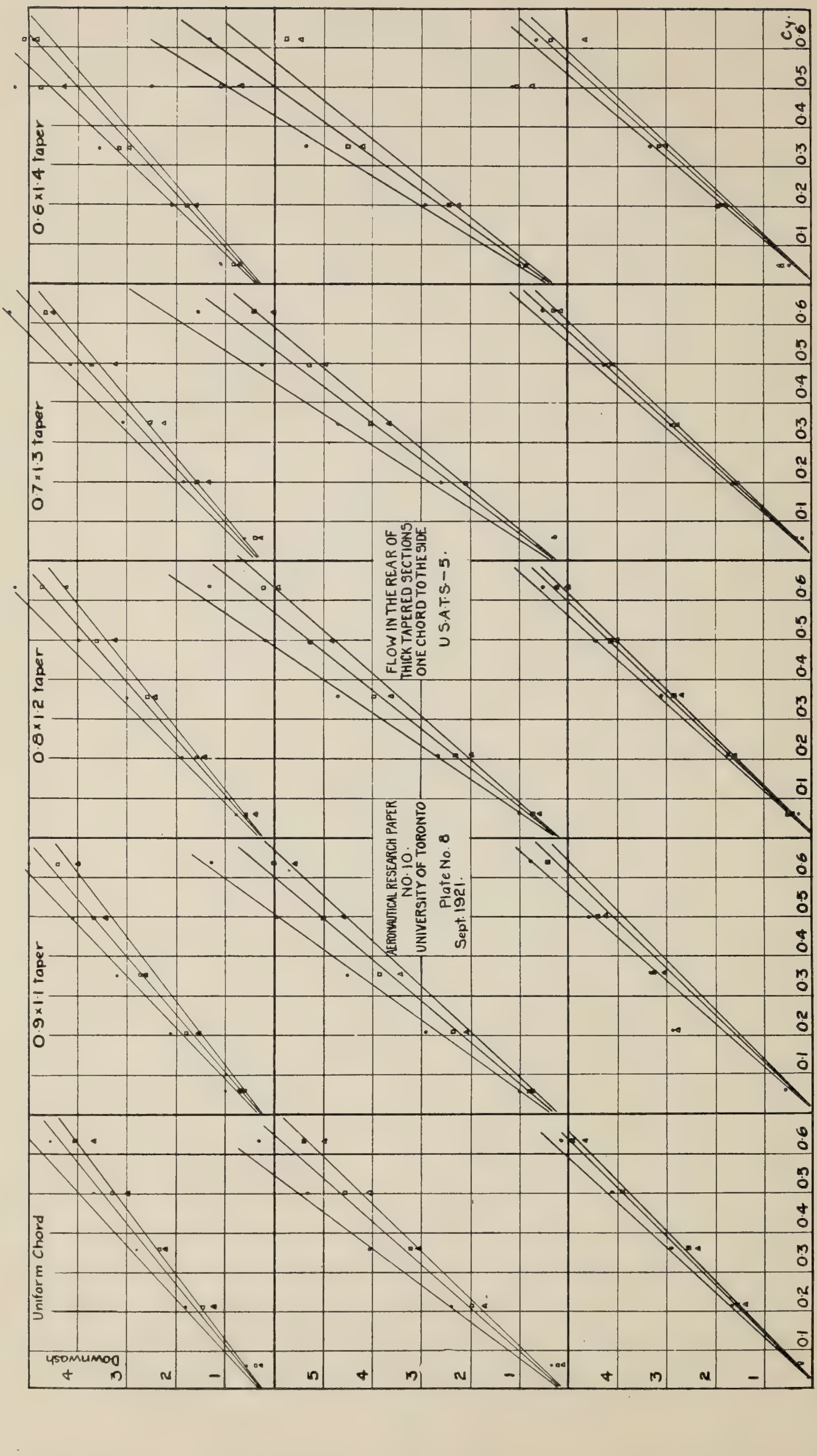
Plate No. 5
Sept. 1921


FLOW IN THE REAR OF
THICK TAPERED SECTIONS
ONE CHORD TO SIDE
N.P.L.A.S.-5.
AERONAUTICAL RESEARCH PAPER
NO-10
UNIVERSITY OF TORONTO
Plate No. 6
Sept 1921.
Uniform Chord
0.9×1.1 taper
0.8×1.2 taper
0.7×1.3 taper
0.6×1.4 taper
Downwash
4
3
2
1
5
4
3
2
1
4
3
2
1
Cy.
0.1
0.2
0.3
0.4
0.5
0.6


FLOW IN THE REAR OF
THICK TAPERED SECTIONS
AT MID SPAN
U·S·A·T·S·-5
AERONAUTICAL RESEARCH PAPER
No. 10.
UNIVERSITY OF TORONTO
Plate No. 7
Sept 1921
Uniform Chord
0·9×1·1 taper
0·8×1·2 taper
0·7×1·3 taper
0·6×1·4 taper
Downwash
4
3
2
1
5
Cy
0·1
0·2
0·3
0·4
0·5
0·6


Uniform Chord
0·9×1·1 taper
0·8×1·2 taper
0·7×1·3 taper
0·6×1·4 taper
Downwash
4
3
2
1
5
4
3
2
1
4
3
2
1
Cy.
0·1
0·2
0·3
0·4
0·5
0·6
FLOW IN THE REAR OF
THICK TAPERED SECTIONS
ONE CHORD TO THE SIDE
U.S.A.T.S.-5.
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NO. 10.
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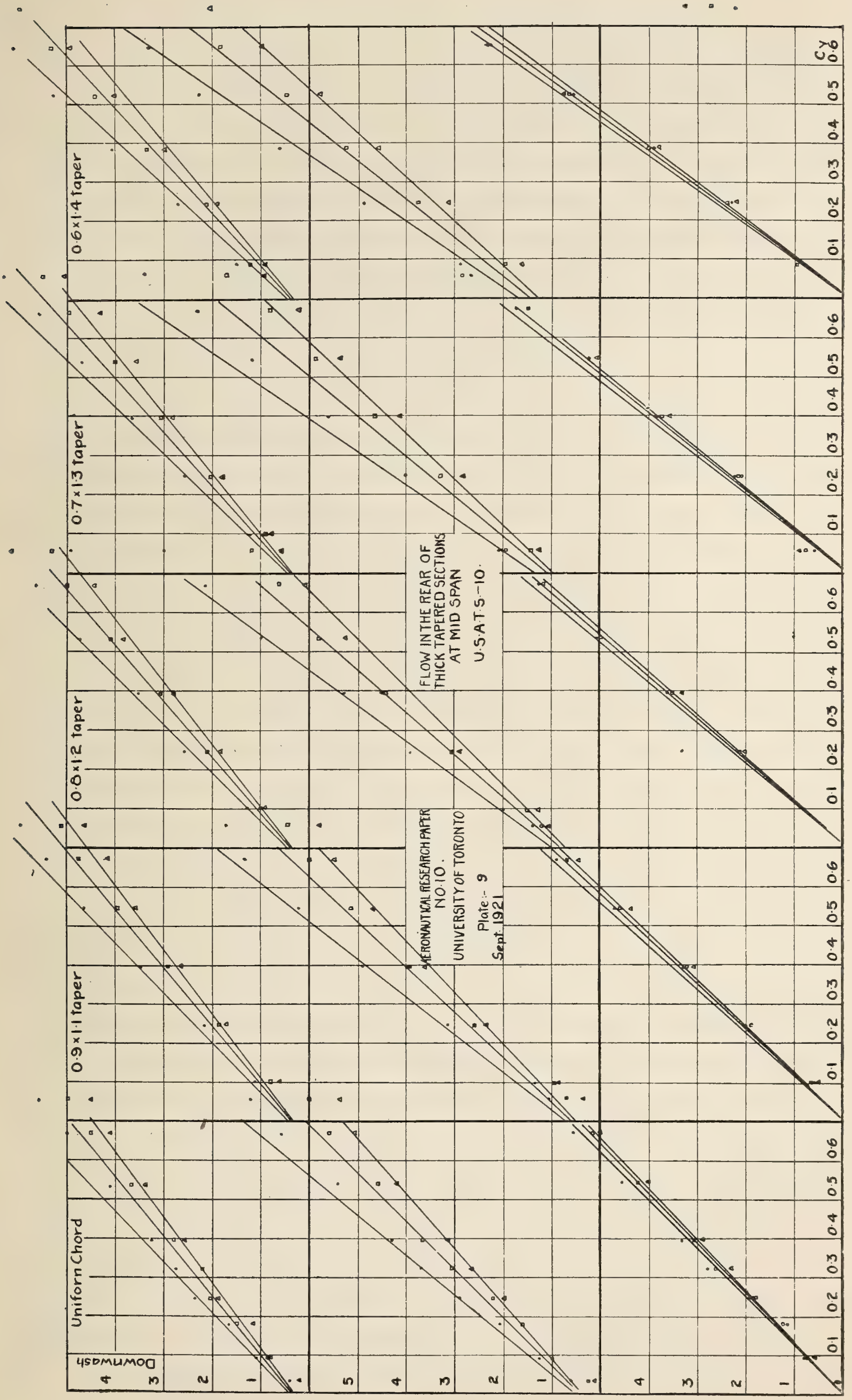

FLOW IN THE REAR OF THICK TAPERED SECTIONS AT MID SPAN
U.S.A.T.S.-10.

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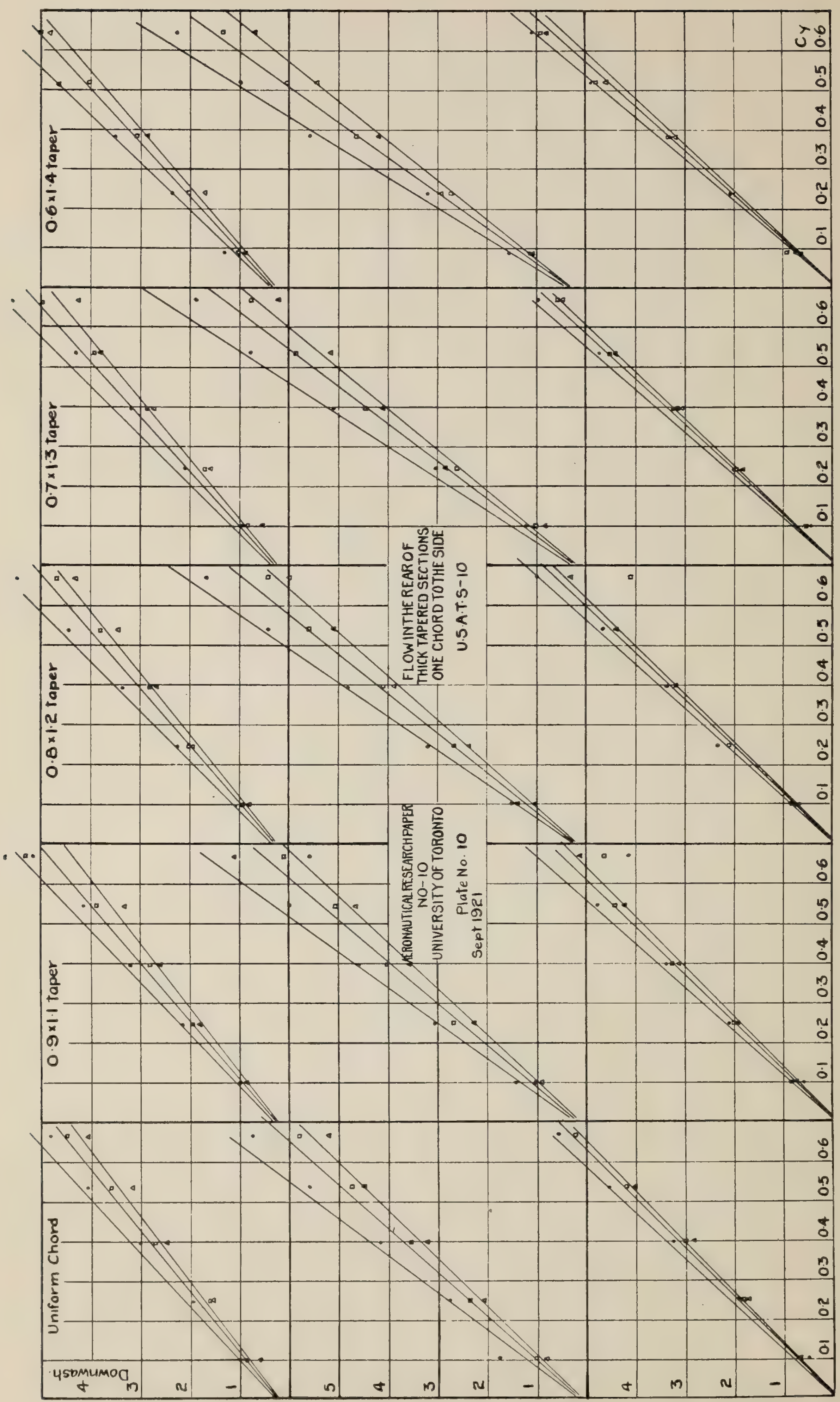
UNIVERSITY OF TORONTO
Plate:- 9
Sept 1921



FLOW IN THE REAR OF
THICK TAPERED SECTIONS
ONE CHORD TO THE SIDE
U.S.A.T.S-10

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NO-10
UNIVERSITY OF TORONTO
Plate No. 10
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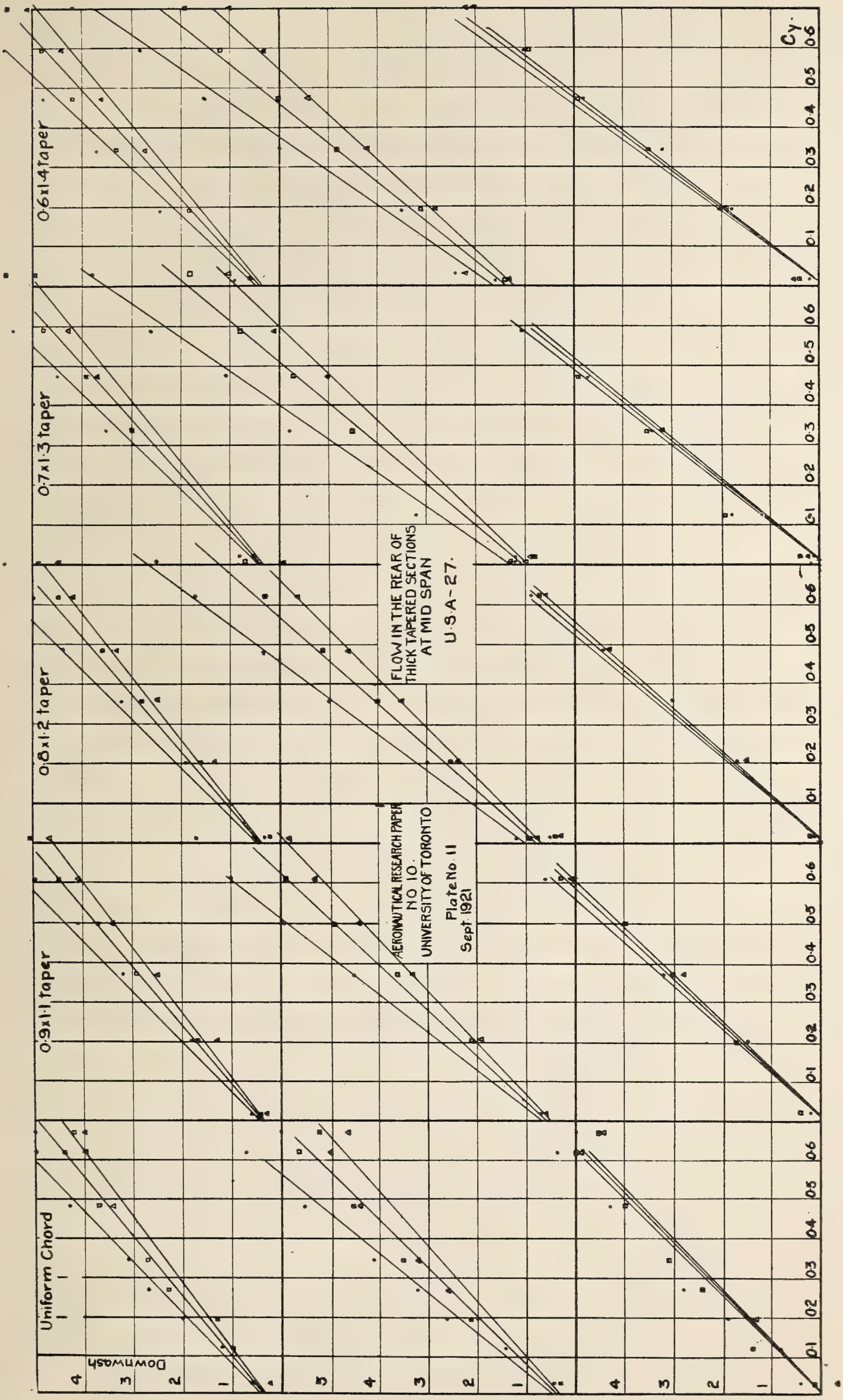

FLOW IN THE REAR OF
THICK TAPERED SECTIONS
AT MID SPAN
U.S.A-27.

AERONAUTICAL RESEARCH PAPER
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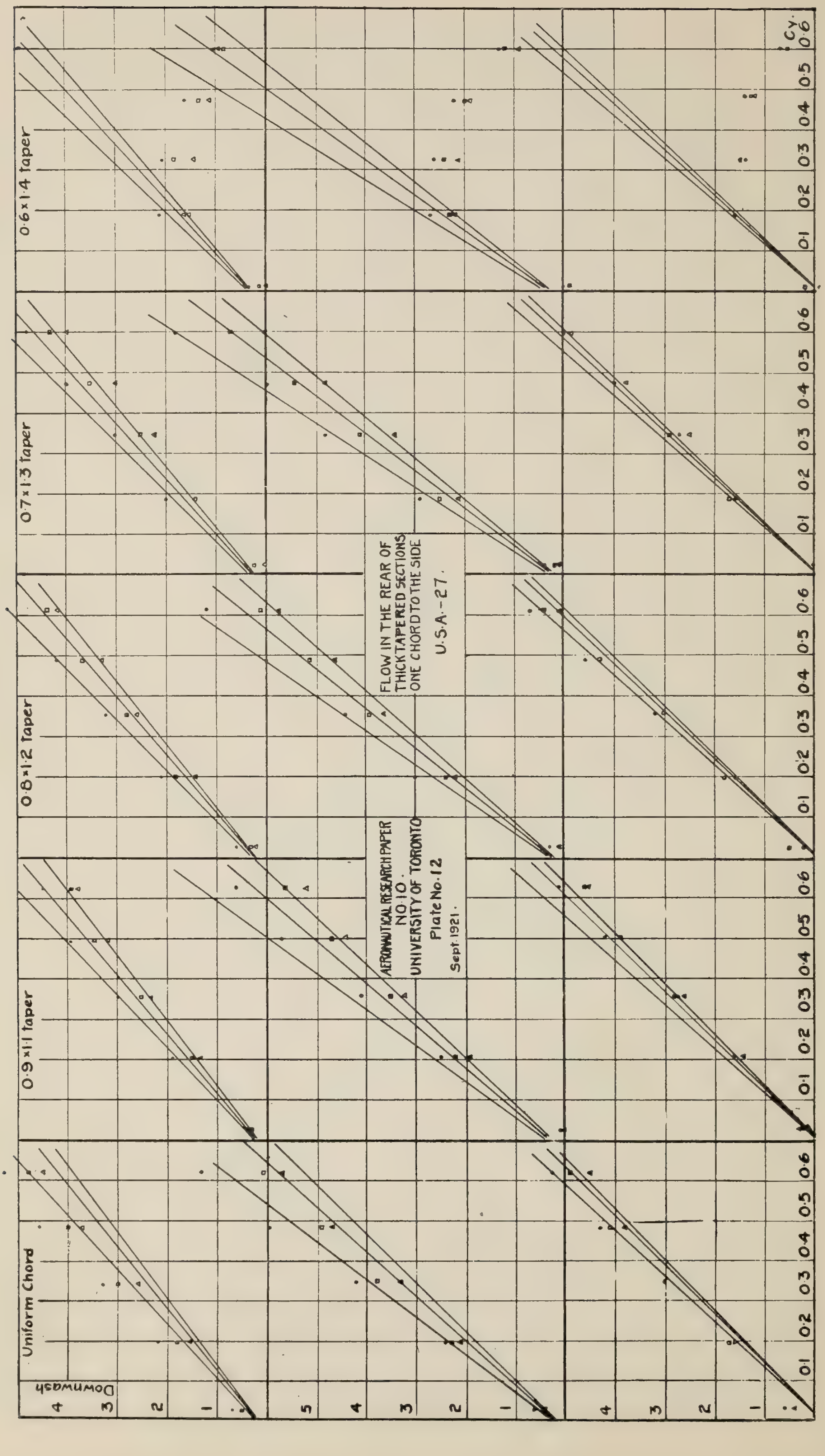
Sept 1921


Uniform Chord
0·9×1·1 taper
0·8×1·2 taper
0·7×1·3 taper
0·6×1·4 taper
Downwash
1
2
3
4
5
Cy
0·1
0·2
0·3
0·4
0·5
0·6
FLOW IN THE REAR OF
THICK TAPERED SECTIONS
ONE CHORD TO THE SIDE
U.S.A.-27.
AERONAUTICAL RESEARCH PAPER
NO·10·
UNIVERSITY OF TORONTO
Plate No·12
Sept. 1921·

Aeronautical Research Paper No. 11.
June, 1922.

GROOVED AEROFOIL

By J. H. Parkin, *Assistant Professor of Mechanical Engineering,*
H. C. Crane and J. S. E. MacAllister, *Research Assistants*

Reason for Investigation

In a certain machine in which a fluid jet, directed between two polished metal disks, was employed to impart rotation to the disks it was observed that the turning effort on the disks was reduced when the surfaces were roughened. This unexpected result seemed to indicate that a reduction in the frictional resistance was effected by the roughening and the following brief investigation was made with the object of determining whether a reduction in the drag of aerodynamic forms might not be secured in the same way.

Models. (See Plate 1)

A brass aerofoil of R.A.F.6.a. section, 3″×18″ being available it was used in this study although a low resistance or streamline form would have been preferable. The aerofoil was first tested and checked with the usual smooth surface. The simplest way of roughening the surface appeared to be by machining grooves parallel to the span. The grooves were milled, using a fly cutter, in different portions of the aerofoil surface as noted below. In the first test owing to a misinterpretation of instructions the grooves were 0.02 inches wide and deep, with semicircular bottoms. In the remaining tests the grooves were of semicircular cross section, 0.02 inches in diameter.

Test	Groove	Spacing	Surface Grooved
A	None	....	....
B	.02×.01	10 per inch	Forward 1/3 of upper surface.
C	.02×.01	30 per inch	Forward 1/3 of upper surface.
D	.02×.01	30 per inch	Entire upper surface.
E	.02×.01	30 per inch	Both surfaces with the exception of the forward 1/3 of the upper.

Where surfaces after grooving had to be rendered smooth again, as in test E, the grooves were filled with wax.

Tests

Each arrangement of grooving listed above was tested for lift and drag under standard conditions, at 40 f.p.s., over the usual range of incidences, namely −6° to + 20°.

Results

The absolute lift and drag coefficients and L/D, corrected for spindle interference, are tabulated in Tables 1, 2 and 3, and are plotted in Plate 1.

Discussion of Results

Grooving the surface of an aerofoil as in this study is apparently ineffective in securing the sought after reduction in drag. On the contrary the grooves generally increase the drag. Grooving the upper surface of the nose has less effect in this respect than grooving the upper trailing surface or the under surface. Whether this difference is due to the relative areas of the surfaces grooved or to the nature of the flow over the surfaces is not known.

There is some slight indication that deep, widely spaced grooves on the upper leading surface results in a reduction in drag particularly at the larger incidences. (See results for test B.)

The minimum drag was in every case increased by the grooves.

The exact nature of the effect of the grooves on lift is rather uncertain but there appears to result a small increase in lift at low incidences which disappears with increase in incidence. The width and spacing of the grooves seems important from this standpoint as is indicated by a comparison of tests B and C. The deep widely spaced grooves decrease the lift, while the shallow closely spaced grooving over the same area increases the lift as compared with the standard aerofoil.

The effect of the grooving on lift is perhaps most marked in the region of the critical angle. The maximum lift is reduced and the lift curve flattened out in this region while there is an indication that a reduction in the critical angle is caused, *i.e.*, the grooves facilitate burbbling. Whether this latter effect, shown in test E, was due to upper or lower surface grooving is not evident.

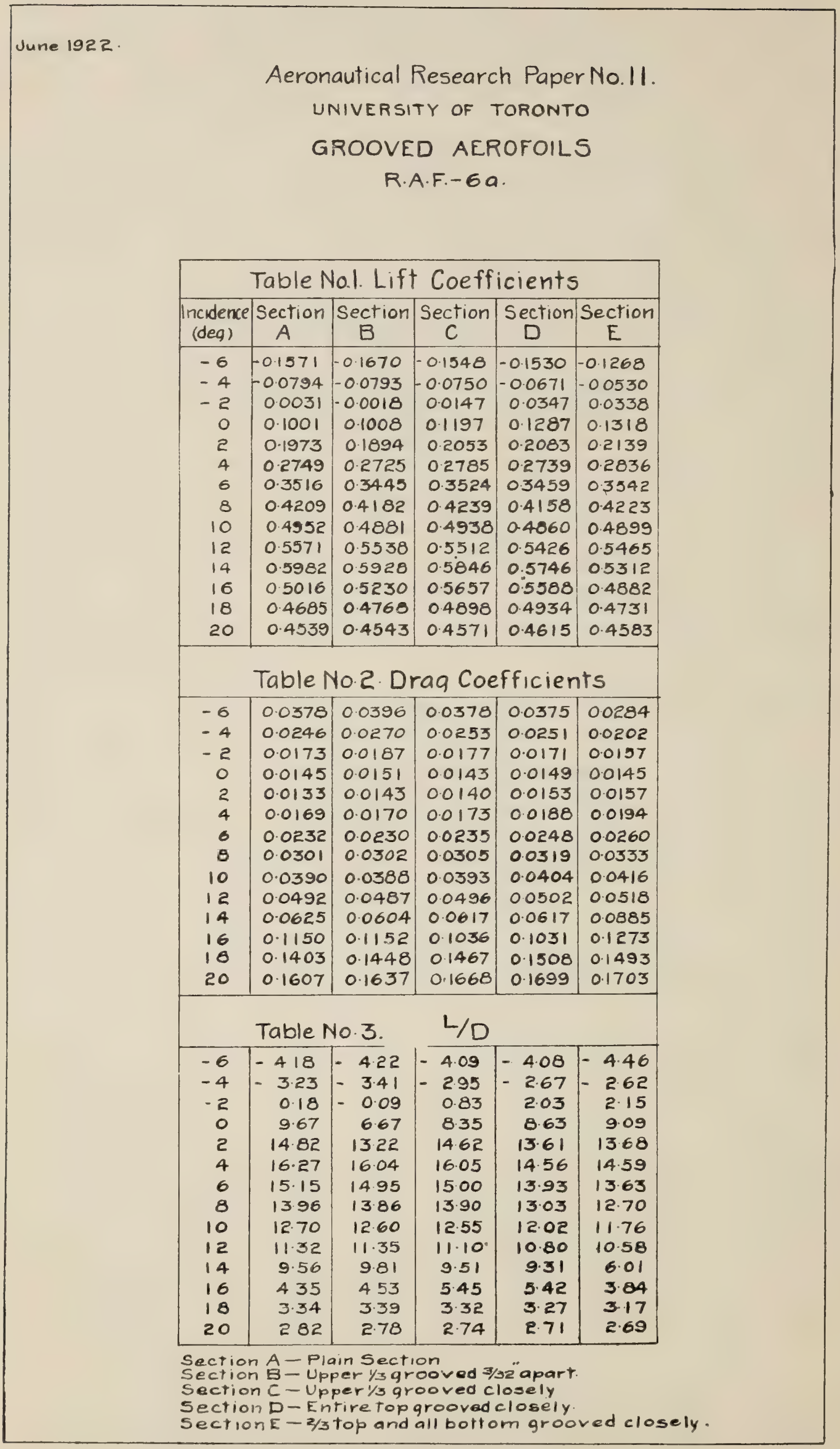
The grooving effect on L/D is apparently largely determined by that on drag. The L/D is reduced at all positive lifts below the critical angle. The reduction in maximum L/D is slight for the upper nose grooving but appreciable for the other arrangements.

The above results for the effect of the grooves on lift are in general agreement with those given in R. & M. No. 72 (pp. 82 and 94) 1912-13 for comparative tests of a varnished and fabric covered aerofoil. The fabric covered model possessed an appreciable advantage in lift at 2° which however decreased rapidly and became zero near the critical angle.

A test of a grooved aerofoil is recorded in R. & M. No. 417 (1917-18) but the conditions were so different as to render a comparison difficult. With two large grooves, 0.03 chord wide and deep, spaced 1/3 chord from leading and trailing edges, in the under surface, a slight reduction in drag was secured and the lift was also reduced, at all positive lifts, by about the same relative amount with the result that the L/D was but little affected. These effects on lift and drag are the same as those secured in this investigation with the deep grooves as used in test B.

It is thought that further tests to investigate the effect of depth and spacing of the grooves, their effect on different portions of the aerofoil surface and the speed or vl. effect on the grooves might be of value and these may be carried out at a later date.

A comparison of the relative aerodynamic effects of this type of grooving and grooving in a fore and aft direction as used in some all metal aeroplanes might be of interest from the standpoint of the corrugated metal wing covering.



June 1922·

Aeronautical Research Paper No. 11.

UNIVERSITY OF TORONTO

GROOVED AEROFOILS

R·A·F.–6a·

Table No. 1. Lift Coefficients

Incidence (deg)	Section A	Section B	Section C	Section D	Section E
- 6	-0·1571	-0·1670	-0·1548	-0·1530	-0·1268
- 4	-0·0794	-0·0793	-0·0750	-0·0671	-0·0530
- 2	0·0031	-0·0018	0·0147	0·0347	0·0338
0	0·1001	0·1008	0·1197	0·1287	0·1318
2	0·1973	0·1894	0·2053	0·2083	0·2139
4	0·2749	0·2725	0·2785	0·2739	0·2836
6	0·3516	0·3445	0·3524	0·3459	0·3542
8	0·4209	0·4182	0·4239	0·4158	0·4223
10	0·4952	0·4881	0·4938	0·4860	0·4899
12	0·5571	0·5538	0·5512	0·5426	0·5465
14	0·5982	0·5928	0·5846	0·5746	0·5312
16	0·5016	0·5230	0·5657	0·5588	0·4882
18	0·4685	0·4768	0·4898	0·4934	0·4731
20	0·4539	0·4543	0·4571	0·4615	0·4583

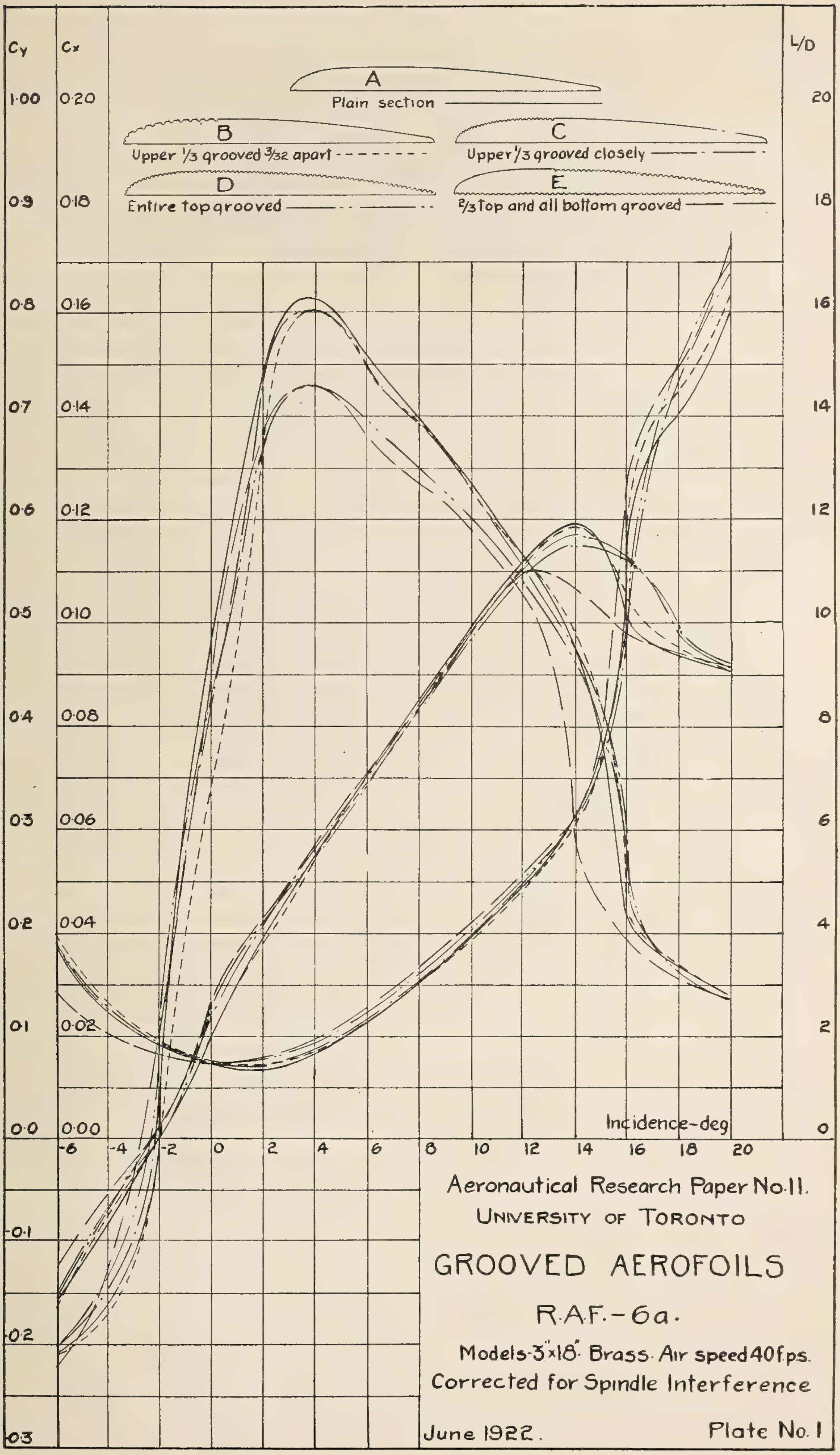
Table No. 2. Drag Coefficients

- 6	0·0378	0·0396	0·0378	0·0375	0·0284
- 4	0·0246	0·0270	0·0253	0·0251	0·0202
- 2	0·0173	0·0187	0·0177	0·0171	0·0157
0	0·0145	0·0151	0·0143	0·0149	0·0145
2	0·0133	0·0143	0·0140	0·0153	0·0157
4	0·0169	0·0170	0·0173	0·0188	0·0194
6	0·0232	0·0230	0·0235	0·0248	0·0260
8	0·0301	0·0302	0·0305	0·0319	0·0333
10	0·0390	0·0388	0·0393	0·0404	0·0416
12	0·0492	0·0487	0·0496	0·0502	0·0518
14	0·0625	0·0604	0·0617	0·0617	0·0885
16	0·1150	0·1152	0·1036	0·1031	0·1273
18	0·1403	0·1448	0·1467	0·1508	0·1493
20	0·1607	0·1637	0·1668	0·1699	0·1703

Table No. 3. L/D

-6	- 4 18	- 4·22	- 4·09	- 4·08	- 4·46
-4	- 3·23	- 3·41	- 2·95	- 2·67	- 2·62
-2	0·18	- 0·09	0·83	2·03	2·15
0	9·67	6·67	8·35	8·63	9·09
2	14·82	13·22	14·62	13·61	13·68
4	16·27	16·04	16·05	14·56	14·59
6	15·15	14·95	15·00	13·93	13·63
8	13·96	13·86	13·90	13·03	12·70
10	12·70	12·60	12·55	12·02	11·76
12	11·32	11·35	11·10	10·80	10·58
14	9·56	9·81	9·51	9·31	6·01
16	4 35	4 53	5·45	5·42	3·84
18	3·34	3·39	3·32	3·27	3·17
20	2 82	2·78	2·74	2·71	2·69

Section A — Plain Section
Section B — Upper 1/3 grooved 3/32" apart.
Section C — Upper 1/3 grooved closely
Section D — Entire top grooved closely.
Section E — 2/3 top and all bottom grooved closely.


Cy
Cx
L/D
1·00 0·20 20
0·9 0·18 18
0·8 0·16 16
0·7 0·14 14
0·6 0·12 12
0·5 0·10 10
0·4 0·08 8
0·3 0·06 6
0·2 0·04 4
0·1 0·02 2
0·0 0·00 0
-0·1
-0·2
-0·3
A
Plain section
B
Upper 1/3 grooved 3/32 apart
C
Upper 1/3 grooved closely
D
Entire top grooved
E
2/3 top and all bottom grooved
Incidence-deg
-6 -4 -2 0 2 4 6 8 10 12 14 16 18 20
Aeronautical Research Paper No. 11.
UNIVERSITY OF TORONTO
GROOVED AEROFOILS
R.A.F.-6a.
Models 3"×18" Brass. Air speed 40 f.p.s.
Corrected for Spindle Interference
June 1922.
Plate No. 1

Aeronautical Research Paper No. 12.
July, 1922.

END SPINDLE CORRECTIONS IN WIND CHANNEL EXPERIMENTS

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Introduction

The data desired from a wind channel test are those pertaining to the aerofoil model alone. Some device must however be used to support the model and being attached to the latter will necessarily modify to a greater or less extent the results obtained. Hence to determine the true data or characteristics, the magnitude of the modification, *i.e.*, the correction, must be found. It is obvious that the correction should be as small as possible as compared with the forces on the model to ensure the greatest accuracy in determining the latter.

The usual method of supporting a model aerofoil in a wind channel equipped with a balance of the N.P.L. type is by means of the "end spindle". The latter has been found to have the smallest effects on the forces on the model of any suporting device for use with this type of balance (see R. and M. 148, pp. 142-3). The end spindle is of turned steel tapering from the larger end which is clamped in the chuck of the balance to the smaller end tapped into the end of the aerofoil, usually at a distance of one-third the chord from the leading edge. The model in the channel thus stands "on one tip", *i.e.*, with span vertical instead of in its normal flight position. The standard of the balance and frequently a portion of the spindle are shielded from the air current by means of a guard of circular or streamline section.

The end spindle effect includes both the effect due to the resistance offered by the spindle to the relative flow of the air and the indirect or interference effect due to the spindle and guard altering the air flow about the tip of the model. The latter effect is apparently much smaller than the former for the standard spindle and guard as described in this report. These effects are found to modify both lift and drag measurements, the latter to a relatively much larger degree than the former, and the effect on drag only is commonly considered (R. and M. 148, N.A.C.A. Tech. Note No. 37, p. 3). Moments, for centre of pressure determinations, being measured about the spindle axis in the N.P.L. balance are not affected. In many cases for routine testing in aerodynamic laboratories spindle effect is entirely neglected while for more precise work it is generally determined only at a single incidence, that of minimum drag (R. and M. 148, p. 142).

It has been the practice at Toronto to determine spindle corrections on both lift and drag over the whole range of incidences. The determination is laborious and difficult, as will be evident from the description following and practically doubles the time required for an aerofoil test. A simple, rapid method for determining the spindle corrections, generally applicable and of sufficient accuracy would thus greatly facilitate the precise determination of aerofoil characteristics. Such a method has been developed at Toronto and is described in this report.

Determination of End Spindle Correction

The method employed for determining the spindle corrections is a combination of those of R. and M. 148 and 198, and has been previously described in App. 1 Aero. Res. Paper No. 4 (see also R. and M. 244, p. 36, N.A.C.A. Tech. Note 37, Tech. Orders No. 12, p. 36). The method will be again outlined here to render this paper more clear and complete.

The model is first tested in the ordinary way for lift and drag over the required range of incidences. The test is repeated but with a duplicate dummy spindle fixed in the upper end of the aerofoil, shielded by a duplicate dummy guard supported from the roof of the channel. The differences between the two sets of readings represent the upper spindle effects on lift and drag. The model remains in the chuck during these two tests in order to eliminate any errors due to minute differences in alignment that might occur if it were removed and replaced. Since the N.P.L. balance measures moments rather than forces, to determine the lower spindle effects the upper spindle effect must be multiplied by the ratio of distance from pivot to centre of pressure of lower spindle to distance from pivot to centre of pressure of upper spindle. A knowledge of the position of the C.P. of the spindle is thus required. The C.P. of the spindle may be either assumed as being located at the centre of projected area of the unshielded portion of spindle, or, subject to certain limitations referred to later, determined experimentally as follows:

The model is supported, in normal flight position, on a spindle attached to its underside at the centre of the span, and a dummy end spindle and guard arranged at one end of the model. Readings are made of torque and drag with and without the dummy spindle and guard, the difference of these then giving the torque and drag due to the spindle. Dividing the former by the latter locates the C.P. of the spindle. This need of course only be done at one angle since, although the C.P. probably moves slightly for varying incidence due to alterations in end flow, the movement could not be detected by this method.

It should be noted that the accuracy necessary in the readings for determining the position of the C.P. of the spindle is much greater than the ordinary limits of accuracy of working. Thus an error of one division only in both drag and torque readings, corresponding to errors of 1% on drag and 1/5% on torque, may result in 1/8″ difference in position of the C.P.

In Toronto it has been possible by making repeated tests to locate the C.P. of the spindle in positions which appear reasonable and consistent for different conditions (see Table 1) but which are of uncertain accuracy for the above reason. However, very precise determinations of C.P. are unnecessary since its location enters into the ratio referred to above and an error of ¼″ in the position of the C.P. (for a 4 ft. channel) results in a difference of less than 2% in value of the ratio. Hence it is sufficiently accurate to consider the C.P. at the centre of projected area of the spindle, as has been done in this report (see R. and M. 148, p. 142, and N.A.C.A. Tech. Note 37, p. 4).

Explanation of Spindle Effects and Basis of Method

The air flow about a wing tip may be considered as made up of two elements, the normal air flow AB Fig. 1 in the direction of the relative motion of aerofoil and air, modified slightly as the air streamlines bend out and around the tip, and the flow AC perpendicular to the chord induced by the pressure difference between the under and upper sides of the aerofoil. The vectorial sum AD represents the actual relative direction of air flow which corresponds with the direction as actually measured as shown in Fig. 2 taken from R. and M. 197. The arrows represent the vectorial difference between actual air flow and that of undisturbed air stream. The pressure distribution curves shown in Fig. 3 from R. and M. 73, are for a somewhat different wing section at the same incidence, 12°.

Due to the element AB of the air flow the spindle introduces a drag AE in the direction AB thus increasing the measured drag of the aerofoil but having no effect on the lift. The element AC causes a spindle drag AF in direction AC which thus affects both lift and drag measurements, the increase in drag being HF and increase, or decrease, in lift AH. Evidently then:

spindle correction on lift—$C_L = HA$
spindle correction on drag—$C_D = HF + AE = HF + FG = HG$.

It will further be evident that the angle HAF is equal to the angle of incidence, i, and therefore $HF = HA \tan i$.

Again the magnitude of the element AC of the air flow and hence the magnitude of the spindle drag AF is proportional to pressure difference between the two surfaces of the tip of the aerofoil at the spindle. The lift of the model depends upon the pressure difference and if it be assumed that the lift is proportional to the p.d. at the wing tip at the spindle, then the spindle drag AF is proportional to the lift of the model. The correction on lift $AH = AF \cos i$. Neglecting $\cos i$ involves a maximum error of 5% in the correction which in any event cannot be determined with this accuracy; hence without serious error the correction on lift may be taken as proportional to lift of model. Also if the distribution of the pressure difference at the wing tip is such that at the spindle the p.d. is zero when the lift is zero then

$$C_L = f(L)$$

and the spindle correction plotted on lift will be a straight line passing through the origin. If at zero lift a p.d. exists at the spindle then

$$C_L = f(L) \pm K$$

K representing the spindle effect on lift at zero lift.

This reasoning assumes that the distribution of pressure difference at the wing tip (form of curve) and variation of p.d. with lift (elevation of p.d. curve) are such that at the spindle the lift is proportional to the p.d. Results given later in this report indicate that such is approximately the case for the aerofoils here considered (see also R. and M. 73 and Aero. Res. Paper No. 6).

Returning to the consideration of the correction on drag it has been shown that

$$C_D = (\text{correction on lift} \times \tan i) + \text{normal drag of spindle}$$
$$= HA \tan i + AE.$$

Further the correction on lift has just been shown to be proportional to the lift and hence

$$C_D = f(L) \tan i + D_S$$
$$= f'(L) + D_S \text{ where } f' = f \times \text{average } \tan i.$$

This latter greatly simplifies the calculation of the drag correction and is sufficiently accurate over the normal flying range of incidences.

Again, neglecting scale effect the lift varies as V^2, the pressure difference at the tips varies as the lift, the velocity component AC as the square root of the pressure difference and the spindle drag AF varies as the square of the velocity component AC hence the spindle drag AF varies as V^2 or the lift. The same equation for the correction on lift should thus hold for all speeds or vl's while for drag corrections the constants D_S should vary as V^2. This ignores the scale effect on lift which is permissible when dealing with quantities of the magnitude of the spindle corrections, and also any possible variation in pressure difference distribution with change in velocity. It should therefore be possible to estimate the corrections at one speed from those determined at some other speed.

The general equations for the corrections on a basis of the above are

$$C_L = f(L) \pm K$$

$$C_D = f'(L) + D_S \left[\frac{V_2}{V_1} \right]^2$$

To check the soundness of the reasoning and general applicability of the equations the spindle corrections for a number of aerofoils of typical sections thin, medium and thick, at various vl's, for models of different sizes and for tapered models have been examined. These have been selected from determinations made from time to time in connection with different investigations with a view to providing a comprehensive range of types and conditions.

Dimensions of the different spindles used and details of the guard are given in Table 2 and Fig. 4.

Plotting of Results

The corrections on lift and drag have been plotted on lift of model on plates 2, 3 and 4. These corrections have been plotted up to the burbble point or critical angle only since above this angle the change in the nature of the flow (to the discontinuous) renders the determinations of the corrections very uncertain. Drag corrections for negative lifts are plotted as though the lifts were positive since the effect on the spindle at negative lifts is the same, as far as the drag is concerned as though lifts were positive. These points may be distinguished on the plates by the accent marks.

On Plate 2 are plotted corrections for models of standard size, 3″×18″ at three vl's and for six aerofoils, RAF-6a, NPLAS-4*, NPLAS-5 USATS-5, USATS-10 and USA-27

Corrections are plotted on Plate 3 for models of four different tapers, average chord 3″, at vl. 10 for five aerofoils. NPLAS-4, NPLAS-5, USATS-5, USATS-10, and USA-27 (see Aero Res. Paper No. 8).

On Plate 4 are plotted corrections for three aerofoils NPLAS-5, USATS-5 and USATS-10 for 1.8″×10.4″ models at vl. 10 and for 4.2″×25.2″ models at three vl's.

Examination of Results

Plate 2—Standard Aerofoils

The corrections on this plate are in most cases the averages of two, in some cases three determinations.

(*See R. & M. 322 National Physical Laboratory Airscrew Section).

Lift Corrections

It will be observed that the points for each aerofoil for all vl's are grouped more or less closely (having proper regard for the difficulties of the method) about a single straight line which for four of the six models passes through the origin and further that the slope of the line is the same for all the aerofoils. Thus for these four aerofoils representing a wide range of types the lift correction for this spindle and guard for vl's from 7.5 to 12.5 is given by the simple equation

$$C_L=0.012L$$

For the aerofoil USATS-10 the equation is

$$C_L=0.012L+0.0011.$$

Evidently the error introduced by using the same equation as for the others is quite small.

For aerofoil NPLAS-5 the equation is

$$C_L=0.012L-0.0020.$$

The deviation of this rule from that of the majority is probably due to the great thickness and blunt curvature of upper surface of the aerofoil.

Drag Corrections

For each aerofoil and vl. the points lie close to a single line whose slope is 0.0012. Now the average critical angle for conditions shown on this plate is about 12° and the average tangent of angles between 0 and 12° is roughly 0.10. Further it was shown that $C_D=f(L)$ tan $i+D_S$ and $f(L)$ has just been found to be .012L hence

$$C_D=.012L\times 0.10+D_S=0.0012L+D_S$$

The straight line of slope 0.0012 approximates most of the points very well but it will be observed that there is a distinct upward trend to the points as the lift increases particularly at the higher lifts, indicating that a variable function tan i should properly be used. D_S represents the normal drag of the spindle and is indicated on the graphs by the intercept of the line on the vertical axis. This drag increases as the square of the velocity hence the intercepts should vary for each aerofoil in proportion to $50^2 : 40^2 :: 40^2 : 30^2$. That they do is shown by the equations on the graphs and in table 3, where measured intercepts and calculated drag (using that at vl. 10 as a basis) are tabulated.

Plate 3—Tapered Aerofoils

The spindle position for tapered aerofoils remained at 1″ from the leading edge for all tapers except the maximum 1.8″×4.2″ in which it was moved forward to ½″ from leading edge in order to place it at the thickest part of the section. In the case of the USA-27, a comparatively thin section, difficulty was experienced with the usual brass spindle socket and in some cases a two-part spindle was used. The location of the spindle was also varied. Particulars of spindle and location are noted on the graphs. The corrections are the results of single determinations only.

Lift Corrections

In most cases the simple general equation $C_L = 0.012L$ found for the majority of the standard models is applicable. For the maximum taper the equation is generally of the form $C_L = 0.012L \pm K$. This is probably due to the location of the spindle well forward where the pressure difference distribution is changing rapidly. The aerofoil NPLAS-5 departs from the general rule probably for the reasons already mentioned.

Drag Corrections

These adhere to the general equation of the standard model the only variation being in the constant D_S. In most cases the latter has a magnitude equal to that for the standard model but there is some variation with taper. This variation may be explained by the fact that the spindle position, while constant at 1″ from the leading edge, is, with respect to the tip section really receding from the leading edge due to the reduction of the chord as the taper is increased. The spindle may thus be shielded to a greater or less extent from the normal air flow thereby varying its resistance and the constant D_S.

An average equation for drag correction for standard and tapered aerofoils at vl. 10 is:

$$C_D = 0.0057 + 0.0012L.$$

Plate 4—Aerofoils of Various Sizes
1.8″×10.4″ Models

In these models the spindle was located ½″ from leading edge. The spindle is necessarily much longer than for the standard model and the corrections particularly on drag are therefore much larger.

The corrections are the results of single determinations only.

Lift Corrections

The slope of the lines is here steeper due to the greater length of spindle offering greater resistance to the air flow.

Drag Corrections

As would be expected the intercept is much larger, the slope being roughly one-tenth that of the lift correction graphs, as for the models already considered.

4.2″×25.2″ Models

These models are larger than can be properly tested in a 4 ft. channel but were, for certain reasons, used in an investigation. The spindle is very short, only 0.71″ exposed, which is about equal to the maximum thickness of the model. The close proximity of the guard to the model interferes to a great extent with the air flow about spindle and wing tip. The corrections are thus made up more largely of guard interference than spindle resistance. For these reasons results are more erratic particularly at vl. 12.5 where the variation of the disturbed flow with the speed is great. A further difficulty in these tests is the very small magnitude of the drag correction compared with the drag of the model. The corrections have been plotted however to show that the method is applicable even under these exceptional conditions.

The spindle was located 1″ from the leading edge which on a 4.2″ chord is well forward.

The corrections are the results of single determinations only.

Lift Correction

For these conditions a single equation agrees with the lines for the aerofoils at vl. 10 while the variation for the different aerofoils at vl. 7.5 is slight. Guard interference renders the results at vl. 12.5 uncertain and prohibits the drawing of conclusions. The slope of the lines is less than for the previous spindles as would be expected due to the smaller exposed spindle area.

Drag Corrections

For vls. 7.5 and 10.0 the results agree very well with those already considered. A single equation is applicable to all three aerofoils at vl. 10 and the variation of the constant in the equations for vl. 7.5 is but little. The slope of the lines is, as in the previous cases, one-tenth the slope of those of lift correction, and the constant approximately proportional to v^2. The actual and calculated values of the constant are tabulated in table 3. Guard interference renders the results at vl. 12.5 of little value.

Conclusions and Recommendations

General average equations for end spindle corrections applicable with considerable accuracy to any standard aerofoil model, 3″×18″ fitted with spindle and guard as herein described and for vl's between 7.5 and 12.5 are:

$$C_L = 0.12L$$

$$C_D = 0.0057\left[\frac{V}{40}\right]^2 + 0.0012L.$$

That is the spindle accounts for 1.2% of the lift or the measured lift of the aerofoil multiplied by 0.988 gives the true no spindle lift. Thus, the lift multiplied by a single constant (product of usual constant taking care of area, speed and density and 0.988) gives the coefficient corrected for spindle resistance. If an approximate drag correction only is required it may be taken as 0.0057 at vl. 10.

For spindle or guard differing from that of these tests a relatively small number of correction determinations will serve to define the position and slope of lines and hence determine the proper equations for the corrections.

If greater accuracy is necessary than the above permit of or for aerofoils of unusual section, using the standard spindle a determination of the corrections at a single angle will define the position of the line the slope being as here given. The measurement should preferably be made at the angle of maximum L/D for greatest accuracy in making the measurement and to determine the maximum L/D with greatest precision. Several check readings should be made at this angle.

The same accuracy is usually not required in the characteristics above the critical angle and hence corrections there are usually of little interest. Corrections above critical angle, on lift may be taken as zero and on drag as being constant and equal to correction at the critical. As a matter of fact this agrees with the majority of corrections as actually found in these tests.

Aeronautical Research Paper No. 12
UNIVERSITY OF TORONTO

END SPINDLE CORRECTIONS.

PLATE- I.

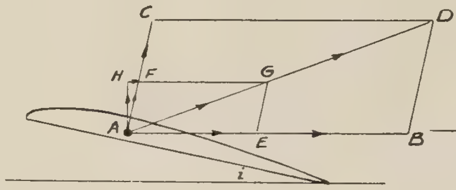


Fig. 1.
Diagram of End Spindle Effects.

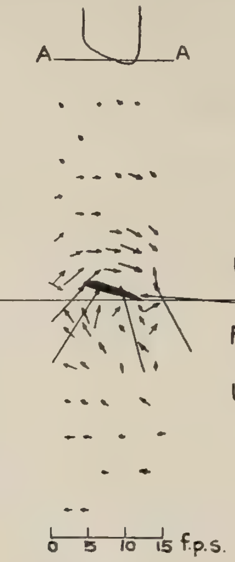


Fig. 2.
Flow Round a Wing Tip.
Components in plane A-A
R&M. 197.

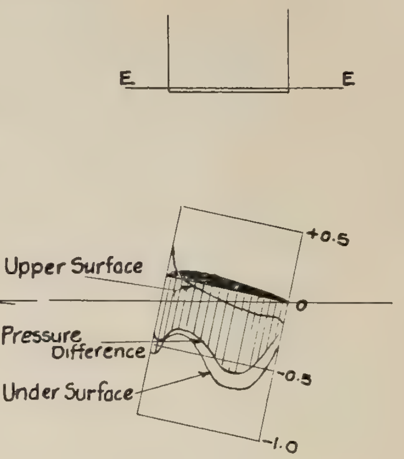


Fig. 3.
Pressures at a Wing Tip.
(At section E-E)
R&M-73

Table No. 1.

POSITION OF CENTRES OF PRESSURE OF SPINDLES
(In inches from end of aerofoil model.)

Aerofoil Section	1.8 chord.	3.0 chord		
	vl. 7.5	vl. 7.5	vl. 10.0	vl. 12.5
NPLAS-5	3.84	1.89	1.62	1.37
USATS-5	3.60	1.34	1.28	1.11
USATS-10	3.67	1.37	1.27	1.14
Centre of Area.	3.36	1.25	1.25	1.25

Table No. 2

END SPINDLE DIMENSIONS

Model.			
chord	1.8	3.0**	4.2
span	10.4	18.0	25.2
Spindle.			
diameter d_1	3/16	3/16	5/16
diameter d_2	5/16*	5/16*	5/16
length-from end of model. L	9.8	6.0	4.4
length-unshielded. l_1	6.11	2.31	0.71
area-projected.	1.49	0.55	0.22
centre of area to end of model.-l_2	3.37	1.25	0.36

*Large end of uniform dia., 5/16, for a length of 3"
**This spindle also used for tapered aerofoils.

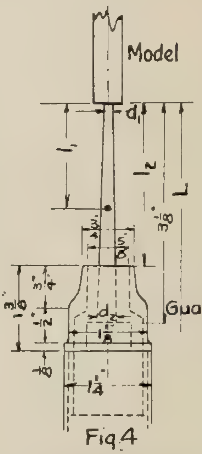
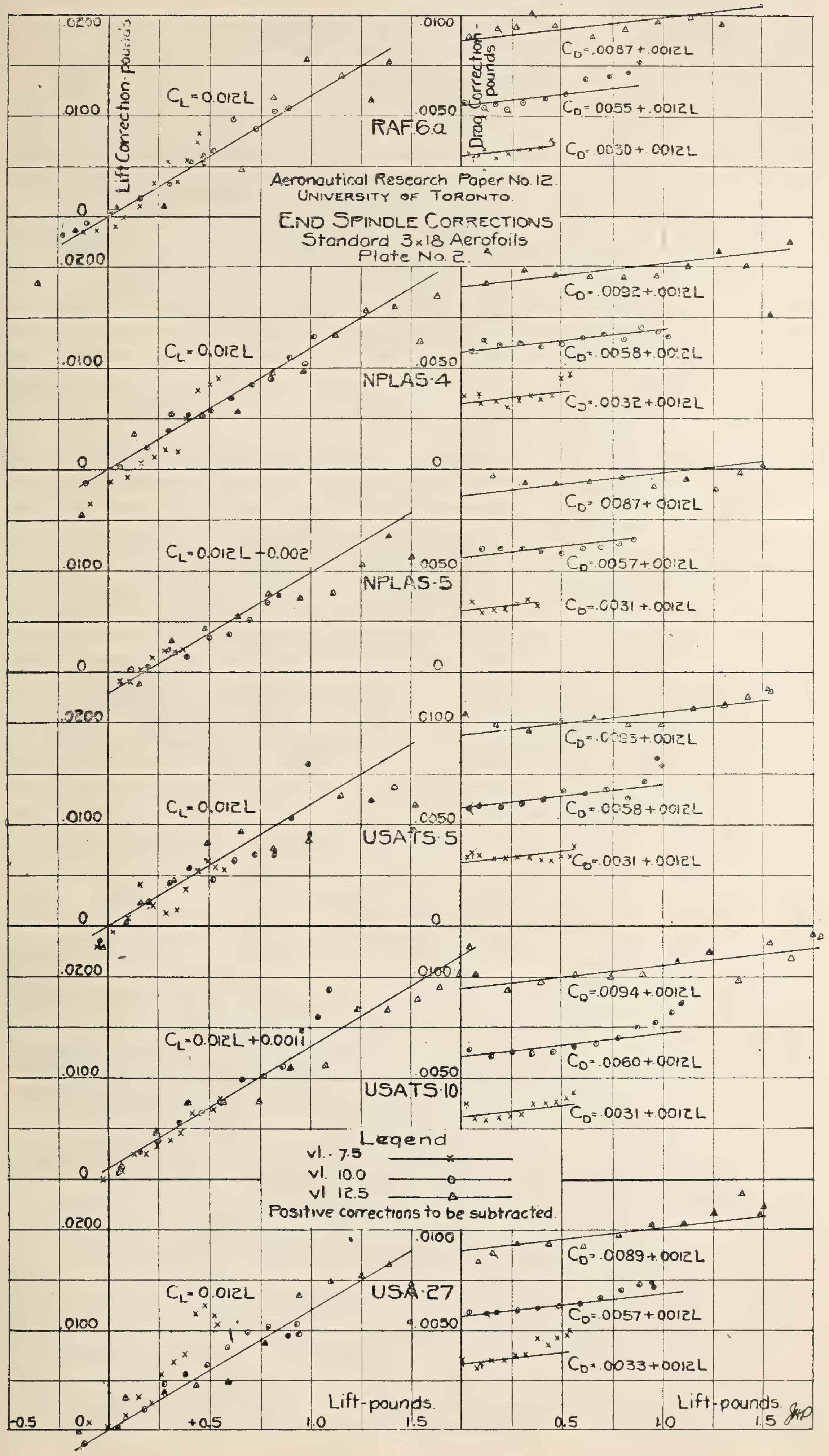


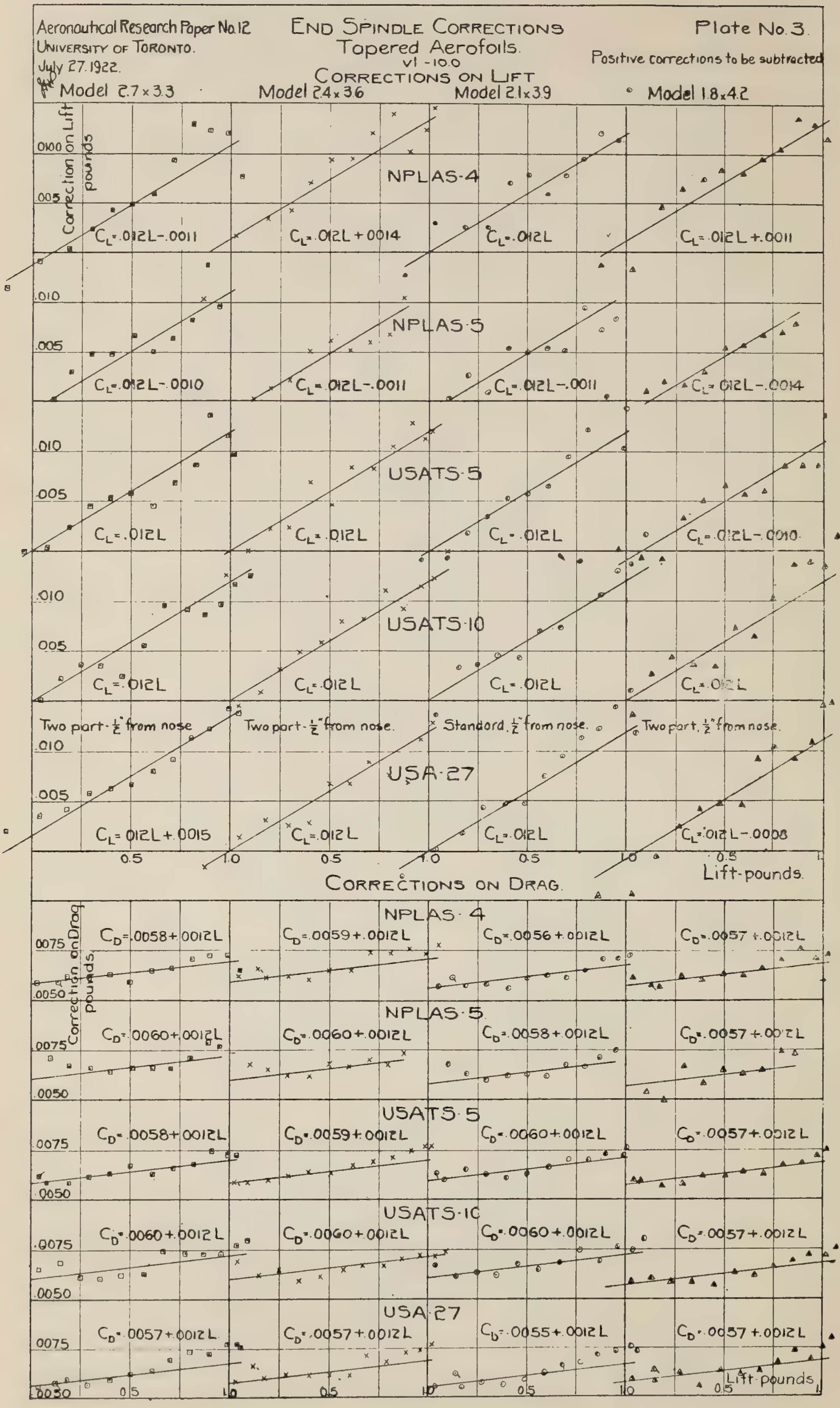
Fig 4
Spindle and Guard.

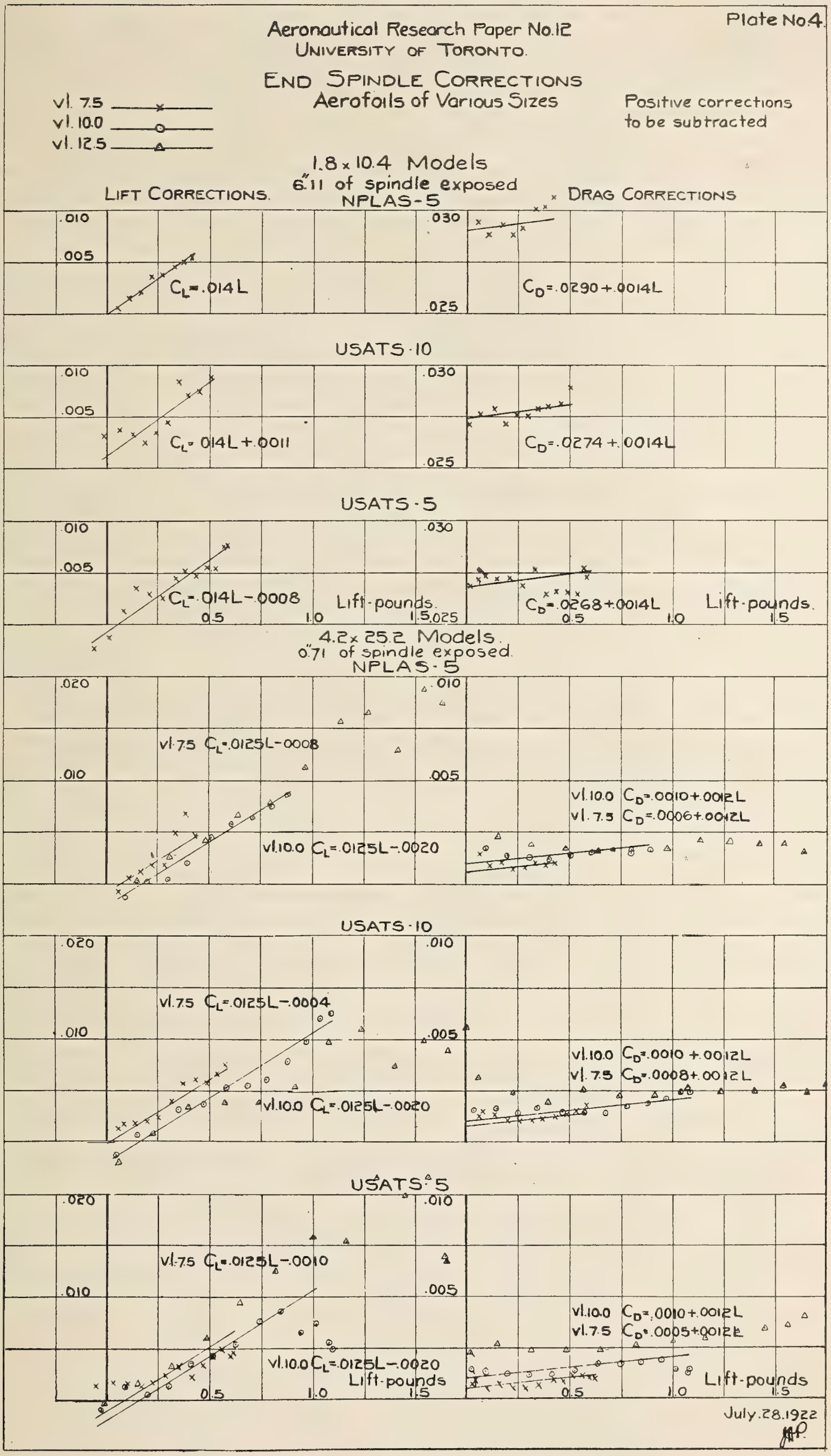
Table No. 3.

COMPARISON OF MEASURED AND CALCULATED CONSTANTS IN EQUATION $C_D=f'(L)+k$ FOR SPINDLE CORRECTION ON DRAG.

Aerofoil	Chord	vl. 12.5		vl. 10.0		vl. 7.5	
		Measured	Calculated	Measured	Calculated	Measured	Calculated
RAF 6a	3	.0087	.0086	.0055	.0055	.0030	.0031
NPLAS-4	3	.0092	.0091	.0058	.0058	.0032	.0033
NPLAS-5	3	.0087	.0089	.0057	.0057	.0031	.0032
USATS-5	3	.0093	.0091	.0058	.0058	.0031	.0033
USATS-10	3	.0094	.0094	.0060	.0060	.0031	.0034
USA 27	3	.0089	.0089	.0057	.0057	.0033	.0032
NPLAS-5	4.2			.0010	.0010	.0006	.0006
USATS-5	4.2			.0010	.0010	.0008	.0006
USATS-10	4.2			.0010	.0010	.0005	.0006


Aeronautical Research Paper No. 12.
UNIVERSITY OF TORONTO.
END SPINDLE CORRECTIONS
Standard 3×18 Aerofoils
Plate No. 2.
Lift Correction-pounds
Drag Correction-pounds
RAF 6a
$C_L = 0.012L$
$C_D = .0087 + .0012L$
$C_D = .0055 + .0012L$
$C_D = .0030 + .0012L$
NPLAS-4
$C_L = 0.012L$
$C_D = .0092 + .0012L$
$C_D = .0058 + .0012L$
$C_D = .0032 + .0012L$
NPLAS-5
$C_L = 0.012L - 0.002$
$C_D = .0087 + .0012L$
$C_D = .0057 + .0012L$
$C_D = .0031 + .0012L$
USATS-5
$C_L = 0.012L$
$C_D = .0095 + .0012L$
$C_D = .0058 + .0012L$
$C_D = .0031 + .0012L$
USATS-10
$C_L = 0.012L + 0.0011$
$C_D = .0094 + .0012L$
$C_D = .0060 + .0012L$
$C_D = .0031 + .0012L$
USA-27
$C_L = 0.012L$
$C_D = .0089 + .0012L$
$C_D = .0057 + .0012L$
$C_D = .0033 + .0012L$
Legend
vl. 7.5 ×
vl. 10.0 ○
vl 12.5 △
Positive corrections to be subtracted.
.0200
.0100
0
.0100
.0050
0
-0.5
0
+0.5
1.0
1.5
0.5
1.0
1.5
Lift-pounds.
Lift-pounds.
JHP


Aeronautical Research Paper No. 12
University of Toronto.
July 27. 1922.
End Spindle Corrections
Tapered Aerofoils.
vl -10.0
Corrections on Lift
Plate No. 3.
Positive corrections to be subtracted
Model 2.7 × 3.3
Model 2.4 × 3.6
Model 2.1 × 3.9
Model 1.8 × 4.2
Correction on Lift pounds
NPLAS-4
C_L = .012L − .0011
C_L = .012L + .0014
C_L = .012L
C_L = .012L + .0011
NPLAS-5
C_L = .012L − .0010
C_L = .012L − .0011
C_L = .012L − .0011
C_L = .012L − .0014
USATS-5
C_L = .012L
C_L = .012L
C_L = .012L
C_L = .012L − .0010
USATS-10
C_L = .012L
C_L = .012L
C_L = .012L
C_L = .012L
Two part - ½" from nose
Two part - ½" from nose.
Standard, ½" from nose.
Two part, ½" from nose.
USA-27
C_L = .012L + .0015
C_L = .012L
C_L = .012L
C_L = .012L − .0008
.0100
.005
.010
0.5
1.0
Lift-pounds.
Corrections on Drag.
Correction on Drag pounds.
NPLAS-4
C_D = .0058 + .0012L
C_D = .0059 + .0012L
C_D = .0056 + .0012L
C_D = .0057 + .0012L
NPLAS-5
C_D = .0060 + .0012L
C_D = .0060 + .0012L
C_D = .0058 + .0012L
C_D = .0057 + .0012L
USATS-5
C_D = .0058 + .0012L
C_D = .0059 + .0012L
C_D = .0060 + .0012L
C_D = .0057 + .0012L
USATS-10
C_D = .0060 + .0012L
C_D = .0060 + .0012L
C_D = .0060 + .0012L
C_D = .0057 + .0012L
USA-27
C_D = .0057 + .0012L
C_D = .0057 + .0012L
C_D = .0055 + .0012L
C_D = .0057 + .0012L
.0075
.0050
0.5
1.0
Lift-pounds


Aeronautical Research Paper No.12
UNIVERSITY OF TORONTO.
Plate No.4.
END SPINDLE CORRECTIONS
Aerofoils of Various Sizes
Positive corrections to be subtracted
vl. 7.5 —×—
vl. 10.0 —○—
vl. 12.5 —△—
1.8 x 10.4 Models
6.11" of spindle exposed
LIFT CORRECTIONS.
DRAG CORRECTIONS
NPLAS-5
.010
.005
C_L=.014L
.030
.025
C_D=.0290+.0014L
USATS-10
.010
.005
C_L=.014L+.0011
.030
.025
C_D=.0274+.0014L
USATS-5
.010
.005
C_L=.014L-.0008
Lift-pounds.
0.5
1.0
1.5
.030
.025
C_D=.0268+.0014L
Lift-pounds.
0.5
1.0
1.5
4.2 x 25.2 Models.
0.71" of spindle exposed.
NPLAS-5
.020
.010
vl.7.5 C_L=.0125L-.0008
vl.10.0 C_L=.0125L-.0020
.010
.005
vl.10.0 C_D=.0010+.0012L
vl.7.5 C_D=.0006+.0012L
USATS-10
.020
.010
vl.7.5 C_L=.0125L-.0004
vl.10.0 C_L=.0125L-.0020
.010
.005
vl.10.0 C_D=.0010+.0012L
vl.7.5 C_D=.0008+.0012L
USATS-5
.020
.010
vl.7.5 C_L=.0125L-.0010
vl.10.0 C_L=.0125L-.0020
Lift-pounds
0.5
1.0
1.5
.010
.005
vl.10.0 C_D=.0010+.0012L
vl.7.5 C_D=.0005+.0012L
Lift-pounds
0.5
1.0
1.5
July.28.1922

Aeronautical Research Paper No. 13.
August 1922.

INVESTIGATION INTO THE EFFECT OF CERTAIN VARIATIONS IN THE LEADING PORTION OF A THICK AEROFOIL

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H. C. Crane, and J. S. E. MacAllister, *Research Assistants*

Introduction.

This research was instituted to gain some information as to the results which would be obtained by varying in a regular manner the forward part of a thick section. A special rear contour was used having a thickness to chord ratio of 15 per cent., and a rear spar depth of 12 per cent. at 70 per cent. of chord from the leading edge. The section is therefore quite thick and has an exceptionally deep rear spar. The ordinates for all sections are given in Table I, and the sections are shown on Plates 10 to 13. Some conception of the thickness may be gained by an inspection of Plate 13, showing *TTS*-10*D** and *TTS*-13 superimposed on *RAF*-6, *NPLAS*-4, *USATS*-5, and *USA*-27.

Sections tested.

The sections tested fall into three series as follows:

Series I, Flat undersurface. As may be seen from Table I, giving the ordinates, or from Plate 10 showing the sections in the series, the series consists of a group of eight aerofoils all having flat undersurfaces and the same rear upper contours, but having noses progressing in thickness from *TTS*-1, having the leading third similar to the rear two-thirds, to *TTS*-5 which has a very blunt nose.

Series II, Elliptical noses. The sections in this series have not flat undersurfaces, but have noses of elliptical form (as far as practicable) with the elevation of the leading edge varied by steps of $2\frac{1}{2}\%$ of chord from an elevation of $7\frac{1}{2}\%$ (or half the maximum thickness, giving a symmetrical nose) to 5% below the datum line.

Series III, Variation of undercamber. This series is a study of undercamber. Several upper contours were selected and various lower contours were tried with each. In the designation of a section the number denotes the upper contour and the letter the lower contour. Series III may be divided into groups as follows:

(1) including *TTS*-2, 2*A*, 2*B*, 2*C*.
(2) " " -3, 3*A*, 3*B*.
(3) " " -13, 13*C*, 13*H*.
(4) " " -10*D*, 10*F*, 10*J*.;
(5) " " 10*E*, 10*L*, 10*K*.;

**TTS*—Toronto Thick Section.

CONDITIONS OF TEST.

The models were constructed of aluminum and wax as described in the appendix. They were of 3″ chord and 18″ span. All tests were made at 40 f.p.s. (*VL* 10) in the No. 1, Four Foot N.P.L. Type Wind Channel of the University of Toronto, for lift, drag, and centre of pressure.

RESULTS.

The characteristics of the sections have been calculated (corrected for spindle interference as described in A.R.P. No. 12), tabulated in tables I to XVI, and plotted on Plates 1 to 7. The characteristics of the sections with regard to landing speed, cruising speed, etc., have been tabulated in Table XVII.

DISCUSSION OF RESULTS.

Series I, Flat undersurface, TTS-1, 12, 13, 2, 14, 3, 4, 5. The results are given in Tables II to V, and the principal characteristics have been plotted on Plate 8, which shows the variation of maximum lift coefficient, maximum lift to drag ratio, lift to drag ratios at *Cy* max/4, at *Cy* max/6.25, and at *Cy* max/9, and centre of pressure travel between the maximum forward position and that at a lift coefficient one-fourth the maximum, all plotted against average thickness of the leading third of the section in per cent. of chord.

The curves show that the principal characteristics as plotted all reach their optimum values for an average ordinate of about 11.65, or in the region of *TTS*-13. These optimum values are fairly well marked in all except the maximum *L/D* curve, the maximum *L/D* being very nearly constant for *TTS*-1, 12, 13, and 2.

Taking the peak of the *Cy* maximum curve as determining the best section of the series, that section will have the following characteristics:

Cy maximum	.797
L/D maximum	13.13
L/D at *Cy* max/4	6.43
L/D at *Cy* max/6.25	3.85
L/D at *Cy* max/9	2.58
C.P. travel between maximum forward position and that at *Cy* max/4	26.5%
Mean spar depth (average of that at 15% and at 60%)	13.05%

Series II, Elliptical noses, TTS-6, 7, 8, 2, 10E, 11G. In this series are six sections with the leading third formed by two ellipses (as far as practicable) with the leading edges at varying elevations above the datum line, as follows:

TTS- 6, leading edge 7½% above datum (nose symmetrical)
7, " " 5% " "
8, " " 2½% " "
2, " " on datum (flat undersurface)
10*E*, " " 2½% below datum
11*G*, " " 5% " "

The characteristic curves for the sections in the series are given on Plate 3, and the principal characteristics have been plotted on Plate 9, against elevation of the leading edge above the datum line. Starting with *TTS*-11*G* with leading edge 5% below datum, the maximum lift coefficient rises rapidly and reaches a maximum of .815 for section 10*E* with leading edge 2½% below datum and then slowly falls off as the leading edge is raised, till with the leading edge 7½% above datum (giving a symmetrical nose), the lift coefficient has fallen to .534.

The maximum *L/D* curve rises rapidly as the nose is raised till section *TTS*-2 (flat undersurface) is reached, after which the maximum *L/D* remains practically constant at 12.88.

The curves of *L/D* at *Cy* max/4, at *Cy* max/6.25, and at *Cy* max/9 are all quite similar and reach their maxima for a leading edge about 3½% above the datum line.

The centre of pressure travel between the maximum forward position and that for *Cy* max/4 appears in this case to be a linear function of the elevation of the leading edge. It drops from about 28% for *TTS*-11*G* to 17% for *TTS*-6.

From Table 17 it may be seen that the speed range and climb also reach their maxima for *TTS*-8 with leading edge 2½% above the datum line.

It would appear that in this series the high maximum lift coefficient obtained by depressing the leading edge 2½% is obtained with a loss in practically all other characteristics. The optimum section in the series is apparently *TTS*-8, with the following characteristics:

Cy maximum	.750
L/D maximum	12.90
L/D at *Cy* max/4	7.70
L/D at *Cy* max/6.25	5.24
L/D at *Cy* max/9	3.50
C.P. travel	22.0%
Mean spar depth	13.05%

Series III (*1*), *TTS-2, 2A, 2B, 2C.* These sections all having the same upper contour, show the effect of certain changes in the forward lower contour. Undercambers *A* and *C* have a depth of 2½% (1/6 of maximum upper contour ordinate), while undercamber *B* has a depth of 5% of chord. All four maximum lift coefficients lie between .773 and .790, a range of only 2.2%. *TTS*-2, 2*A*, and 2*C* have maximum *L/D's* of 12.83, 12.72, and 12.90 respectively while the heavily undercambered 2*B* has a maximum *L/D* of only 11.83. At lift coefficients below the maximum *L/D* the order of merit is 2*A*, 2*C*, 2, 2*B*. With regard to C.P. travel the order of merit is 2, 2*A*, 2*C*, 2*B*.

Series III (2), TTS-3, 3A, 3B. Because of the low lift coefficient the results for this series were not plotted.

Series III (3), TTS-13, 13C, 13H. Compared with *TTS*-13, 13*C* and 13*H* gave lower maximum lift coefficients. 13 and 13*C* have about the same maximum *L/D* ratios and have quite similar characteristics in general, while 13*H* is markedly inferior in all respects.

The use of a moderate undercamber results in Series III, (1) and (3), in a slight reduction in maximum lift coefficient and in maximum *L/D*, but greatly increases the forward spar depth.

Series III (4), TTS-10D, 10F, 10J. This series shows the remarkable effect which small changes in the leading 15 to 20% of the under contour may cause. These sections have exactly the same upper contour and practically the same under contour except for the leading 20% of the chord. The *L/D* curve for *TTS-10D* completely envelopes that for *TTS*-10*F*, which in turn envelopes that for *TTS*-10*J*. At *Cy* max/4 the *L/D's* are 7.78, 5.82, and 4.67 respectively. However, with regard to C.P. travel the order of merit is reversed. Except for C.P. travel *TTS*-10*D* is a good "thick section", having the following characteristics:

Cy maximum	.817
L/D maximum	12.13
L/D at *Cy* max/4	7.78
L/D at *Cy* max/6.25	4.31
L/D at *Cy* max/9	2.44
C.P. travel	32.6%
Mean spar depth	15.2%

Although the maximum *L/D* is rather low, in the case of cantilever construction where full utilization of the great spar depth could be made, this section would probably prove excellent.

Series III (5), TTS-10E, 10L, 10K. Of this group *TTS*-10*E* is undoubtedly the best aerodynamically, but it has not as good a forward spar depth as the others. *TTS*-10*L* and *TTS*-10-*K* have about the same *L/D*'s below a lift coefficient of .15 and about the same maximum *L/D*. Between a lift coefficient of .15 and that for maximum *L/D*, *TTS*-10*K* is much better than *TTS*-10*L*, but it has a lower maximum lift coefficient, and a greater C.P. travel.

In general the results for Series III, under camber variation, are not as instructive as those for the flat undersurface and the elliptical series. One reason probably is that in trying to develope higher maximum lift coefficients attention was not directed toward a systematic variation of undersurface. No general conclusions may be drawn from this series but it is hoped that the fragmentary evidences here presented may be of benefit to the designer.

APPENDIX I

ADVANCES IN TEMPLATE MAKING.

The templates were for the most part produced by the photographic methods described in A.R.P.-8. However, in this case a saving was effected by drawing several curves on one plate. The negative obtained from this drawing was used to make the required number of prints on steel. The template maker then took the several blanks and cut each to the required contour line. In this manner several templates were produced from a single negative.

The chief innovation, however, was that of cutting templates to contours traced from other templates with a scriber and modified slightly when filing. The research described in this report was carried out under great pressure due to lack of time, as the space taken up in the hydraulic laboratory by the wind channel was required for new hydraulic equipment, and a definite date had been set for the removal of the wind channel for whose housing no provision had been made. Consequently when a model gave promising results, instead of waiting to make a drawing and have the template produced from it, resort was had to the expedient of making a modified template by hand. Upper template 11 and lower template *C*, *D*, *E*, *F*. *G*, *H*, *J*, *K*, and *L* were produced in this manner. The ordinates were later obtained by placing the template in a "thread projector" and tracing the magnified image on the screen. This method proved very satisfactory indeed.

APPENDIX II

ALUMINUM AND WAX MODELS.

The tests described in this report were all run on models constructed of aluminum and wax by a method developed in the Toronto laboratory.

In the case of Series I, flat undersurface, the aluminum model was such as would ordinarily be used, except that the leading third of the upper surface was cut low, and the various noses produced by painting on ordinary melted beeswax and removing the surplus beeswax with a heated template cut from steel and without rake. It was only necessary to have the leading third of the template and the trailing edge cut accurately as the rear two-thirds of the model had been previously finished to a master template. The model was clamped on a surface plate before painting with beeswax. The template was screwed to a brass block with wooden handles, before heating, as it was found that there was not sufficient metal in the template to hold the temperature while the template was being drawn over the model. The action of the template is a combination of cutting and melting. If the proper temperature is not used, or if the template is drawn too quickly or too slowly, poor models are obtained. If the temperature is too high, or if the speed is too low, holes are liable to result; while if the temperature is too low, or the speed too high, tearing occurs. However after a few trials the speed can be adjusted according to the temperature of the template and good models cut within a considerable range of temperatures.

For the elliptical nosed models two-part templates were employed. The procedure outlined above was used with the upper part of the template for the upper surface. The model was then turned over and the rear portion of the flat under surface and the trailing edge served to guide the other half of the template while the under surface was being cut.

In cutting models with the leading lower third below the datum, that portion was allowed to overhang the edge of the surface plate.

This method of producing models was found to work exceedingly well. It requires only common beeswax and a template cut without rake. The models were found to be quite accurate and to have a good smooth hard finish when cold. The advantages in the way of saving in time, and consequent low cost are perfectly obvious.

Aug. 1922.

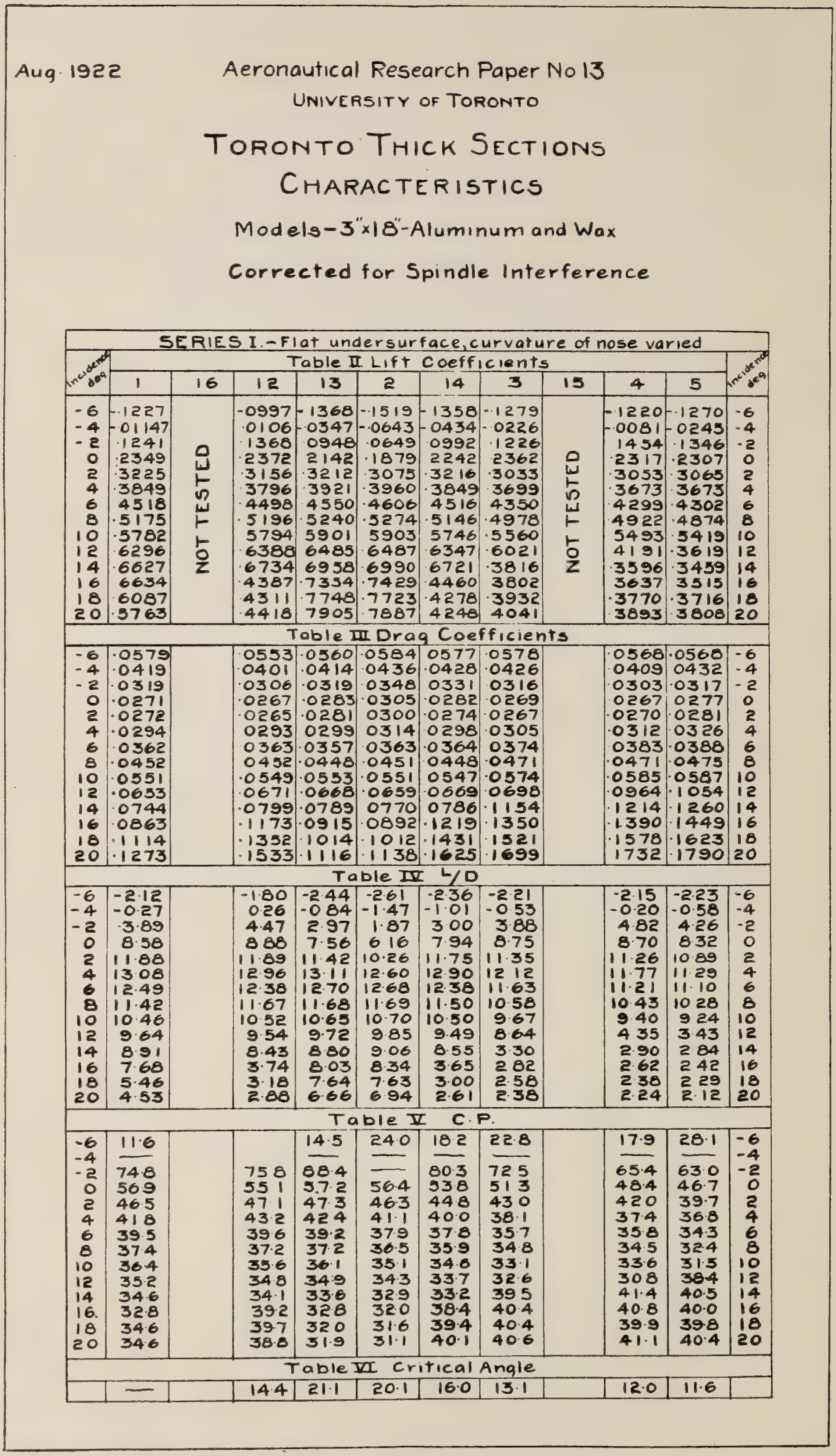
Aeronautical Research Paper No 13.

UNIVERSITY of TORONTO

ORDINATES OF TORONTO THICK SECTIONS

Table I.

	% of chord	SERIES-I — Flat Undersurface-curvature of nose varied										SERIES II-Elliptical.				SERIES-III Under camber variation										% of chord
		1	16	12	13	2A	14	15	3B	4	5	6	7	8	2	10M	10D	10F	10J	10E	10L	10K	11G	13C	13H	
UPPER SURFACE	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.50	5.00	2.50	0.00	-2.50	-2.50	-2.50	-2.50	-2.50	-2.50	-2.50	-5.00	0.00	0.00	0
	1¼	1.60	2.13	2.76	3.33	4.13	4.06	4.73	5.13	5.67	6.67	9.67	7.86	5.98	4.13	2.37	2.37	2.37	2.37	2.37	2.37	2.37	-1.74	3.33	3.33	1¼
	2½	2.98	3.65	4.33	4.99	5.77	6.03	6.52	7.03	7.76	8.99	10.45	8.89	7.38	5.77	4.30	4.30	4.30	4.30	4.30	4.30	4.30	+1.33	4.99	4.99	2½
	5	5.45	6.07	6.77	7.34	7.98	8.45	8.99	9.66	10.40	11.30	11.51	10.30	9.20	7.98	6.81	6.81	6.81	6.81	6.81	6.81	6.81	4.45	7.34	7.34	5
	7½	7.51	8.47	8.62	9.05	9.47	10.13	10.71	11.37	11.98	12.59	12.31	11.39	10.51	9.47	8.71	8.71	8.71	8.71	8.71	8.71	8.71	6.77	9.05	9.05	7½
	10	9.25	9.65	10.04	10.37	10.73	11.40	11.95	12.53	13.03	13.39	12.87	12.14	11.45	10.73	10.07	10.07	10.07	10.07	10.07	10.07	10.07	8.60	10.37	10.37	10
	15	11.89	12.05	12.26	12.43	12.64	13.19	13.66	13.93	14.19	14.33	13.79	13.39	12.97	12.64	12.15	12.15	12.15	12.15	12.15	12.15	12.15	11.33	12.43	12.43	15
	20	13.51	13.54	13.67	13.74	13.80	14.12	14.39	14.58	14.51	14.53	14.43	14.20	13.95	13.80	13.53	13.53	13.53	13.53	13.53	13.53	13.53	13.10	13.74	13.74	20
	30	14.84	14.86	14.88	14.89	14.90	14.91	14.93	14.95	14.97	14.97	14.99	14.93	14.90	14.90	14.81	14.81	14.81	14.81	14.81	14.81	14.81	14.87	14.89	14.89	30
	40	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	14.90	40
	50	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	14.41	50
	60	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	13.58	60
	70	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	70
	80	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	9.30	80
	90	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	90
	95	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	95
	100	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	100
LOWER SURFACE	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.50	5.00	2.50	0.00	-2.50	-2.50	-2.50	-2.50	-2.50	-2.50	-2.50	-5.00	0.00	0.00	0
	1¼	"	"	"	"	-1.20	"	"	-2.34	"	"	5.47	3.66	1.73	"	-3.01	-4.20	-5.08	-5.70	-2.23	-3.70	-5.20	-7.10	-1.90	-2.83	1¼
	2½	"	"	"	"	-1.62	"	"	-3.21	"	"	4.67	3.12	1.46	"	-4.14	-4.60	-5.16	-6.33	-2.10	-4.30	-5.70	-7.40	-2.28	-3.23	2½
	5	"	"	"	"	-2.18	"	"	-4.33	"	"	3.61	2.56	1.19	"	-4.67	-4.96	-5.94	-6.50	-1.80	-4.17	-5.94	-7.33	-2.47	-3.40	5
	7½	"	"	"	"	-2.40	"	"	-4.81	"	"	2.86	1.88	0.95	"	-4.91	-5.10	-5.76	-6.24	-1.50	-3.78	-5.89	-7.18	-2.46	-3.27	7½
	10	"	"	"	"	-2.45	"	"	-4.94	"	"	2.21	1.47	0.73	"	-4.94	-5.12	-5.38	-5.73	-1.30	-3.47	-5.75	-7.00	-2.33	-3.03	10
	15	"	"	"	"	-2.27	"	"	-4.62	"	"	1.37	0.91	0.45	"	-4.62	-4.67	-4.67	-5.22	-0.88	-2.83	-5.18	-5.50	-2.08	-2.42	15
	20	"	"	"	"	-1.75	"	"	-3.90	"	"	0.73	0.47	0.25	"	-3.87	-3.83	-3.90	-3.90	-0.48	-2.18	-4.49	-4.10	-1.67	-1.80	20
	30	"	"	"	"	-0.33	"	"	-1.71	"	"	0.13	0.12	0.07	"	-1.71	-2.15	-2.27	-2.25	-0.07	-1.10	-3.15	-1.90	-0.75	-0.88	30
	40	"	"	"	"	0.00	"	"	0.00	"	"	0.00	0.00	0.00	"	0.00	-0.75	-0.83	-0.87	0.00	-0.47	-1.94	-0.63	-0.17	-0.26	40
	50	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	0.00	-0.09	-0.13	"	0.00	-1.03	-0.20	0.00	0.00	50
	60	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	0.00	0.00	"	"	-0.42	0.00	"	"	60
	70	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	-0.08	"	"	"	70
	80	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	0.00	"	"	"	80
	90	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	90
	95	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	95
	100	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	100



Aug· 1922

Aeronautical Research Paper No 13

UNIVERSITY OF TORONTO

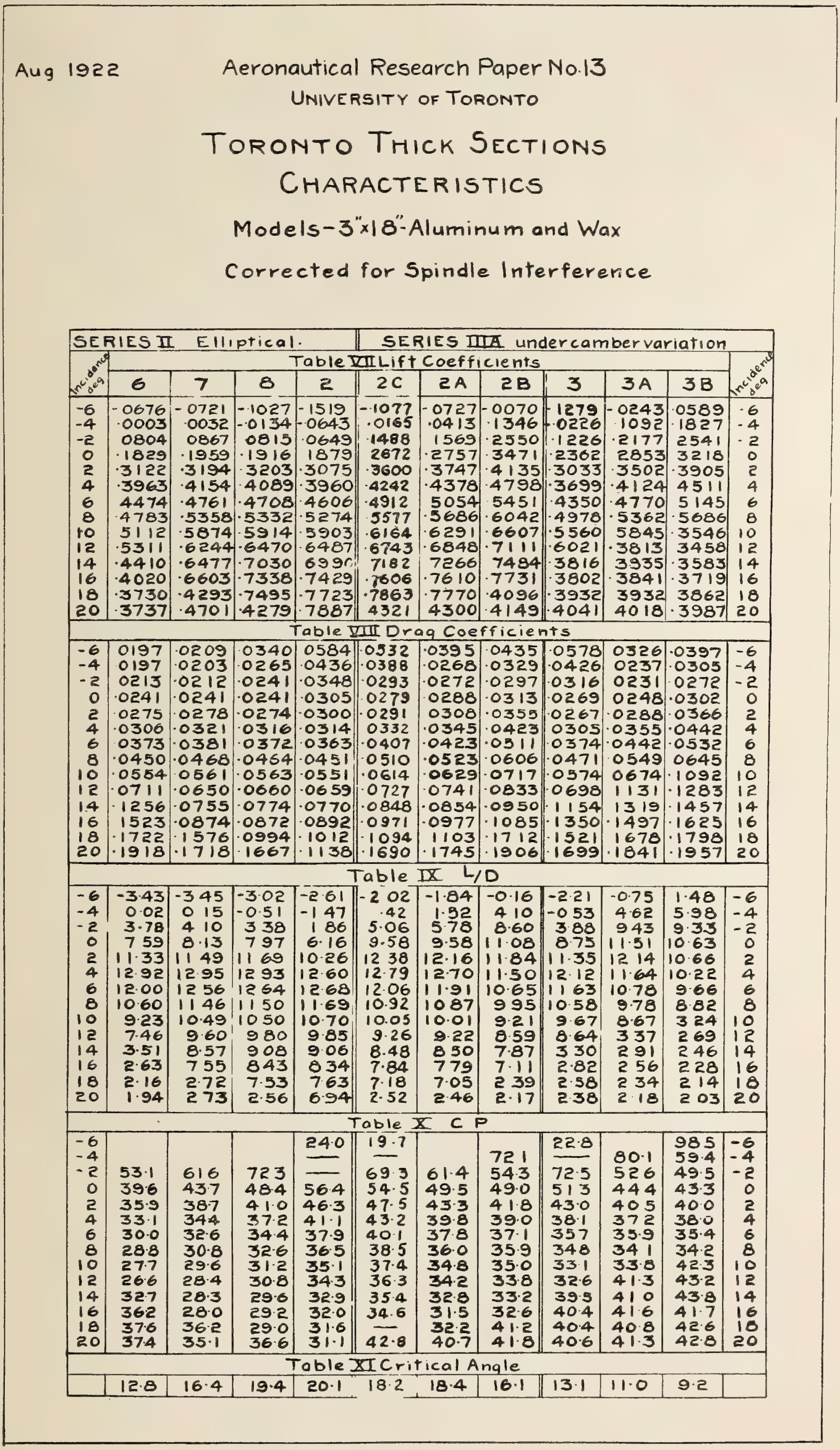
TORONTO THICK SECTIONS CHARACTERISTICS

Models–3″×18″–Aluminum and Wax

Corrected for Spindle Interference

SERIES I.–Flat undersurface, curvature of nose varied

Incidence deg	1	16	12	13	2	14	3	15	4	5	Incidence deg
	Table II Lift Coefficients										
-6	-·1227	NOT TESTED	-·0997	-·1368	-·1519	-·1358	-·1279	NOT TESTED	-·1220	-·1270	-6
-4	-·01147		·0106	-·0347	-·0643	-·0434	-·0226		-·0081	-·0245	-4
-2	·1241		·1368	·0948	·0649	·0992	·1226		·1454	·1346	-2
0	·2349		·2372	·2142	·1879	·2242	·2362		·2317	·2307	0
2	·3225		·3156	·3212	·3075	·3216	·3033		·3053	·3065	2
4	·3849		·3796	·3921	·3960	·3849	·3699		·3673	·3673	4
6	·4518		·4498	·4550	·4606	·4516	·4350		·4299	·4302	6
8	·5175		·5196	·5240	·5274	·5146	·4978		·4922	·4874	8
10	·5782		·5794	·5901	·5903	·5746	·5560		·5493	·5419	10
12	·6296		·6388	·6485	·6487	·6347	·6021		·4191	·3619	12
14	·6627		·6734	·6958	·6990	·6721	·3816		·3596	·3459	14
16	·6634		·4387	·7334	·7429	·4460	·3802		·3637	·3515	16
18	·6087		·4311	·7748	·7723	·4278	·3932		·3770	·3716	18
20	·5763		·4418	·7905	·7887	·4248	·4041		·3893	·3808	20
	Table III Drag Coefficients										
-6	·0579		·0553	·0560	·0584	·0577	·0578		·0568	·0568	-6
-4	·0419		·0401	·0414	·0436	·0428	·0426		·0409	·0432	-4
-2	·0319		·0306	·0319	·0348	·0331	·0316		·0303	·0317	-2
0	·0271		·0267	·0283	·0305	·0282	·0269		·0267	·0277	0
2	·0272		·0265	·0281	·0300	·0274	·0267		·0270	·0281	2
4	·0294		·0293	·0299	·0314	·0298	·0305		·0312	·0326	4
6	·0362		·0363	·0357	·0363	·0364	·0374		·0383	·0388	6
8	·0452		·0452	·0448	·0451	·0448	·0471		·0471	·0475	8
10	·0551		·0549	·0553	·0551	·0547	·0574		·0585	·0587	10
12	·0653		·0671	·0668	·0659	·0669	·0698		·0964	·1054	12
14	·0744		·0799	·0789	·0770	·0786	·1154		·1214	·1260	14
16	·0863		·1173	·0915	·0892	·1219	·1350		·1390	·1449	16
18	·1114		·1352	·1014	·1012	·1431	·1521		·1578	·1623	18
20	·1273		·1533	·1116	·1138	·1625	·1699		·1732	·1790	20
	Table IV L/D										
-6	-2·12		-1·80	-2·44	-2·61	-2·36	-2·21		-2·15	-2·23	-6
-4	-0·27		0·26	-0·84	-1·47	-1·01	-0·53		-0·20	-0·58	-4
-2	3·89		4·47	2·97	1·87	3·00	3·88		4·82	4·26	-2
0	8·58		8·88	7·56	6·16	7·94	8·75		8·70	8·32	0
2	11·88		11·89	11·42	10·26	11·75	11·35		11·26	10·89	2
4	13·08		12·96	13·11	12·60	12·90	12·12		11·77	11·29	4
6	12·49		12·38	12·70	12·68	12·38	11·63		11·21	11·10	6
8	11·42		11·67	11·68	11·69	11·50	10·58		10·43	10·28	8
10	10·46		10·52	10·65	10·70	10·50	9·67		9·40	9·24	10
12	9·64		9·54	9·72	9·85	9·49	8·64		4·35	3·43	12
14	8·91		8·43	8·80	9·06	8·55	3·30		2·90	2·84	14
16	7·68		3·74	8·03	8·34	3·65	2·82		2·62	2·42	16
18	5·46		3·18	7·64	7·63	3·00	2·58		2·38	2·29	18
20	4·53		2·88	6·66	6·94	2·61	2·38		2·24	2·12	20
	Table V C·P.										
-6	11·6			14·5	24·0	18·2	22·8		17·9	28·1	-6
-4	—			—	—	—	—		—	—	-4
-2	74·8		75·8	88·4	—	80·3	72·5		65·4	63·0	-2
0	56·9		55·1	57·2	56·4	53·8	51·3		48·4	46·7	0
2	46·5		47·1	47·3	46·3	44·8	43·0		42·0	39·7	2
4	41·8		43·2	42·4	41·1	40·0	38·1		37·4	36·8	4
6	39·5		39·6	39·2	37·9	37·8	35·7		35·8	34·3	6
8	37·4		37·2	37·2	36·5	35·9	34·8		34·5	32·4	8
10	36·4		35·6	36·1	35·1	34·6	33·1		33·6	31·5	10
12	35·2		34·8	34·9	34·3	33·7	32·6		30·8	38·4	12
14	34·6		34·1	33·6	32·9	33·2	39·5		41·4	40·5	14
16	32·8		39·2	32·8	32·0	38·4	40·4		40·8	40·0	16
18	34·6		39·7	32·0	31·6	39·4	40·4		39·9	39·8	18
20	34·6		38·8	31·9	31·1	40·1	40·6		41·1	40·4	20
	Table VI Critical Angle										
	—		14·4	21·1	20·1	16·0	13·1		12·0	11·6	



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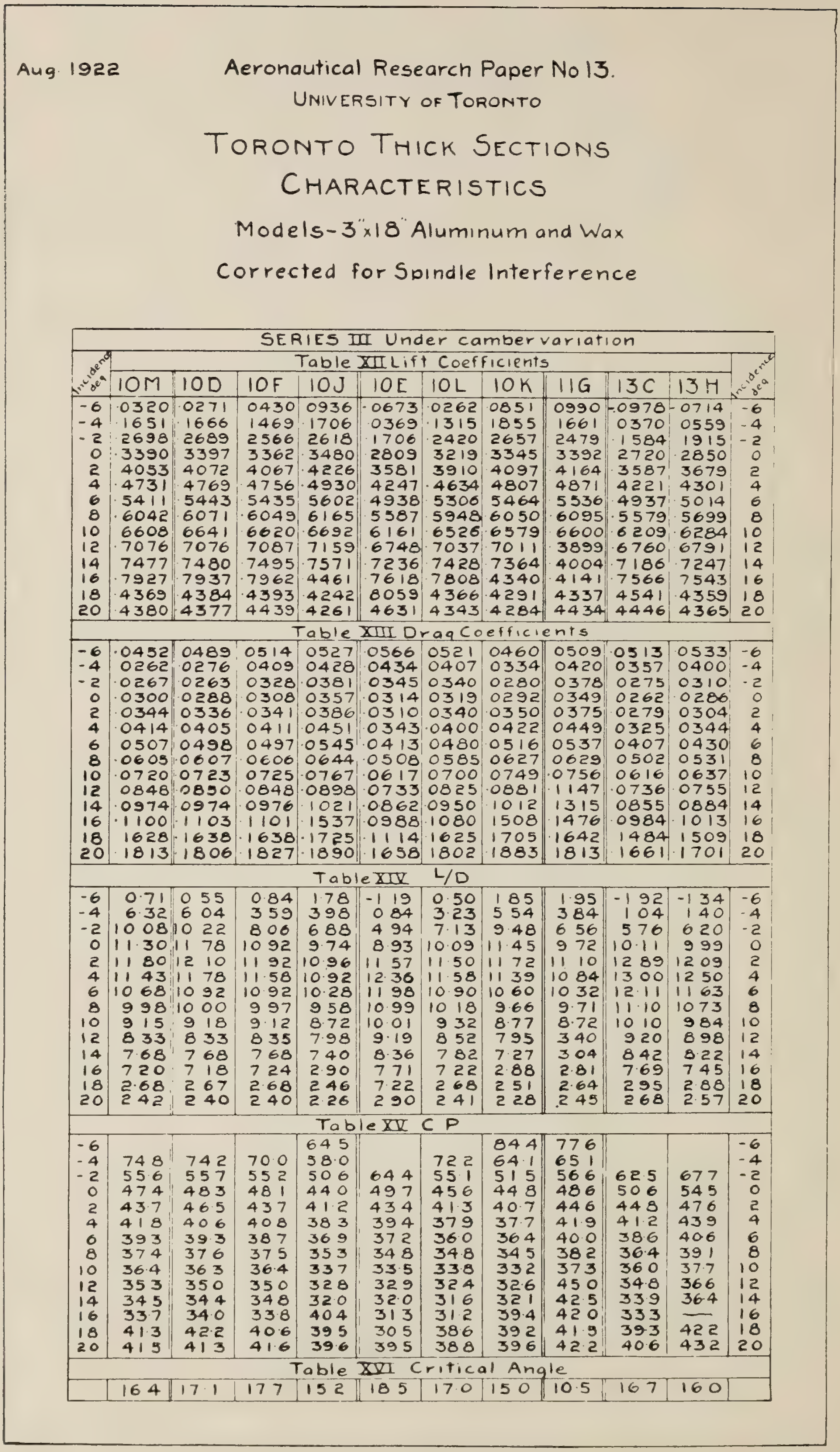
TORONTO THICK SECTIONS CHARACTERISTICS

Models–3"×18"–Aluminum and Wax

Corrected for Spindle Interference

Incidence deg	SERIES II Elliptical·				SERIES IIIA undercamber variation						Incidence deg
	Table VII Lift Coefficients										
	6	7	8	2	2C	2A	2B	3	3A	3B	
-6	-0676	-0721	-·1027	-1519	-·1077	-0727	-0070	-1279	-0243	·0589	-6
-4	·0003	·0032	-·0134	-·0643	·0165	·0413	·1346	-·0226	1092	·1827	-4
-2	0804	0867	0815	·0649	·1488	1569	·2550	·1226	·2177	2541	-2
0	·1829	·1959	·1916	1879	2672	·2757	·3471	·2362	2853	3218	0
2	·3122	·3194	·3203	·3075	·3600	·3747	·4135	·3033	·3502	·3905	2
4	·3963	·4154	·4089	·3960	·4242	·4378	·4798	·3699	·4124	4511	4
6	4474	·4761	·4708	·4606	·4912	5054	5451	·4350	·4770	5145	6
8	·4783	·5358	·5332	·5274	5577	·5686	·6042	·4978	·5362	·5686	8
10	5112	·5874	·5914	·5903	·6164	·6291	·6607	·5560	5845	·3546	10
12	·5311	·6244	·6470	·6487	·6743	·6848	·7111	·6021	·3813	3458	12
14	·4410	·6477	·7030	6990	7182	7266	7484	·3816	3935	·3583	14
16	·4020	·6603	·7338	·7429	·7606	·7610	·7731	·3802	·3841	·3719	16
18	·3730	·4293	·7495	·7723	·7863	·7770	·4096	·3932	3932	3862	18
20	·3737	·4701	·4279	·7887	4321	4300	·4149	·4041	4018	·3987	20
	Table VIII Drag Coefficients										
-6	0197	·0209	·0340	0584	·0532	·0395	·0435	·0578	0326	·0397	-6
-4	0197	·0203	·0265	·0436	·0388	·0268	·0329	·0426	0237	·0305	-4
-2	0213	·0212	·0241	·0348	·0293	·0272	·0297	·0316	0231	·0272	-2
0	·0241	·0241	·0241	·0305	0279	·0288	·0313	·0269	0248	·0302	0
2	·0275	·0278	·0274	·0300	·0291	0308	·0355	·0267	·0288	·0366	2
4	·0306	·0321	·0316	·0314	0332	·0345	·0423	0305	·0355	·0442	4
6	0373	·0381	·0372	·0363	·0407	·0423	·0511	·0374	·0442	·0532	6
8	·0450	·0468	·0464	·0451	·0510	·0523	·0606	·0471	0549	0645	8
10	·0554	·0561	·0563	·0551	·0614	·0629	·0717	·0574	0674	·1092	10
12	·0711	·0650	·0660	·0659	·0727	·0741	·0833	·0698	1131	·1283	12
14	·1256	·0755	·0774	·0770	·0848	·0854	·0950	·1154	1319	·1457	14
16	1523	·0874	·0872	·0892	·0971	·0977	·1085	·1350	·1497	·1625	16
18	·1722	·1576	·0994	·1012	·1094	1103	·1712	·1521	1678	·1798	18
20	·1918	·1718	·1667	·1138	·1690	·1745	·1906	·1699	·1841	·1957	20
	Table IX L/D										
-6	-3·43	-3 45	-3·02	-2·61	-2 02	-1·84	-0·16	-2·21	-0·75	1·48	-6
-4	0·02	0 15	-0·51	-1 47	·42	1·52	4 10	-0 53	4·62	5·98	-4
-2	3·78	4 10	3 38	1 86	5·06	5·78	8·60	3·88	9·43	9·33	-2
0	7 59	8·13	7 97	6·16	9·58	9·58	11·08	8·75	11·51	10·63	0
2	11·33	11 49	11 69	10·26	12 38	12·16	11·84	11·35	12 14	10·66	2
4	12·92	12·95	12 93	12·60	12·79	12·70	11·50	12·12	11·64	10·22	4
6	12·00	12 56	12 64	12·68	12·06	11·91	10·65	11 63	10·78	9·66	6
8	10·60	11 46	11 50	11·69	10·92	10 87	9 95	10·58	9·78	8·82	8
10	9·23	10·49	10 50	10·70	10.05	10·01	9·21	9·67	8·67	3 24	10
12	7·46	9·60	9 80	9·85	9·26	9·22	8·59	8·64	3 37	2 69	12
14	3·51	8·57	9 08	9·06	8·48	8 50	7·87	3 30	2 91	2 46	14
16	2·63	7 55	8 43	8 34	7·84	7 79	7·11	2·82	2 56	2 28	16
18	2·16	2·72	7·53	7 63	7·18	7·05	2 39	2·58	2 34	2 14	18
20	1·94	2 73	2·56	6·94	2·52	2·46	2·17	2·38	2 18	2 03	20
	Table X C P										
-6				24·0	19·7			22·8		98 5	-6
-4				—	—		72 1	—	80·1	59·4	-4
-2	53·1	61 6	72 3	—	69 3	61·4	54·3	72·5	52 6	49·5	-2
0	39·6	43·7	48·4	56·4	54·5	49·5	49·0	51 3	44 4	43·3	0
2	35·9	38·7	41·0	46·3	47·5	43·3	41·8	43·0	40 5	40·0	2
4	33·1	34 4	37·2	41·1	43·2	39·8	39·0	38·1	37 2	38·0	4
6	30·0	32·6	34·4	37·9	40·1	37·8	37·1	35 7	35·9	35·4	6
8	28·8	30·8	32·6	36·5	38·5	36·0	35·9	34 8	34 1	34·2	8
10	27·7	29·6	31·2	35·1	37·4	34·8	35·0	33·1	33·8	42·3	10
12	26·6	28·4	30·8	34·3	36·3	34·2	33·8	32·6	41·3	43·2	12
14	32·7	28·3	29·6	32·9	35·4	32·8	33·2	39·5	41 0	43·8	14
16	36 2	28·0	29·2	32·0	34·6	31·5	32·6	40·4	41·6	41·7	16
18	37·6	36·2	29·0	31·6	—	32·2	41·2	40·4	40·8	42·6	18
20	37·4	35·1	36·6	31·1	42·8	40·7	41·8	40·6	41·3	42·8	20
	Table XI Critical Angle										
	12·8	16·4	19·4	20·1	18·2	18·4	16·1	13·1	11·0	9·2	



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CHARACTERISTICS

Models – 3" x 18" Aluminum and Wax

Corrected for Spindle Interference

SERIES III Under camber variation

Incidence deg	10M	10D	10F	10J	10E	10L	10K	11G	13C	13H	Incidence deg
					Table XII Lift Coefficients						
-6	·0320	·0271	0430	0936	-·0673	·0262	·0851	0990	-·0978	-·0714	-6
-4	·1651	·1666	1469	·1706	·0369	·1315	1855	1661	0370	0559	-4
-2	2698	2689	2566	2618	·1706	·2420	2657	2479	·1584	1915	-2
0	·3390	3397	3362	·3480	2809	3219	·3345	3392	2720	·2850	0
2	4053	4072	4067	·4226	3581	3910	·4097	·4164	·3587	3679	2
4	·4731	4769	·4756	·4930	4247	·4634	4807	4871	4221	4301	4
6	·5411	·5443	5435	5602	4938	5306	5464	5536	·4937	·5014	6
8	·6042	6071	·6049	6165	5587	5948	·6050	·6095	·5579	5699	8
10	6608	6641	·6620	·6692	6161	·6526	·6579	6600	·6209	·6284	10
12	·7076	7076	7087	7159	·6748	7037	·7011	3899	·6760	6791	12
14	7477	7480	·7495	·7571	·7236	7428	·7364	4004	·7186	·7247	14
16	·7927	7937	·7962	4461	·7618	7808	·4340	·4141	·7566	7543	16
18	·4369	·4384	·4393	·4242	8059	4366	·4291	4337	4541	·4359	18
20	·4380	·4377	4439	·4261	4631	4343	·4284	4434	4446	4365	20
					Table XIII Drag Coefficients						
-6	·0452	0489	0514	0527	·0566	0521	0460	0509	·0513	·0533	-6
-4	0262	·0276	0409	0428	·0434	0407	0334	0420	0357	0400	-4
-2	·0267	0263	0328	·0381	·0345	0340	0280	0378	0275	0310	-2
0	·0300	·0288	0308	0357	·0314	0319	0292	0349	0262	·0286	0
2	·0344	0336	·0341	0386	·0310	0340	·0350	0375	·0279	0304	2
4	·0414	0405	0411	·0451	·0343	·0400	0422	0449	0325	0344	4
6	0507	0498	0497	·0545	·0413	0480	·0516	0537	0407	0430	6
8	·0605	·0607	·0606	0644	·0508	0585	0627	0629	0502	0531	8
10	·0720	·0723	0725	·0767	·0617	0700	0749	·0756	0616	0637	10
12	0848	·0850	·0848	·0898	·0733	0825	·0881	·1147	·0736	·0755	12
14	·0974	·0974	·0976	1021	·0862	·0950	1012	1315	0855	0884	14
16	·1100	·1103	1101	1537	·0988	·1080	1508	·1476	·0984	·1013	16
18	1628	·1638	·1638	·1725	·1114	·1625	1705	·1642	1484	1509	18
20	·1813	·1806	·1827	·1890	·1658	1802	·1883	1813	·1661	·1701	20
					Table XIV L/D						
-6	0·71	0 55	0·84	1·78	-1 19	0·50	1 85	1·95	-1 92	-1 34	-6
-4	6·32	6 04	3 59	3 98	0 84	3·23	5 54	3 84	1 04	1 40	-4
-2	10 08	10 22	8 06	6 88	4 94	7·13	9·48	6 56	5 76	6 20	-2
0	11·30	11 78	10 92	9·74	8·93	10·09	11·45	9 72	10·11	9 99	0
2	11 80	12 10	11 92	10·96	11 57	11·50	11 72	11 10	12 89	12 09	2
4	11 43	11 78	11·58	10·92	12·36	11 58	11 39	10 84	13 00	12 50	4
6	10 68	10 92	10 92	10·28	11 98	10·90	10 60	10 32	12·11	11 63	6
8	9 98	10 00	9 97	9 58	10·99	10 18	9·66	9·71	11·10	10 73	8
10	9 15	9 18	9·12	8·72	10·01	9 32	8·77	8·72	10 10	9 84	10
12	8 33	8 33	8 35	7·98	9·19	8 52	7 95	3 40	9 20	8 98	12
14	7·68	7 68	7 68	7 40	8·36	7 82	7·27	3 04	8 42	8·22	14
16	7 20	7 18	7 24	2·90	7 71	7 22	2·88	2·81	7·69	7 45	16
18	2·68	2 67	2·68	2 46	7·22	2 68	2 51	2·64	2 95	2·88	18
20	2 42	2 40	2 40	2·26	2 90	2 41	2 28	.2 45	2 68	2·57	20
					Table XV C P						
-6				64 5			84 4	77 6			-6
-4	74 8	74 2	70·0	58·0		72 2	64·1	65 1			-4
-2	55·6	55 7	55 2	50 6	64 4	55·1	51 5	56 6	62·5	67 7	-2
0	47 4	48 3	48 1	44 0	49 7	45 6	44 8	48 6	50 6	54 5	0
2	43·7	46·5	43 7	41·2	43 4	41·3	40·7	44 6	44 8	47 6	2
4	41 8	40 6	40 8	38 3	39 4	37 9	37·7	41·9	41·2	43 9	4
6	39 3	39·3	38 7	36 9	37 2	36·0	36 4	40·0	38·6	40 6	6
8	37 4	37 6	37 5	35 3	34 8	34·8	34 5	38 2	36·4	39 1	8
10	36·4	36 3	36·4	33 7	33·5	33·8	33 2	37 3	36 0	37·7	10
12	35 3	35 0	35 0	32 8	32 9	32 4	32·6	45 0	34·8	36 6	12
14	34 5	34 4	34 8	32·0	32·0	31 6	32 1	42·5	33·9	36·4	14
16	33·7	34·0	33·8	40 4	31 3	31·2	39·4	42 0	33 3	—	16
18	41·3	42·2	40·6	39 5	30 5	38·6	39 2	41·5	39·3	42 2	18
20	41 5	41 3	41·6	39·6	39 5	38 8	39 6	42·2	40·6	43 2	20
					Table XVI Critical Angle						
	16 4	17·1	17 7	15 2	18 5	17·0	15 0	10·5	16 7	16 0	

TABLE XVII
ARP 13

PRINCIPAL CHARACTERISTICS OF TORONTO THICK SECTIONS

												Spar Depths in Percent of Chord					
Group	Section	Landing Speed Maximum C_Y	Cruising Speed Maximum L/D	High Speed, Pursuit L/D at $C_{Ymax}/9$	High Speed {Reconnaisance Fighter} L/D at $C_{Ymax}/6.25$	High Speed {Bomber or Commercial} L/D at $C_{Ymax}/4$	Speed Range $\sqrt{C_{Ymax}} \div \sqrt[3]{C_{X\,min}}$	Ceiling and Climb $(C_Y^{3/2} \div C_X)_{max}$	Maximum Forward Position of C.P. (% of Chord)	C.P. Travel between Max Fwd Position and that at $C_{Ymax}/6.25$	C.P. Travel between Max. Fwd. Position and that at $C_{Ymax}/4$	At 10% from L E	At 15% from L.E.	At 60% from L.E.	At 70% from L.E.	Mean. (Average, Col's 14&15)	Section
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Series I	TTS-1	·672	13 11	2·18	3 34	5·60	2·13	8 40	32·8	44 7	34 7	9 25	11 89	13 58	12·00	12·78	TTS 1
	" 12	·673	12 94	2 20	3 35	5·76	2 75	8 40	34 0	54 5	34 2	10 04	12 26	"	"	12 92	" 12
	" 13	·792	13 20	2 69	4·23	6 95	2 94	8 65	31 6	47 1	28 9	10 37	12 43	"	"	13 00	" 13
	" 2	·790	12 83	2 68	4 00	6 50	2 86	8 65	30 9	38 6	23 9	10 13	12 64	"	"	13·11	" 2
	" 14	·713	12 92	2 37	3 68	6 15	2 81	8 33	32 6	43 9	27 6	11 40	13 19	"	"	13 38	" 14
	" 3	616	12 12	2 00	2 95	5 00	2 64	7 66	32 4	45 1	33 9	12 53	13 93	"	"	13 45	" 3
	" 4	·586	11 83	1·90	2 90	4 75	2 58	7 39	33 0	47 0	32 0	13 03	14 19	"	"	13 88	" 4
	" 5	·565	11 31	1·80	2·78	4 54	2 48	7 32	31·2	45 8	30·3	13·39	14 33	"	"	13 95	" 5
Series II	TTS-6	·534	12 84	2 60	3 82	5·80	2 73	8 23	27·1	23 7	16 7	10 66	12 42	13 58	12·00	13 00	TTS 6
	" 7	·662	12 90	3 41	4 92	7 20	2·99	8 65	28·2	32 0	20 9	10 67	12 48	"	"	13 03	" 7
	" 8	·750	12 90	3 50	5 24	7 70	3 01	8 58	29 2	30 8	20·3	10 72	12 52	"	"	13 05	" 8
	" 2	·790	12 83	2 68	4 00	6 50	2 86	8 65	30 9	38 6	23 9	10 73	12 64	"	"	13 11	" 2
	" 10E	·812	12 37	2 44	3 66	6 15	2 87	8 40	30 5	42·0	27 8	11 37	13 03	"	"	13 30	" 10E
	" 11G	·665	11·10	1 30	2 15	3 90	2 50	7 69	37 3	37 7	27 7	15 60	16 83	"	"	15 20	" 11G
Series III (·)	TTS-2	·790	12 83	2 68	4 00	6 50	2 86	8 65	30 9	38 6	23 9	10 73	12 64	13·58	12 00	13 11	TTS 2
	" 2A	·784	12 72	3·20	4 56	7 10	2 96	8 53	31 6	34 9	25 9	13 18	14 91	"	"	14 24	" 2A
	" 2B	·773	11 83	2 41	3 68	6 43	2 84	7 97	32 6	41 7	29 7	15 67	17 26	"	"	15·42	" 2B
	" 2C	·788	12 90	2 70	4 16	6 97	2 94	8 47	34 4	40 3	27 1	13 06	14 72	"	"	14 15	" 2C
(3)	TTS-13	·792	13 20	2 69	4 23	6 95	2 94	8 65	31 6	47 1	28 9	10 37	12 43	13 58	12 00	13 00	TTS 13
	" 13C	·764	13 14	2 67	4 16	7 02	2 95	8 56	33 0	36 7	24 3	12 70	14 51	"	"	14 04	" 13C
	" 13H	·754	12 50	2 19	3 43	6 04	2 84	8 36	35 9	46 6	31·4	13 40	14 85	"	"	14 21	" 13H
(4)	TTS-10D	·817	12 13	2 44	4 31	7 78	2 74	8 14	33 9	50 5	32 6	15 19	16 82	13 58	12 00	15 20	TTS 10D
	" 10F	·827	11 93	2 00	3 14	5 82	2 90	8 10	34·4	37 8	27 1	15 45	16 82		"	15 20	" 10F
	" 10J	·780	11 05	1·64	2·60	4 67	2 68	7 71	31 8	30 5	24 5	15 80	17·37		"	15 47	" 10J
(5)	TTS-10E	·812	12 37	2 44	3 66	6 15	2 87	8 40	30 5	42 0	27 8	11 37	13 03	13 58	12 00	13 30	TTS 10E
	" 10L	·800	11 64	1 96	3 18	5·71	2 82	7 97	32 6	37 8	26 4	13 54	14 98	"	"	14 28	" 10L
	" 10K	·756	11 80	1 92	3 17	5 90	2 86	7 89	31 6	44·6	31 4	15·82	17 33	14 00	12 00	15 66	" 10K

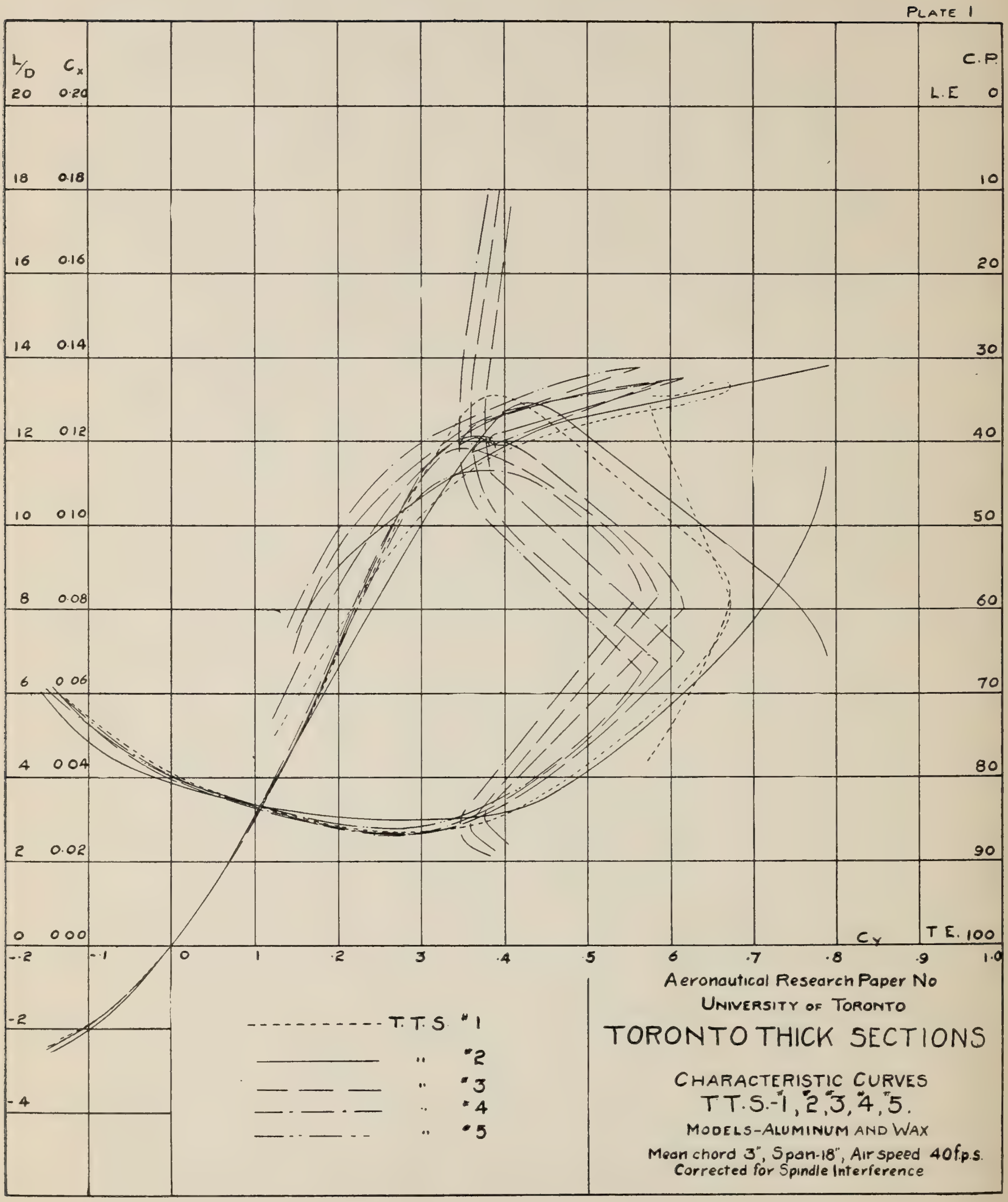
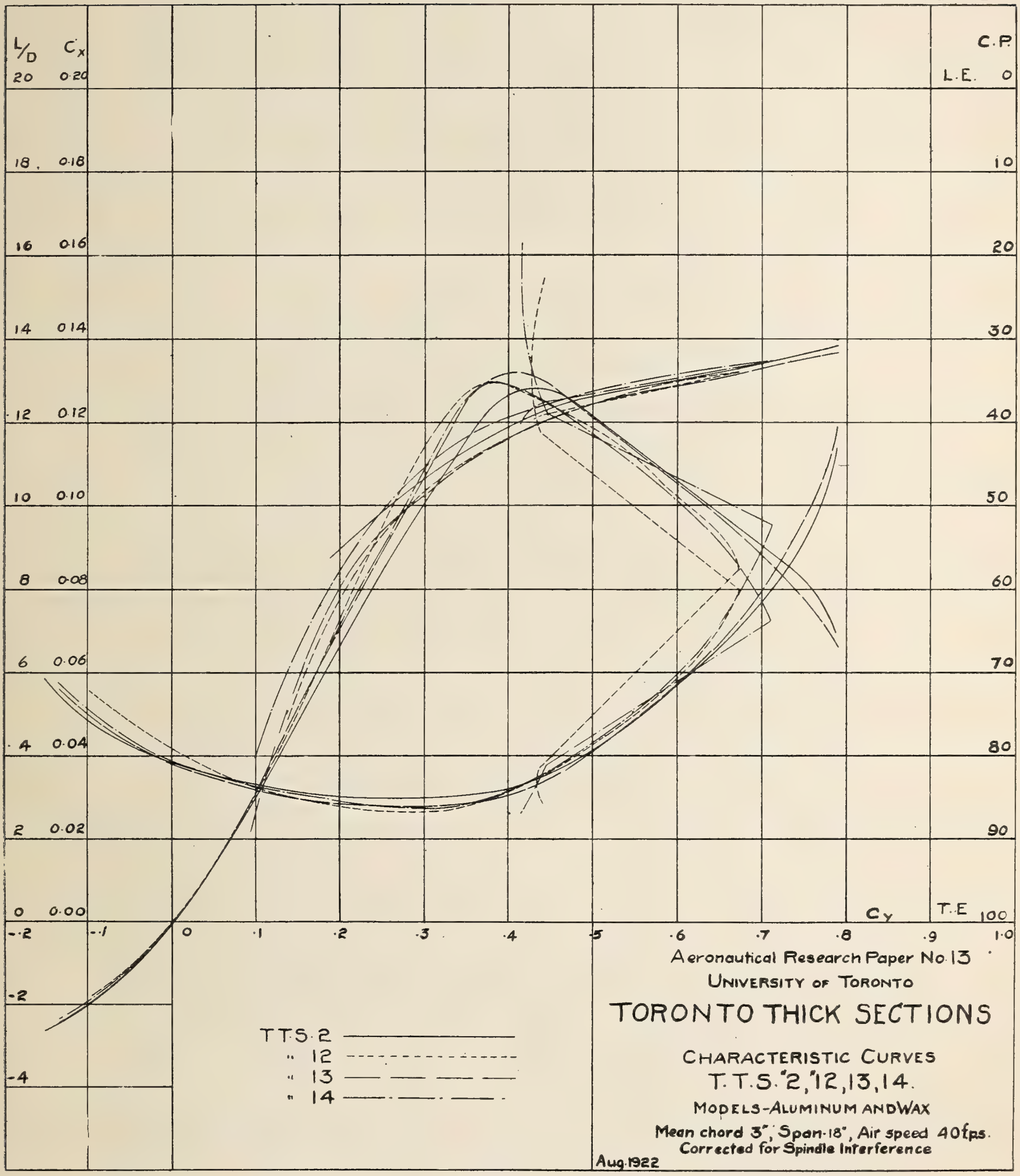


PLATE I
L/D
C_x
C.P.
20 0.20
L.E 0
18 0.18
10
16 0.16
20
14 0.14
30
12 0.12
40
10 0.10
50
8 0.08
60
6 0.06
70
4 0.04
80
2 0.02
90
0 0.00
T.E. 100
C_y
-.2
-.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
-2
-4
T.T.S #1
" #2
" #3
" #4
" #5
Aeronautical Research Paper No
UNIVERSITY OF TORONTO
TORONTO THICK SECTIONS
CHARACTERISTIC CURVES
T.T.S.-#1, #2, #3, #4, #5.
MODELS-ALUMINUM AND WAX
Mean chord 3", Span-18", Air speed 40 f.p.s.
Corrected for Spindle Interference

PLATE 2

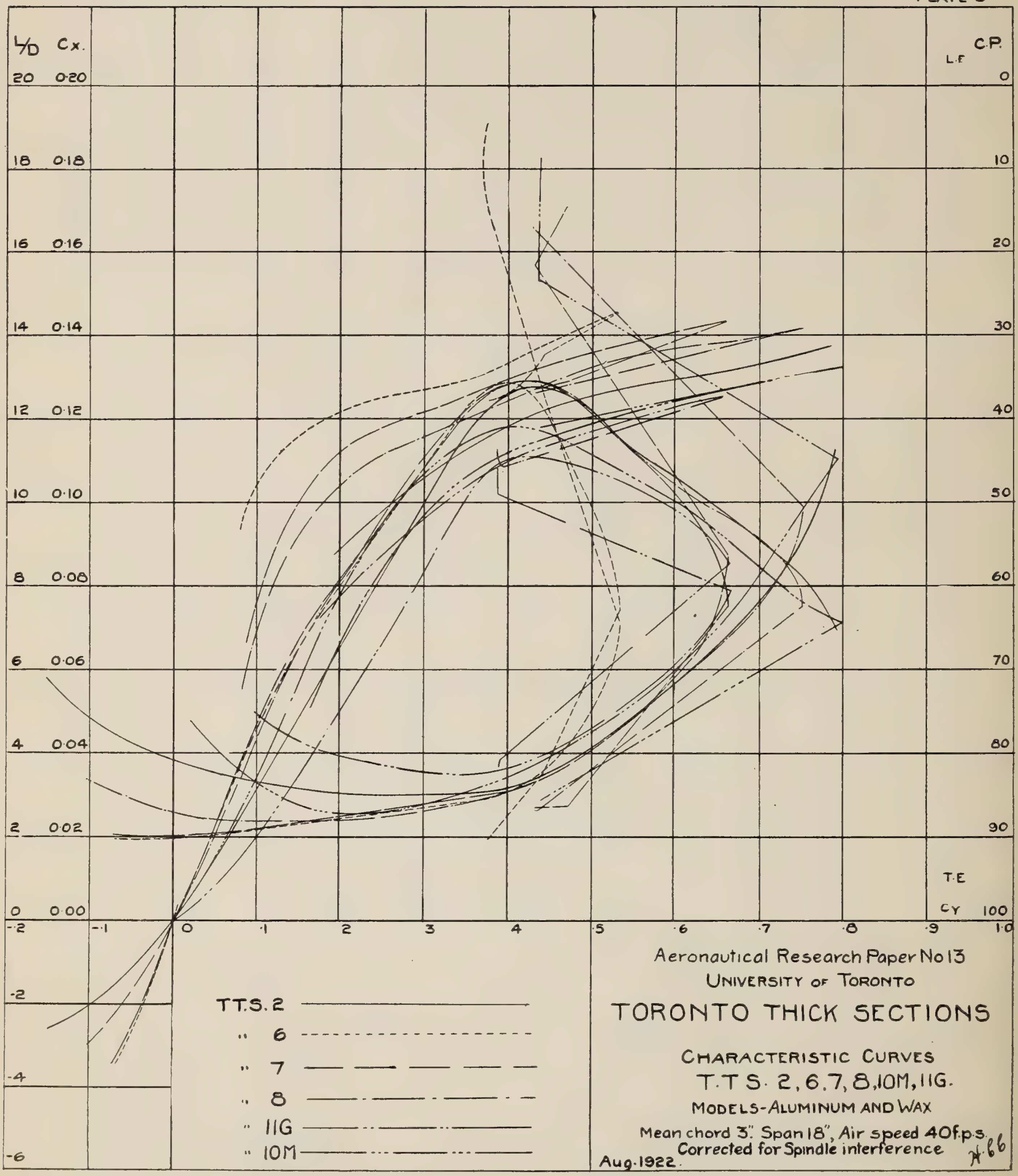


Aeronautical Research Paper No 13
UNIVERSITY OF TORONTO

TORONTO THICK SECTIONS

CHARACTERISTIC CURVES
T.T.S. 2, 12, 13, 14.
MODELS-ALUMINUM AND WAX
Mean chord 3", Span-18", Air speed 40 f.p.s.
Corrected for Spindle Interference
Aug 1922

PLATE 3



Aeronautical Research Paper No 13
UNIVERSITY OF TORONTO
TORONTO THICK SECTIONS
CHARACTERISTIC CURVES
T.T.S. 2, 6.7, 8, 10M, 11G.
MODELS-ALUMINUM AND WAX
Mean chord 3". Span 18", Air speed 40f.p.s.
Corrected for Spindle interference
Aug. 1922.
H.66

PLATE 4

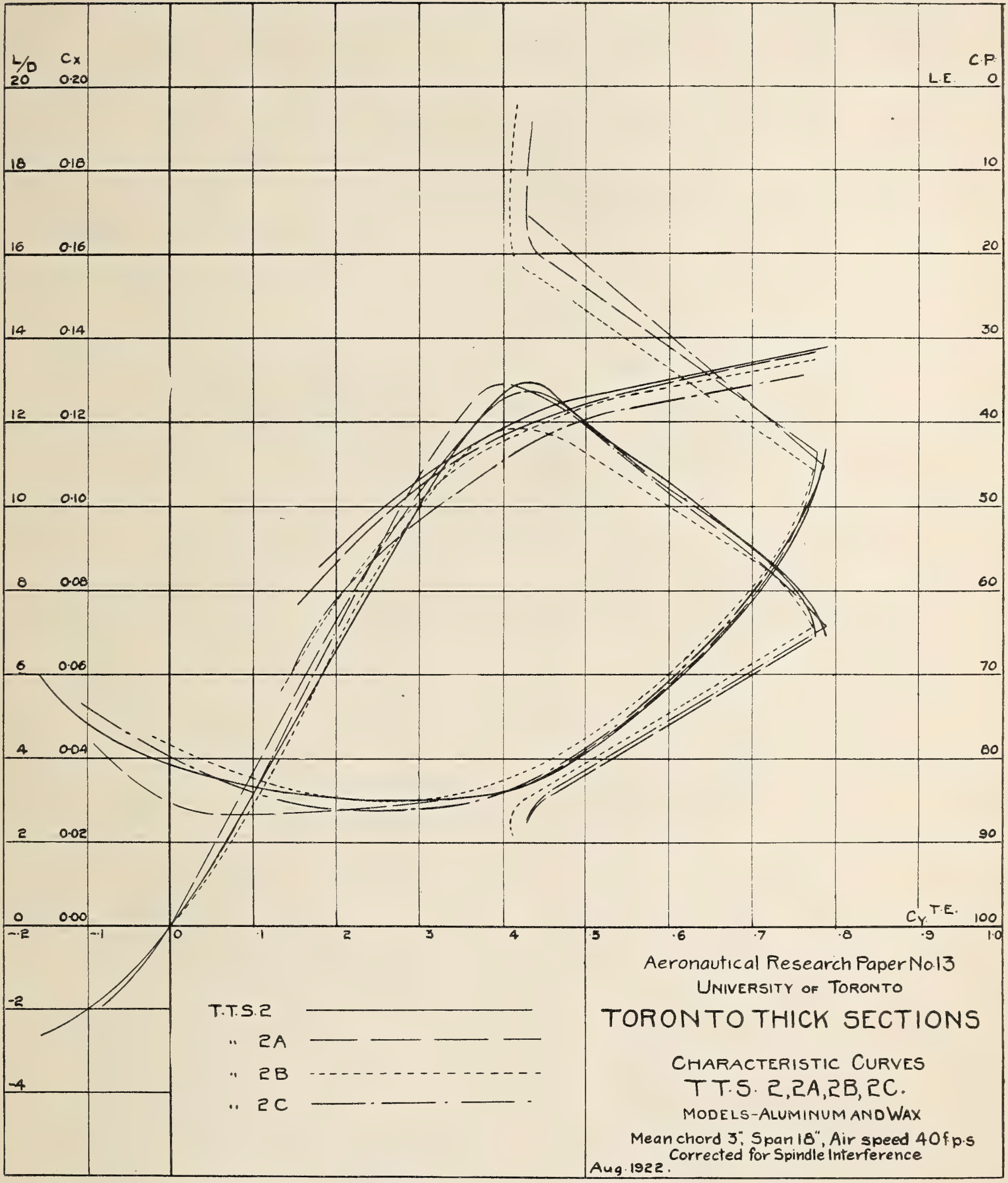
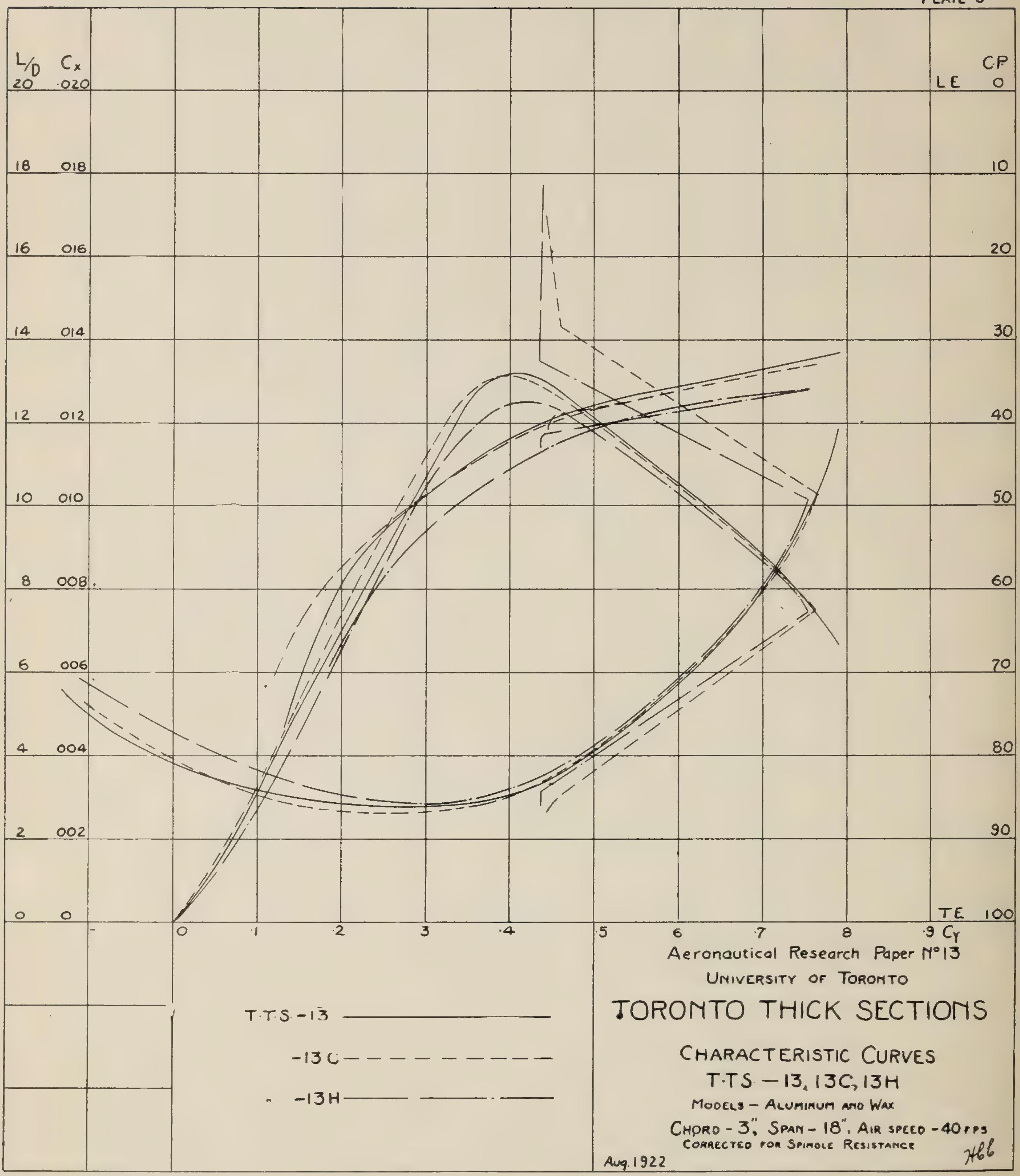

L/D
Cx
C.P.
L.E.
20 0.20
18 0.18
16 0.16
14 0.14
12 0.12
10 0.10
8 0.08
6 0.06
4 0.04
2 0.02
0 0.00
0
10
20
30
40
50
60
70
80
90
100
T.E.
Cy
-.2
-.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
-2
-4
T.T.S.2
" 2A
" 2B
" 2C
Aeronautical Research Paper No.13
UNIVERSITY OF TORONTO
TORONTO THICK SECTIONS
CHARACTERISTIC CURVES
T.T.S. 2,2A,2B,2C.
MODELS-ALUMINUM AND WAX
Mean chord 3", Span 18", Air speed 40 f.p.s
Corrected for Spindle Interference
Aug. 1922.

PLATE 5

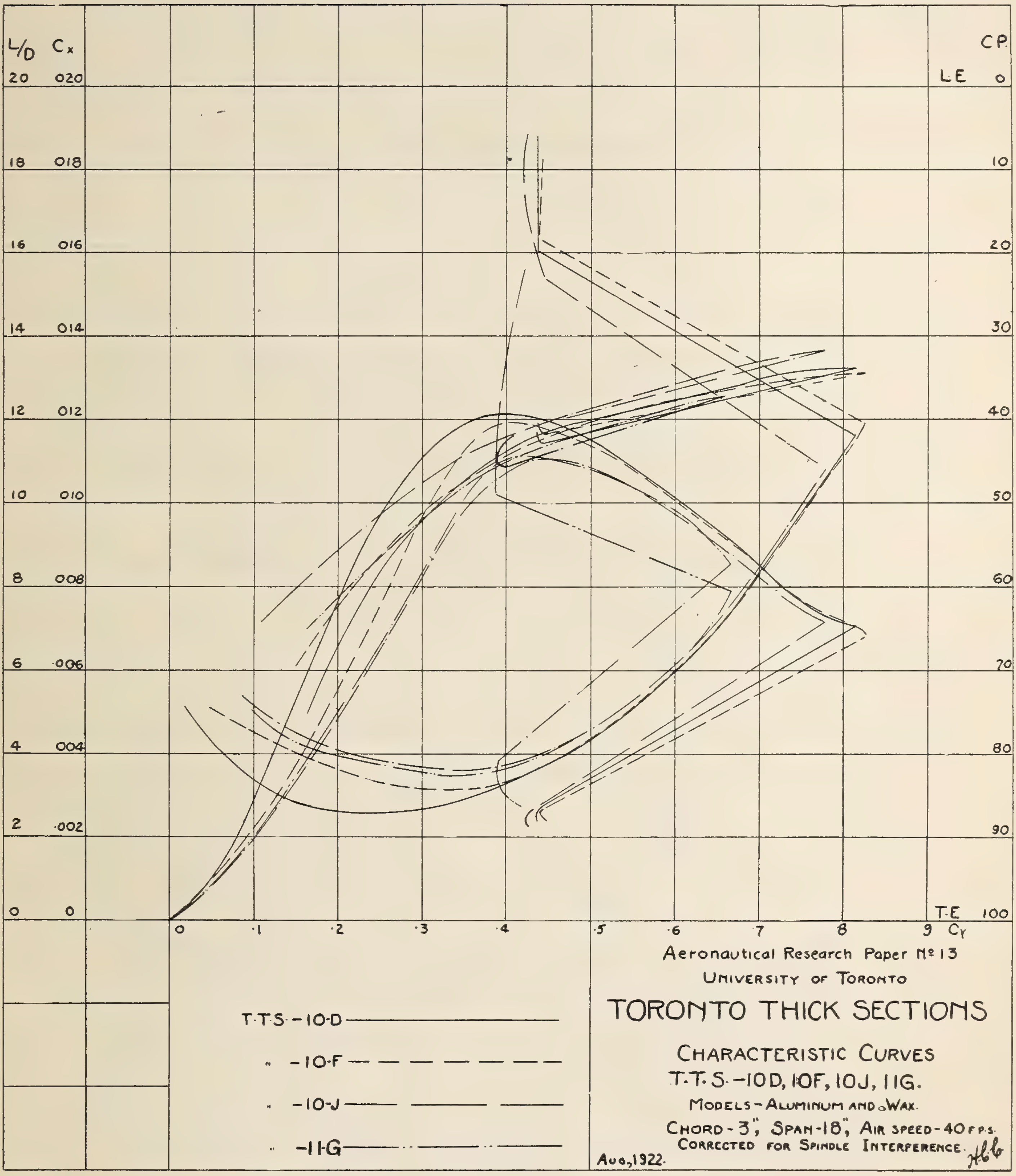


Aeronautical Research Paper N°13
UNIVERSITY OF TORONTO

TORONTO THICK SECTIONS

CHARACTERISTIC CURVES
T·TS —13, 13C, 13H
MODELS — ALUMINUM AND WAX
CHORD - 3", SPAN - 18", AIR SPEED - 40 FPS
CORRECTED FOR SPINDLE RESISTANCE
Aug. 1922
Hbl

PLATE 6



Aeronautical Research Paper No 13
University of Toronto

TORONTO THICK SECTIONS

Characteristic Curves
T.T.S.–10D, 10F, 10J, 11G.
Models–Aluminum and Wax.
Chord-3", Span-18", Air speed-40 f.p.s.
Corrected for Spindle Interference.
Aug., 1922.
H.G.

PLATE 7

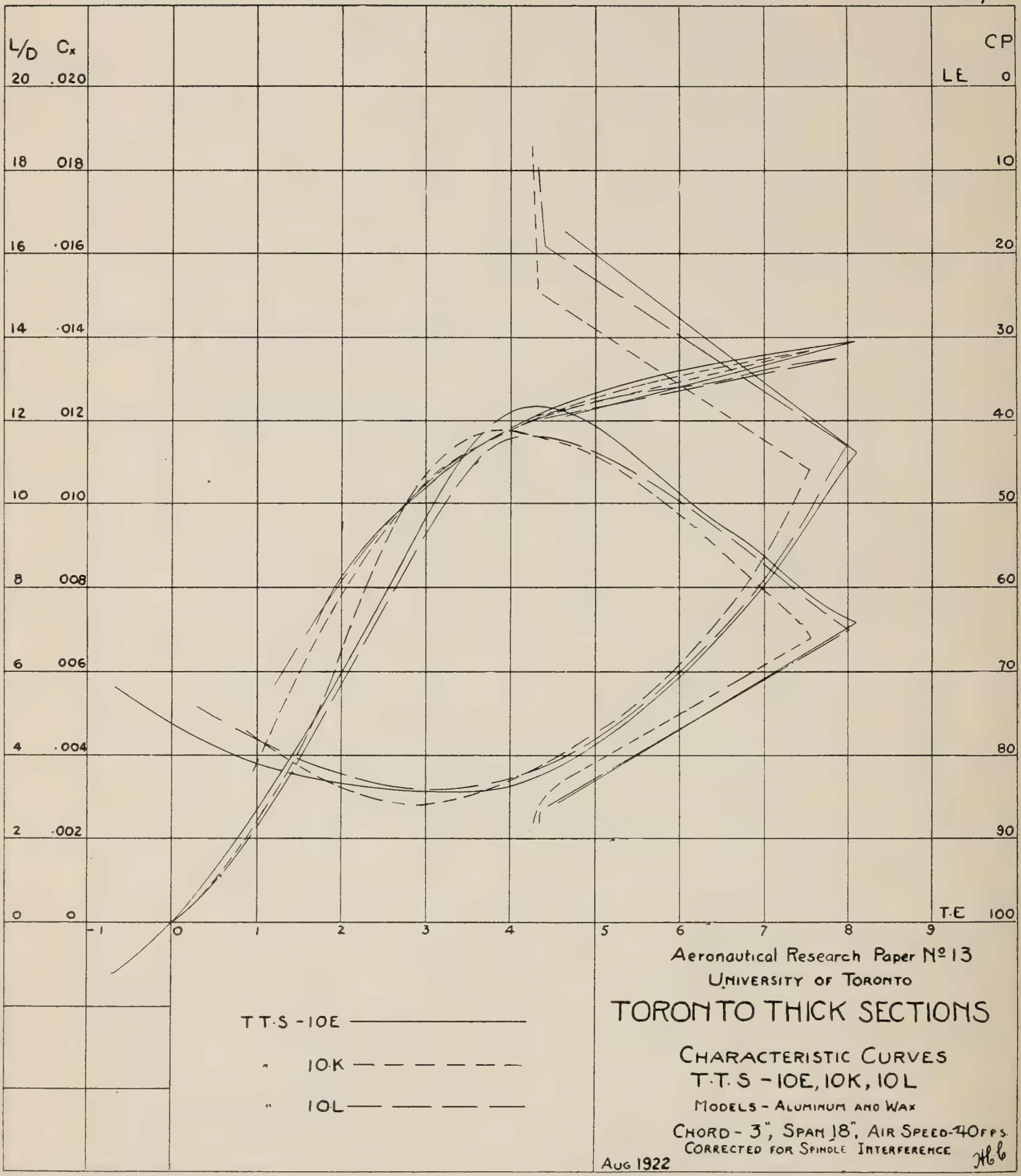
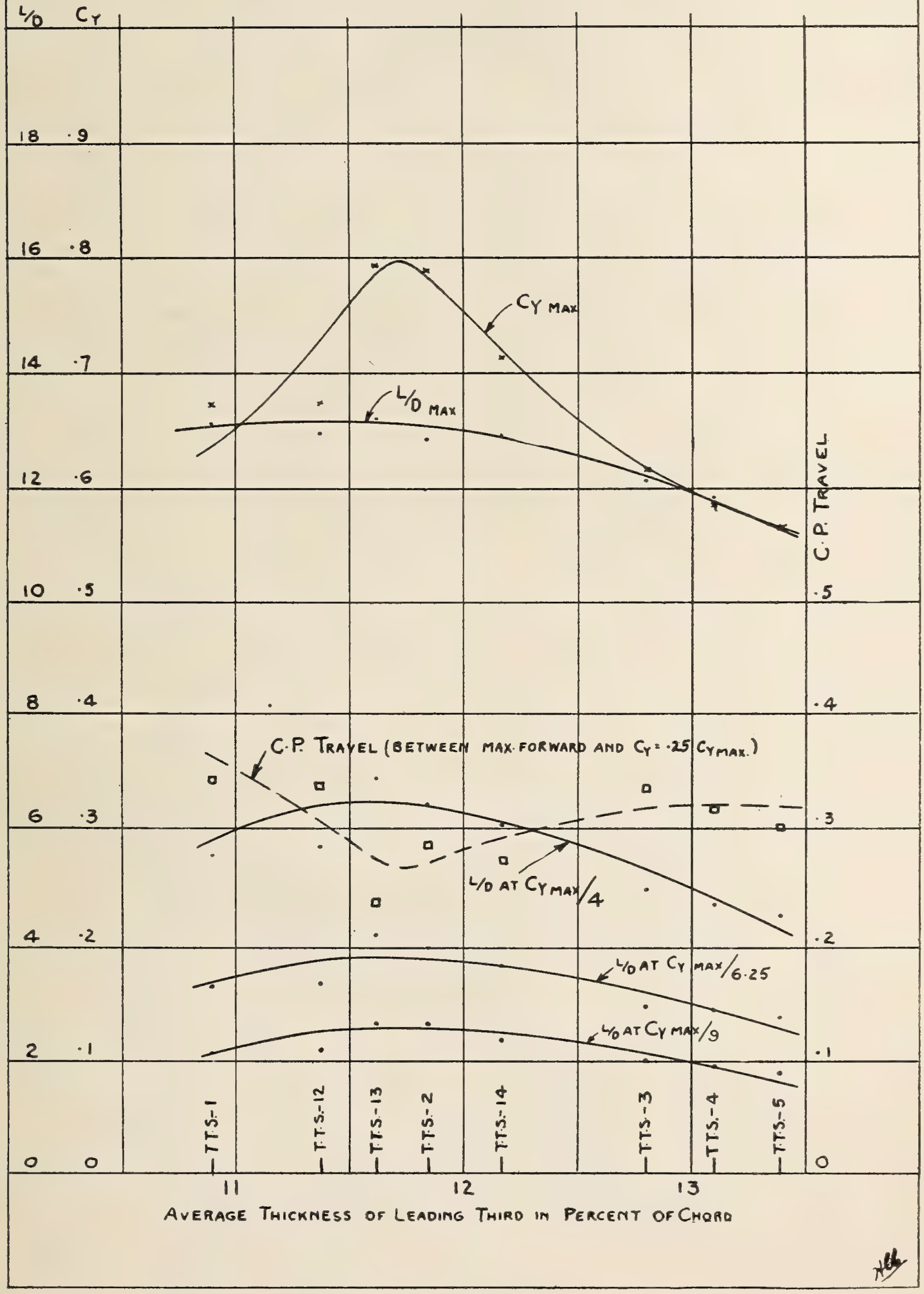

L/D
Cx
CP
LE 0
20 .020
18 018
16 ·016
14 ·014
12 012
10 010
8 008
6 006
4 ·004
2 ·002
0 0
10
20
30
40
50
60
70
80
90
T.E 100
-1
0
1
2
3
4
5
6
7
8
9
T.T.S - 10E
" 10K
" 10L
Aeronautical Research Paper Nº 13
UNIVERSITY OF TORONTO
TORONTO THICK SECTIONS
CHARACTERISTIC CURVES
T.T.S - 10E, 10K, 10L
MODELS - ALUMINUM AND WAX
CHORD - 3", SPAN 18", AIR SPEED - 40 F.P.S.
CORRECTED FOR SPINDLE INTERFERENCE
AUG 1922

PLATE Nº 8

PRINCIPAL CHARACTERISTICS

T.T.S.—1, 12, 13, 2, 14. 3, 4, 5.



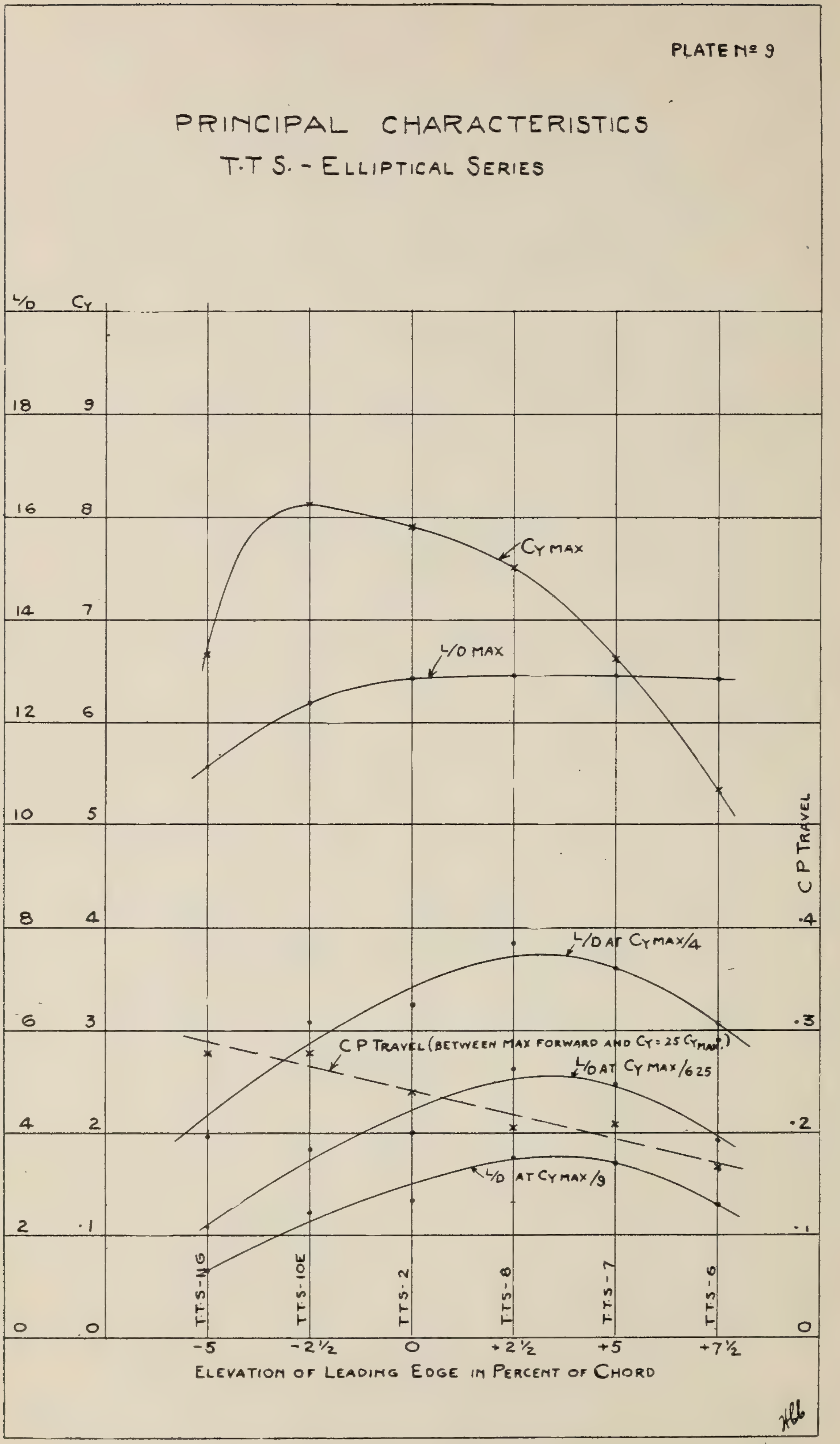


PLATE No 9

PRINCIPAL CHARACTERISTICS

T.T.S. - ELLIPTICAL SERIES

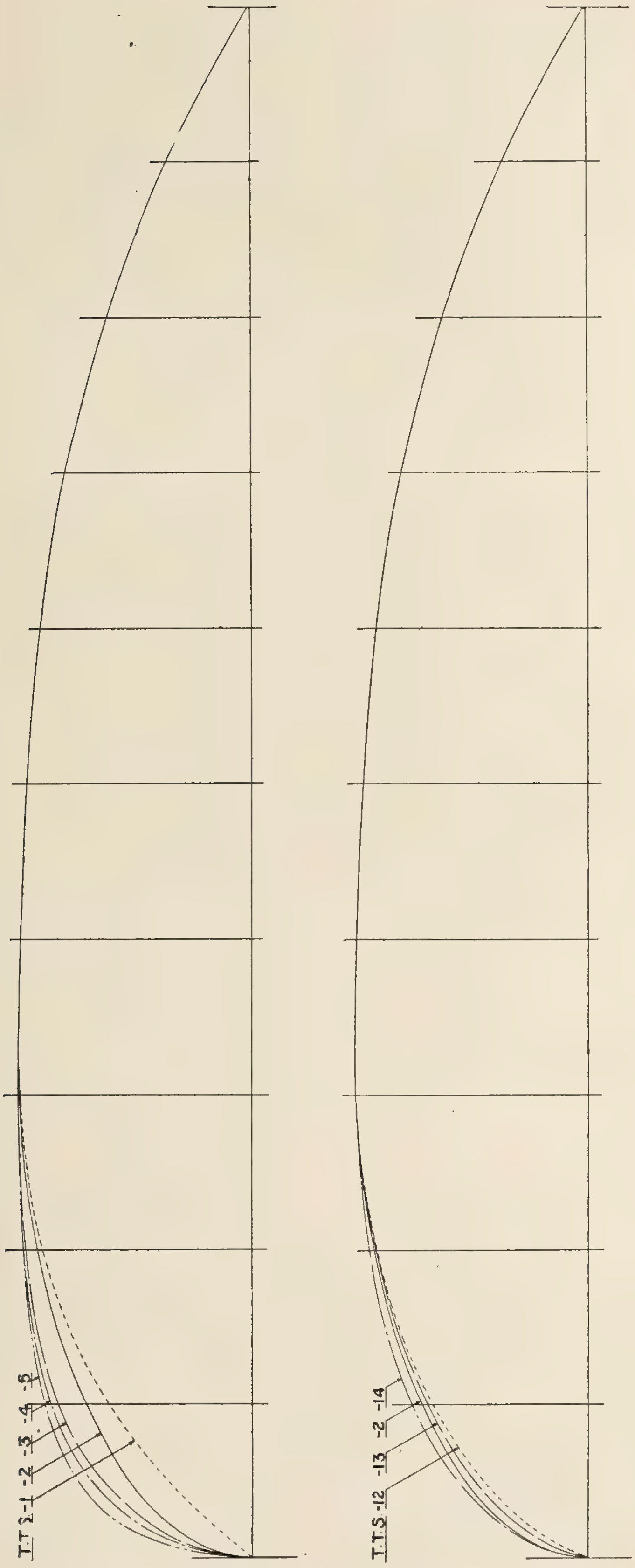


PLATE NO. 10

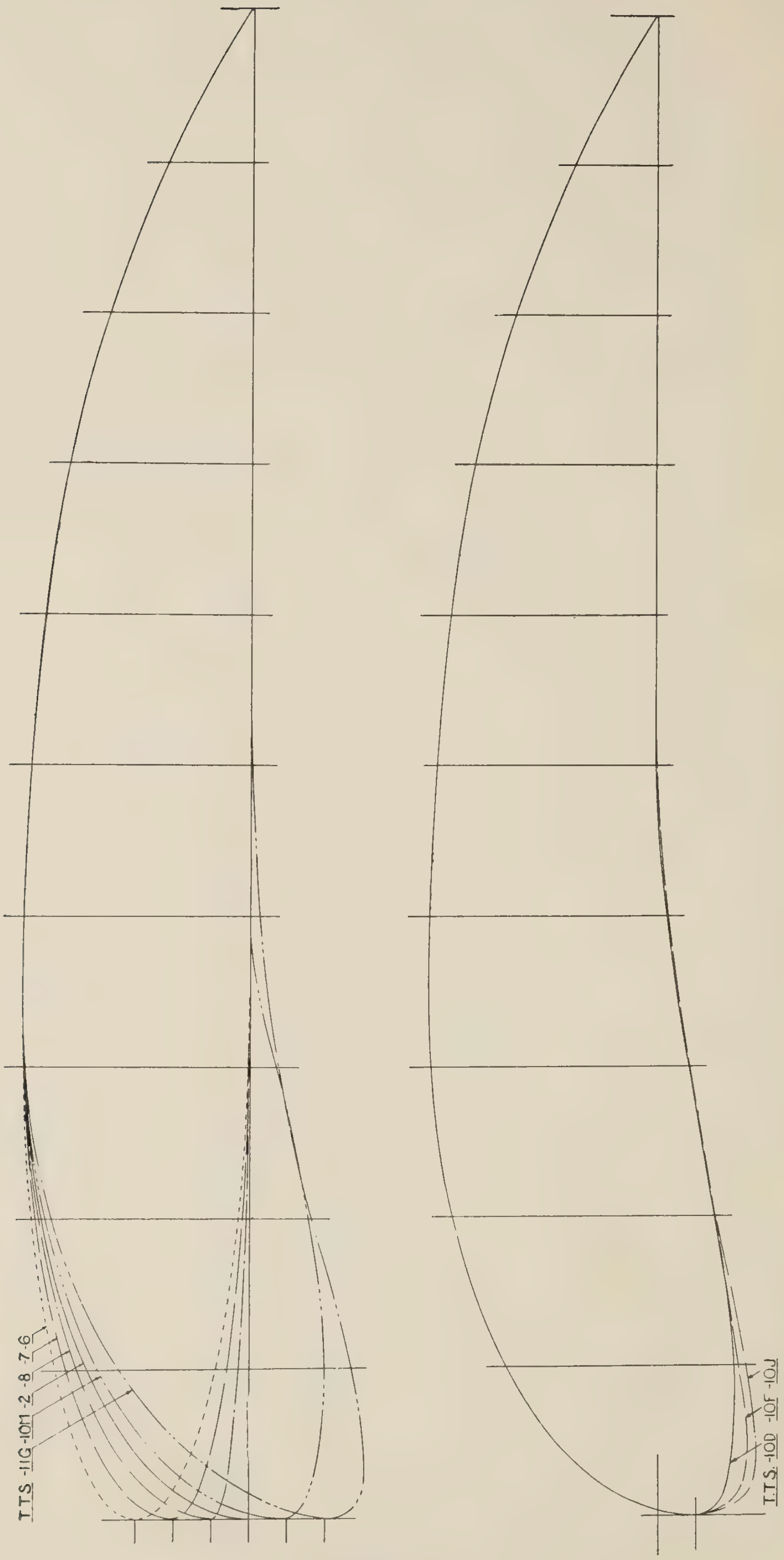


PLATE No. 11

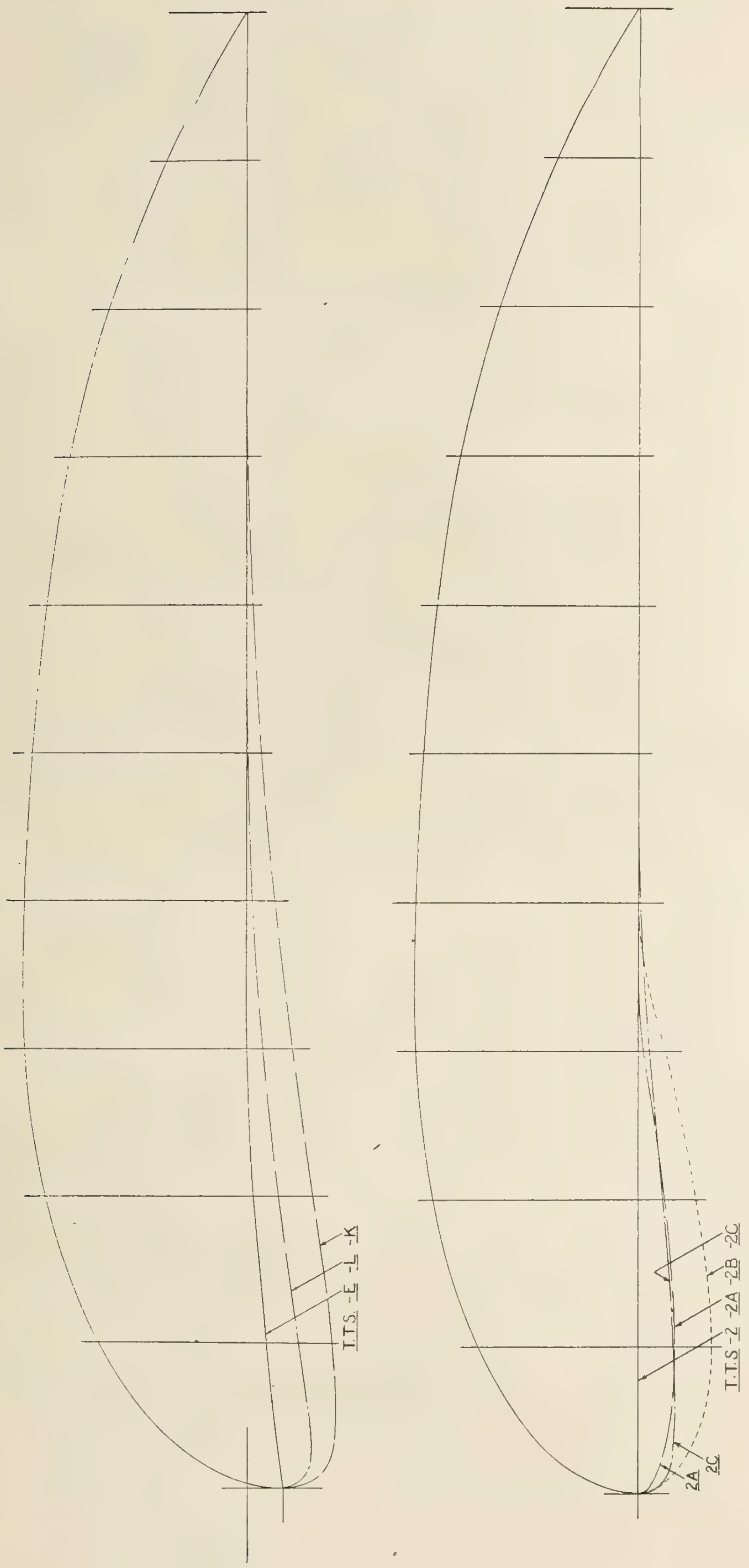


PLATE NO. 12

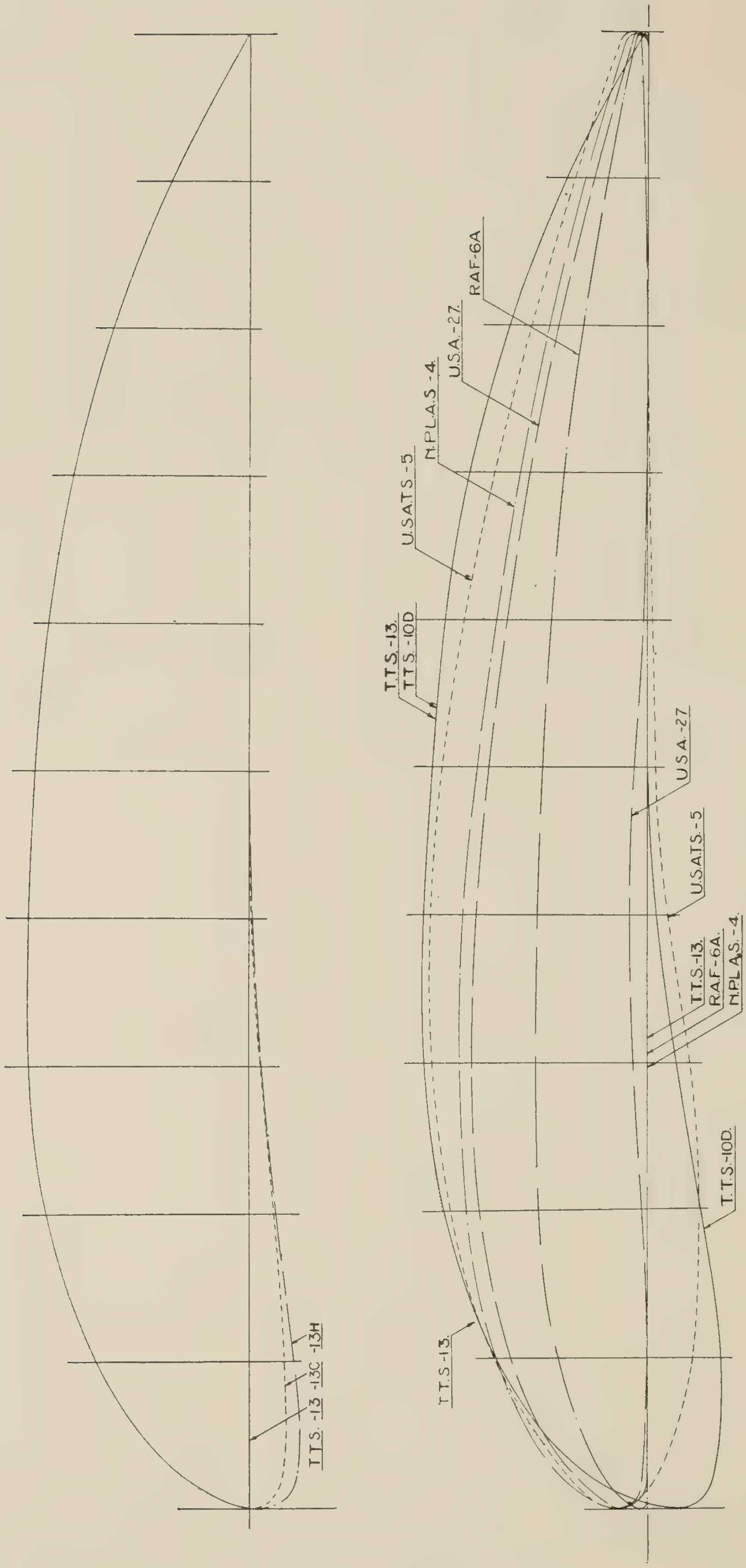

T.T.S. -13 -13C -13H
R.A.F-6A
U.S.A.-27.
N.P.L.A.S.-4
U.S.A.T.S.-5
T.T.S.-13.
T.T.S.-10D
U.S.A.-27
U.S.A.T.S.-5
T.T.S.-13.
R.A.F.-6A.
N.P.L.A.S.-4
T.T.S.-10D.
T.T.S.-13.

PLATE No. 13

Aeronautical Research Paper No. 15.
September, 1922.

CENTRE OF PRESSURE DETERMINATION IN WIND CHANNEL EXPERIMENTS

By J. H. PARKIN, *Assistant Professor of Mechanical Engineering,*
and H. C. CRANE, *Research Assistant*

Introduction

In testing aerofoils in a tunnel of the N.P.L. type the aerofoil is supported in the vertical position by means of a spindle screwed into one end of it and clamped in the chuck of the balance as described in Aeronautical Research Paper No. 12. With the velocity of the air stream held constant, observations of lift (the cross wind component), drag (the down stream component) and torque about the spindle axis are made for angles of incidence uniformly spaced between −6° and +20°. As described in the paper to which reference has just been made, the calculation of the lift and drag coefficient, *Cy* and *Cx*, corrected for spindle resistance is not a very laborious task but the calculation of the centre of pressure analytically or its determination graphically, while easily understood, has required considerable time. To obviate this difficulty the methods to be described have been evolved. Their value has been proven in the calculation of a large number of sets of results.

The Toronto Method

The usual graphical method will first be described as the new method is a development from it. The aerofoil section is laid out to a large scale and the angle of incidence θ laid off (see figure 1, plate 1). The values of D and L (corrected for spindle interference) are then laid off along and at right angles, respectively, with the wind and the magnitude of the resultant vector sum R scaled off. The radius of action r from the spindle is found:

$$r = \frac{t}{R} \qquad (1)$$

$$= \frac{K'T}{R} \qquad (2)$$

where r = radius in inches
t = torque in inch pounds
T = torque spring reading
K' = constant of torque spring
R = resultant = $\sqrt{L^2+D^2}$.

With centre S (the spindle axis) and radius r an arc is drawn and a line tangent to it and parallel to the line from which R was scaled gives at its intersection with the chord the centre of pressure.

The first short-cut in the new method is in the determination of the radius r. The first step as described above was to determine R. Now

$$R=\sqrt{L^2+D^2} \tag{3}$$

For values of L/D greater than 4, R equals L within 3% and for values of L/D greater than 7, R equals L within 1%. (See Table No. 1.)

A convenient method of calculating R is obtained by expanding $\sqrt{L^2+D^2}$

$$\begin{aligned}
R &= \sqrt{L^2+D^2} \\
&= (L^2+D^2)^{\frac{1}{2}} \\
&= L\left(1+\left(\frac{D}{L}\right)^2\right)^{\frac{1}{2}} \\
&= L\left(1+\tfrac{1}{2}\left(\frac{D}{L}\right)^2-\frac{1}{8}\left(\frac{D}{L}\right)^4+\ldots\ldots\ldots\right. \\
&= L\left(1+\frac{1}{2\left(\frac{L}{D}\right)^2}-\frac{1}{8\left(\frac{L}{D}\right)^4}+\ldots\ldots\ldots\right. \\
&= L+\frac{D}{2\,\mathrm{L/D}}\quad - \qquad (4)
\end{aligned}$$

This gives the rule that to find the resultant R, to the lift L add the drag D divided by 2 L/D. For values of L/D greater than 4 this gives results within 1/20 of one per cent. This method is here given, not because it is in use at present, but because it is sometimes convenient.

The exact value of R is

$$R=\frac{L}{\sin\,(\tan^{-1}\,(\mathrm{L/D}))} \tag{5}$$

Inserting this value in equation (2) above:

$$r=\frac{K'T\sin\,(\tan^{-1}\,(\mathrm{L/D}))}{L} \tag{6}$$

$$=\frac{T}{L}\left[\,(K'\sin\,(\tan^{-1}\,(\mathrm{L/D}))\right] \tag{7}$$

For a given spring K' is constant and the quantity within the square brackets is a function of L/D; hence:

$$r=\frac{T}{L}.f(\mathrm{L/D}) \tag{8}$$

f (L/D) may be read from a curve such as is shown on plate No. 2. This curve was prepared by multiplying values of sin ($\tan^{-1}$ (L/D)) given in table No. 1 by the spring constant $K'(=.2154)$ and plotting on the corresponding values of L/D. Equation 8 may then be solved by one setting on a slide rule with the aid of the curve.

The chart shown in plate No. 5 was also constructed for the determination of the radius r. Its solution requires the use of two lines. The one joins the values of L and f (L/D); the other parallel to it through the value of T gives the value of r. The chart is not as convenient as the curve and slide rule method for two reasons. In the first place the use of parallel lines is not as convenient as a slide rule. In the second place, as may be seen from the chart, the working range of f (L/D) is only about one third of the maximum value. Hence the curve may be plotted to a large scale on a small sheet making the reading of the curve easy.

Two diagrams A, Plate No. 3, and B, Plate No. 4 are used in finding the centre of pressure. These are also shown diagrammatically on Plate No. 1. Chart A is simply a system of concentric circles drawn five times the natural scale, the circles being one twentieth of an inch apart but marked in hundredths. The centre represents the spindle axis S (Fig. 1, Plate 1). The chord of the aerofoil is laid out by measuring on the model the lengths $SP=m$ and $SO=n$; the intersection of any radial line on chart A and the circle of radius $SO=n$ gives O the rear point on the chord and the line OPT tangent to the circle of radius $SP=m$ is the chord. Only a short portion of this chord near T the leading edge is drawn in and it with a very soft pencil.

Diagram B is drawn on transparent material such as a good grade of tracing linen or celluloid. If of tracing linen it should be reinforced around O with celluloid secured by metal eyelets, to prevent wear. The lines VO and OJ are laid out at right angles to each other, VO being the true wind direction and OJ the lift direction. Lines OT are laid out making angles $-10°-9°\ldots.+19°$, $+20°$ with OV. These lines are chord lines. OT is made equal to the chord to the same scale as diagram A and divided by arcs into per cent of chord, the point O being the trailing edge or 100%. Starting from O, OJ is uniformly divided and provided with L/D scales A, B and C corresponding to the points A B and C respectively on the line OV such that any line drawn through A B or C to intersect the line OJ has a slope given by its intercept on OJ to the corresponding scale.

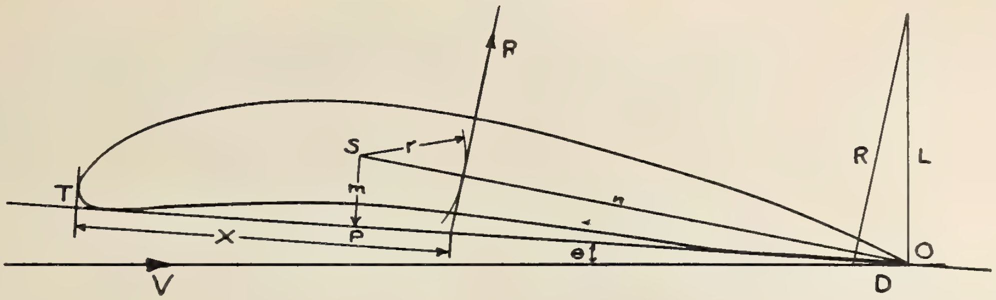
To find a C.P. given the incidence ϕ, radius r and L/D. Having found the chord OT on chart A superimpose chart B on A in such a way that O on B is coincident with O (the trailing edge) on A. Push in a fine needle to keep O on B coincident with O on A and allow chart B

to rotate about O. Rotate chart B till the line OT of proper angle of incidence on B coincides with the chord line OT on chart A. Now the condition of affairs is essentially the same as in Fig. 1, Plate No. 1. VO is the line of zero incidence on chart B. TO, the chord, is the line of $\theta°$ incidence on chart B. The centre S of the concentric circles on A is the spindle axis. All that remains is to draw a line of the slope L/D with respect to VO and tangent to the circle of radius r and find its intersection with the chord TO and from the arcs giving per cent of chord read off the required C.P. The last step is conveniently performed with two set squares. One square is laid down with one edge passing through one of the points A B or C and through the point on OJ giving the L/D to the proper scale and by means of the other square is shifted parallel to itself till the edge is tangent to the circle of radius r. The C.P. is then read off at the intersection of the edge and the chord OT or the line of $\theta°$ incidence.

Conclusion

Although from the description, owing to the relative motion of the two charts, the method may seem complicated it is really quite simple in practice and including the calculation of r the whole process of determining the C.P. for a complete test (14 angles of incidence) not including time taken in observation can be done in fifteen minutes. With the usual graphical method the time required is at least two hours. An even greater advantage is the fact that the elimination of the close work on the drawing board, with the corresponding reduction in mental fatigue is conducive to the reduction of errors due to fatigue.

Plate No 1



$C.P. = x/c$

FIG I THE USUAL GRAPHICAL SOLUTION

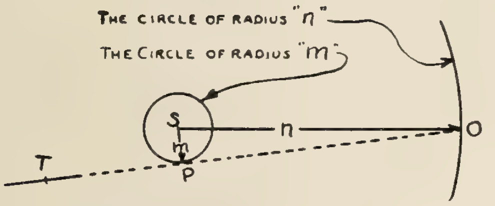


CHART A

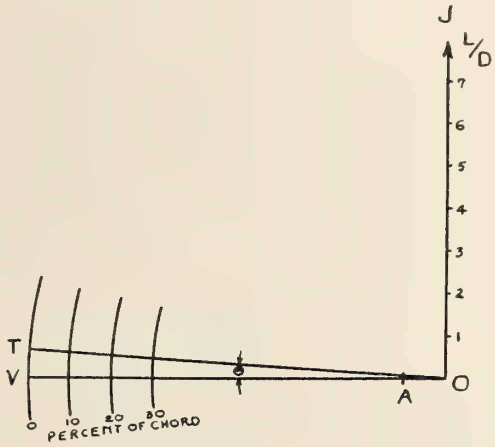


CHART B

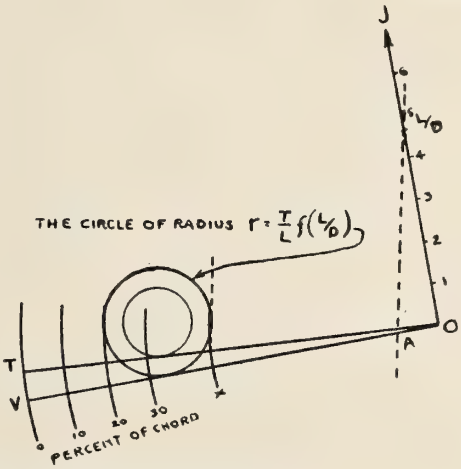


CHART B SUPERIMPOSED ON CHART A

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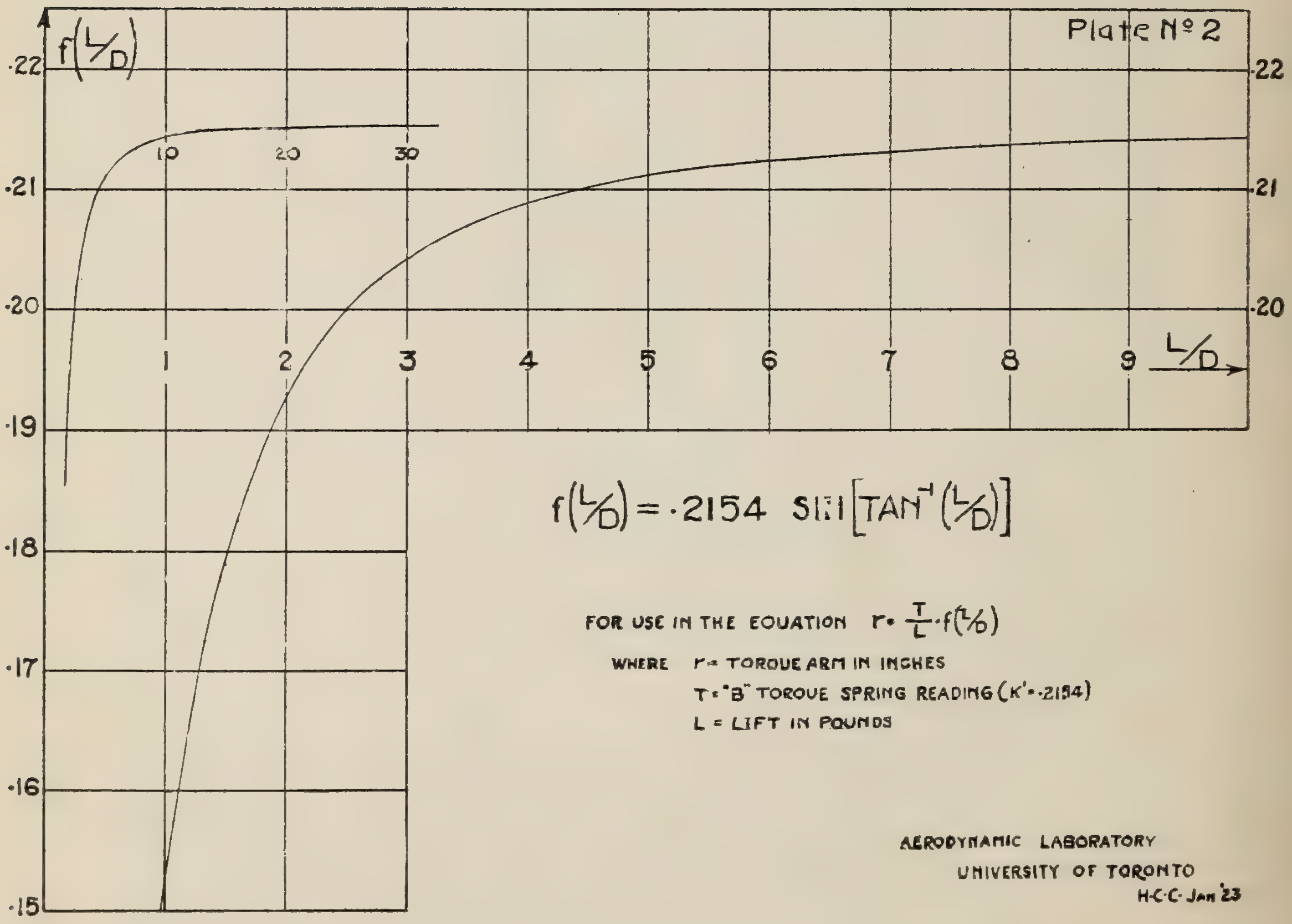
UNIVERSITY OF TORONTO

C.P. DETERMINATION

TABLE #1

L/D	$\text{Sin}\,\text{Tan}^{-1}(L/D)$	L/D	$\text{Sin}\,\text{Tan}^{-1}(L/D)$
1·0	·7071	2·8	·9418
1·1	·7400	3·0	·9487
1·2	·7683	3·5	·9615
1·3	7926	4·0	·9702
1·4	·8138	5·0	·9806
1·5	·8321	6·0	·9864
1·6	·8480	7·0	·9899
1·7	·8619	8·0	·9923
1·8	·8742	9·0	·9939
1·9	·8850	10·0	·9951
2·0	·8944	15·0	·9978
2·2	·9103	20·0	·9987
2·4	·9231	30·0	·9995
2·6	·9334	∞	1·0000



$$f(L/D) = \cdot 2154\ \text{SIN}\left[\text{TAN}^{-1}(L/D)\right]$$

FOR USE IN THE EQUATION $r = \frac{T}{L} \cdot f(L/D)$

WHERE r = TORQUE ARM IN INCHES
T = "B" TORQUE SPRING READING (K' = ·2154)
L = LIFT IN POUNDS

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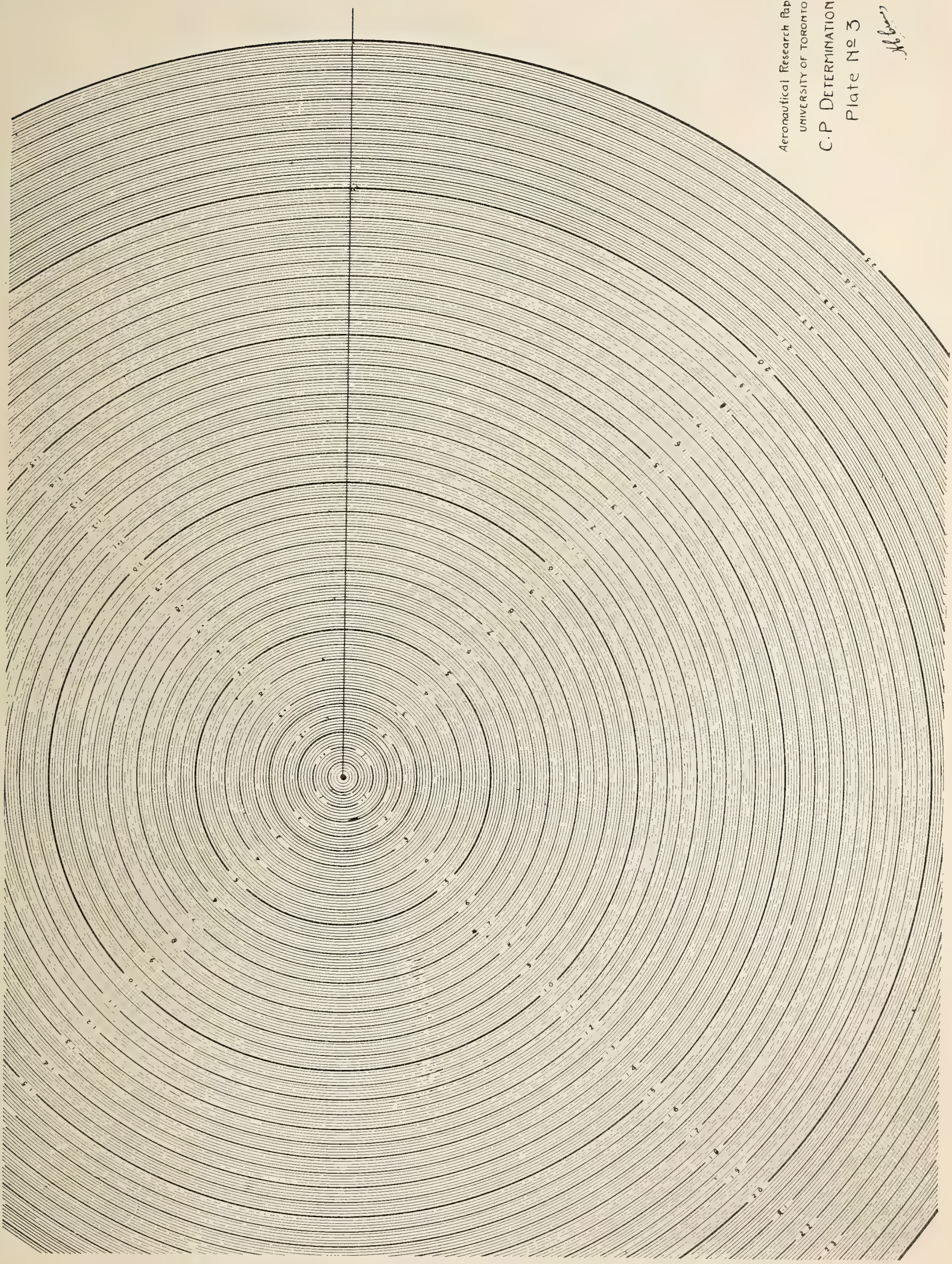
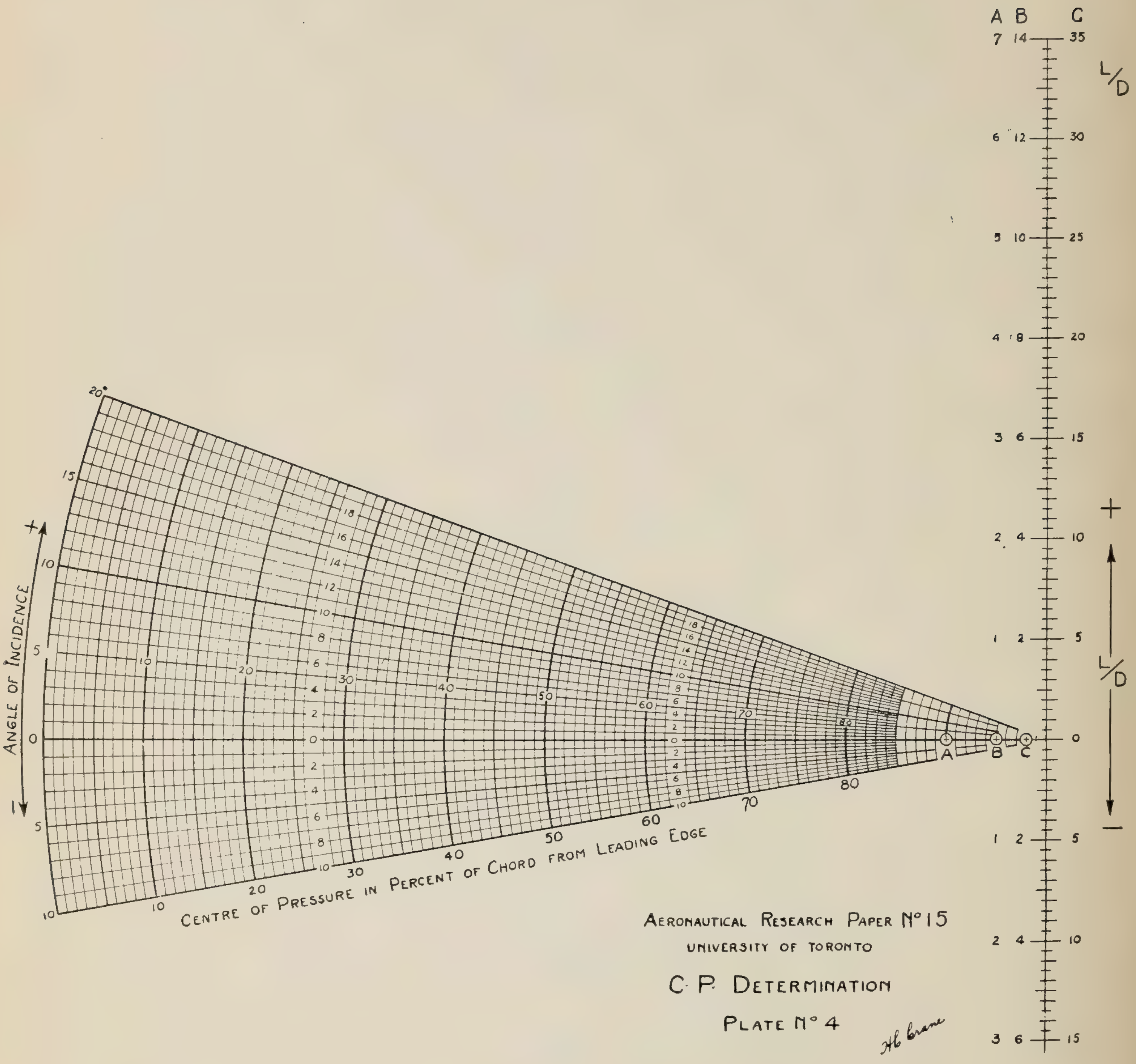
H.C.C. JAN '23



Aeronautical Research Paper N° 13
UNIVERSITY OF TORONTO
C.P DETERMINATION
Plate Nº 3



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C. P. Determination

Plate No 4

PLATE 5

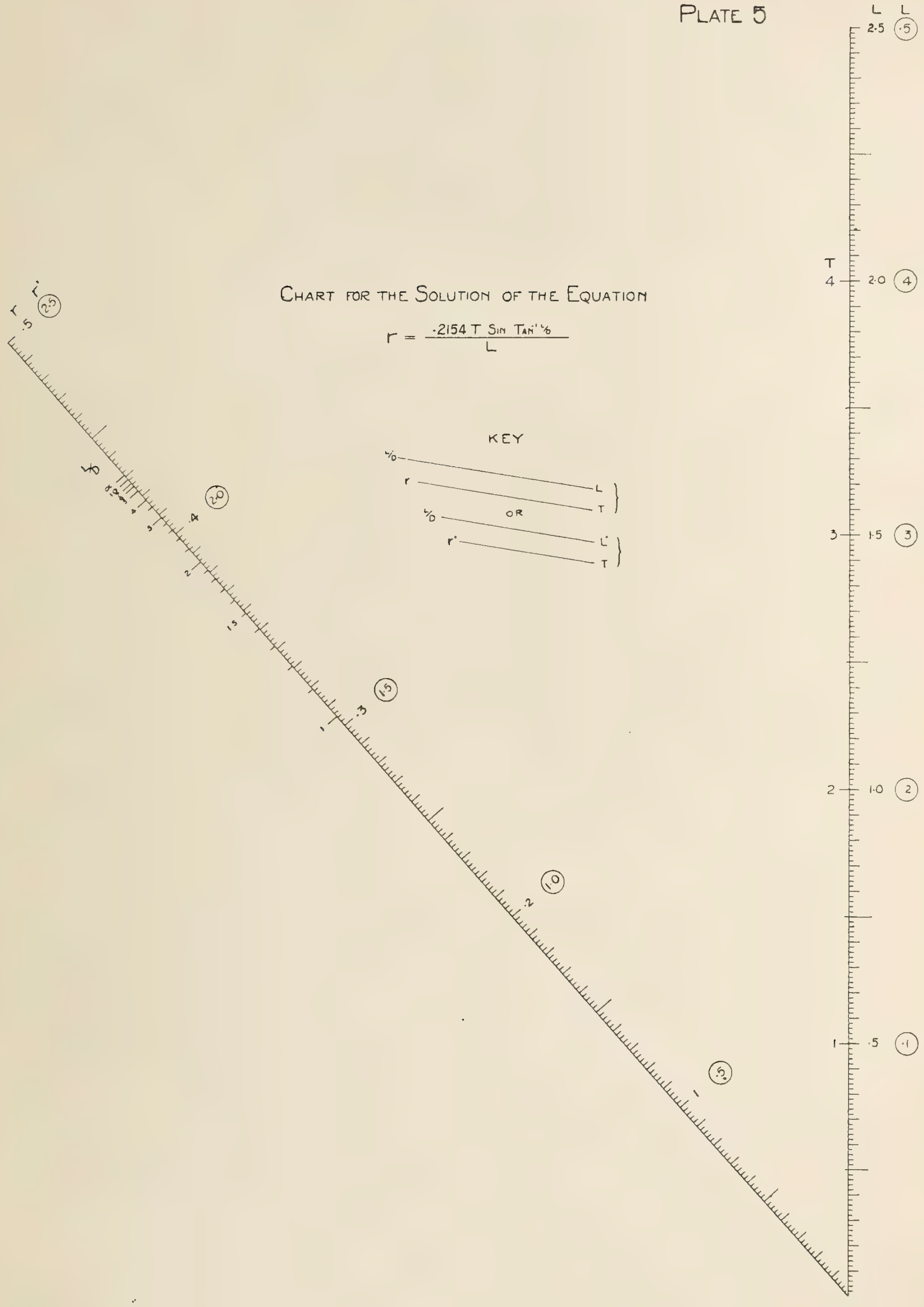

CHART FOR THE SOLUTION OF THE EQUATION

$$r = \frac{\cdot 2154\,T\,\mathrm{Sin}\,\mathrm{Tan}^{-1}\,L/D}{L}$$

GUNITE AS A PROTECTION FOR STEEL STRUCTURES

By PETER GILLESPIE, *Professor of Civil Engineering*,
and P. J. CULLITON, *Research Assistant*

In the month of December, 1921, two steel girder bridges in the city of Hamilton, Ont., forming part of the main highway between that city and Toronto, were covered with gunite in order to protect them from the corrosive action of locomotive blast since both pass over tracks where locomotive traffic is moderately heavy. Sometime in the month of May next following, it was observed that on both structures numerous small cracks in the gunite had developed. Their widths varied from that of a hair up to one-sixteenth of an inch and in isolated cases this latter limit was exceeded. It was almost impossible to measure their depths but it was believed that generally they did not penetrate past the plane of the reinforcing fabric. They did not lie in any general direction, some being vertical, others horizontal and still others having the diagonal direction of the reinforcing strands.

The specifications for this work called for a coating of gunite 1½ in. thick in which an expanded metal fabric weighing 34 lb. per hundred square feet was to be imbedded ¾ in. from the steel plates. The aggregate was to be a graded silicious sand free from deleterious matter and capable of passing a ¼ in. mesh. The mixture was to be 3 to 1. Metal surfaces were to be thoroughly cleaned of rust and scale before covering and gunite was not to be applied during a temperature lower than 35° F.

On bridge No. 4, the first to receive the coating of gunite, the cracks had not developed to the same extent as they had on bridge No. 5. On the former, cracks had extended over the flat areas between the web stiffeners and in more than one instance had continued across the stiffeners into the next panel. In many panels several cracks appeared and in each panel examined there was at least one visible crack.

The cause of the cracking, the remedy for it and the extent, if any, to which it exposed the metal beneath to corrosion were questions that naturally suggested themselves. In the effort to answer them, it was proposed to experiment on a scale somewhat larger than would be possible in a small laboratory. To that end The Canadian National Railways donated two superseded 42-ft. steel plate girders. Through the further co-operation of the Honorary Advisory Council for Scientific and Industrial Research, the Canada Cement Company, and the Cement Gun Company of Allentown, Pa., the experiment was made possible.

The girders were accordingly set up at Leaside, Ont., and to them gunite was applied, the thickness of coating, the reinforcing fabric, the ingredients and the proportions constituting the variables of the problem.

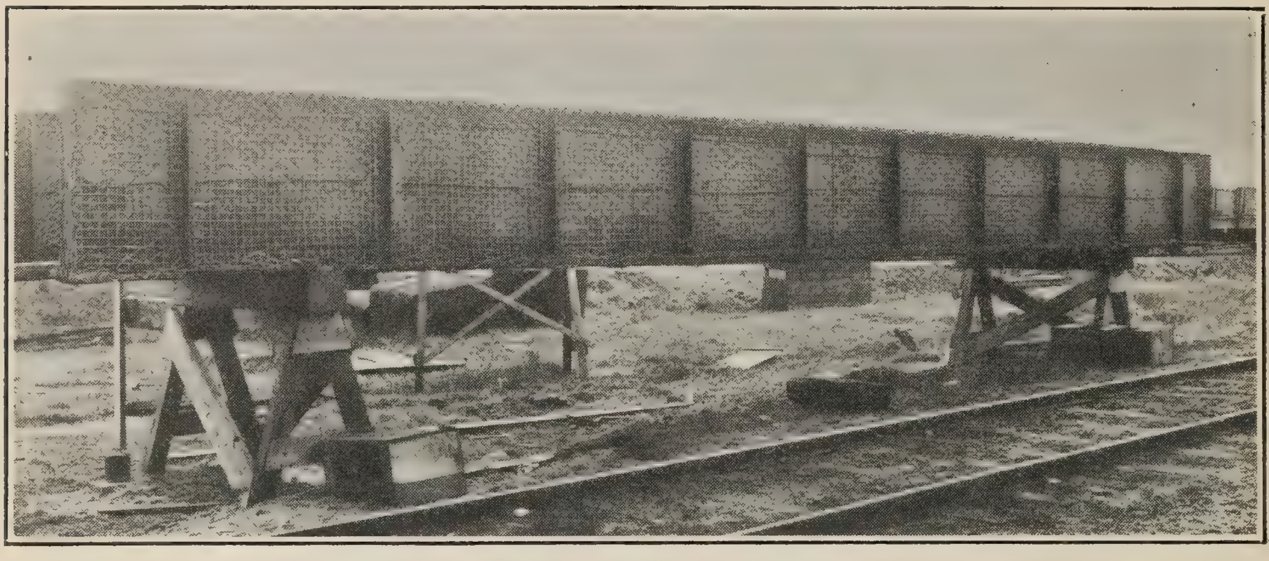

Fig. 1.—Leaside Girders prior to application of Gunite.

The observations made on the bridges at Hamilton, on the girders at Leaside and on numerous small scale specimens in the laboratory are briefly recorded here together with such conclusions as appear warranted by the evidence deduced.

THE PROGRAMME

After the girders had been cleaned of old paint, the webs were drilled and three horizontal lines of 3/8 in. round rod attached to either face in order to keep the reinforcement away from the plate. To these rods the fabric was wired. In each girder there were eleven panels. The north girder was covered with expanded metal commercially designated as 1¾-14-20. The north side of the south girder was covered also with expanded metal of larger mesh designated 3-14-10. The south side of this girder, with the exception of one panel, was covered with Greening square wire mesh. Three series of tests, A, B and C, were planned according to the following schedules.

Series "A"—North Girder. Two-inch thickness of coat using 1¾-14-20 mesh. Weight of steel per sq. ft., .68 lb. Two panels, one on either side of the girder, constitute a test.

West											East
Test No.	1	2	3	4	5	6	7	7	8	10	11
Mix:	1:4	1:3	1:4	1:4	1:3	1:3	1:3	1:4	1:4	1:3	1:4
Sand:	mixed	coarse	coarse	coarse	coarse	fine	fine	fine	mixed	mixed	fine
Adulterant:	lime		loam		clay		loam				clay

The percentages of adulterants were as follows:
A1 —lime, 6% of volume of cement
A3 —loam, 17% of volume of cement
A5 —clay, 10% of volume of cement
A7 —loam, 17% of volume of cement
A11—clay, 15% of volume of cement

Series "B"—South girder, north side. One and a half inch thickness of coat using 3-14-10 mesh. Weight of steel per sq. ft., .34 lb. One panel constitutes a test.

West											East
Test No.	12	13	14	15	16	17	18	19	20	21	22
Mix:	1:3	1:4	1:3	1:4	1:3	1:4	1:5	1:5	1:2½	1:2½	1:4
Sand:	coarse	coarse	fine	fine	mixed	mixed	mixed	mixed	mixed	mixed	Ottawa

Series "C"—South girder, south side. One and a half inch thickness of coat using two-inch square wire mesh. Weight of metal per sq. ft., .57 lb. An exception is test No. 32, where there is no reinforcing but to which two coats of gunite were applied.

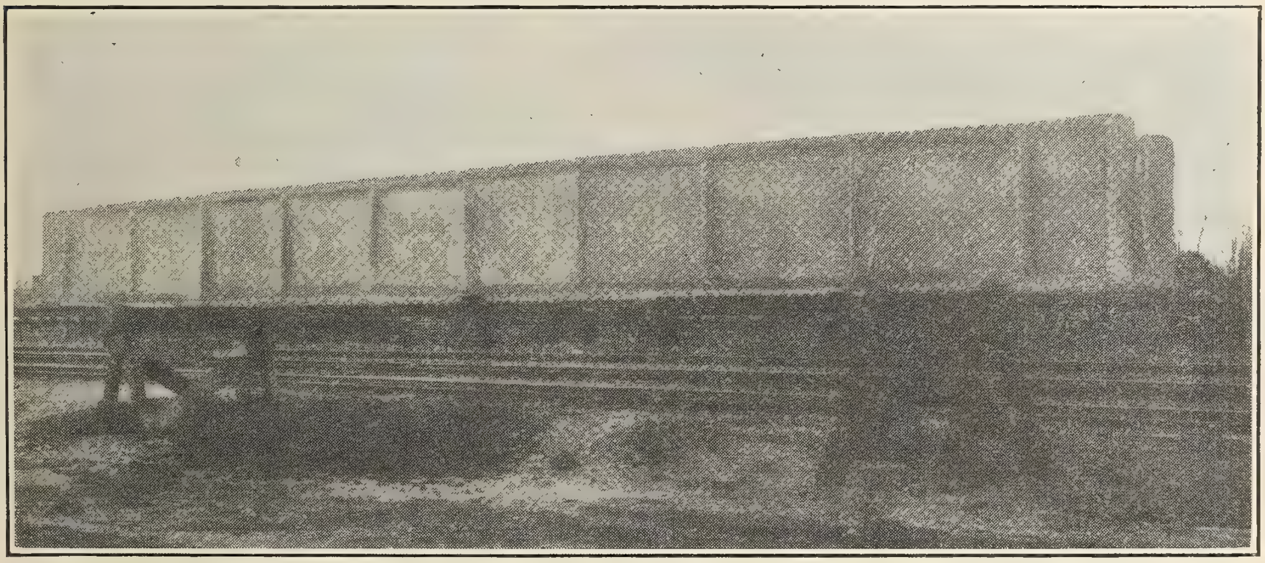

Fig. 2.—Leaside Girders after application of Gunite.

West											East
Test No.	23	24	25	26	27	28	29	30	31	32	33
Mix:	1:3	1:4	1:3	1:4	1:3	1:4	1:5	1:5	1:2½	1:4	1:4
Sand:	coarse	coarse	fine	fine	mixed	mixed	mixed	mixed	mixed	Mixed, no reinforcing on web	Ottawa

In addition to the above, eight small steel plates varying in dimensions from 18″ × 24″ to 24″ × 24″ stiffened on the edges by having light steel T's riveted to them were provided. These were coated in the field and afterwards transferred to the testing laboratory for observation. They are known as series "D."

MATERIALS USED

The sand used was a good grade of water-washed sand. The coarse variety was purchased in Toronto while the fine material was obtained from the Canadian National Railway stores. On certain of the tests the sands were mixed in equal proportions. Mechanical analysis by Tyler sieves is given herewith:

Coarse sand—500 gram sample.

Sieve	Percent Retained
4	.8
8	15.8
14	39.4
35	80.2
40	83.2
100	96.6

Fineness modulus, 3.16

Fine sand—500 gram sample.

Sieve	Percent Retained
4	.0
8	.0
14	.4
35	48.8
40	57.0
100	98.0

Fineness modulus, 2.04

Mixed sand—equal parts coarse and fine—500 gram sample.

Sieve	Percent Retained
4	.6
8	9.0
14	21.0
35	67.0
40	71.0
100	96.6

Fineness modulus, 2.65.

The cement was No. 1 Portland from the Canada Cement Co.

The water was taken from the mains of the City of Toronto at a pressure of 60 pounds per square inch.

The air was dried as completely as possible, having been passed through a dryer before it reached the gun. The pressure at the gun was 25 pounds per square inch except in test A2, where it was 20 pounds per square inch.

The lime was a good grade of slaked lime.

The clay was obtained from the Canadian Fireclay Products at New Toronto and was well-dried and finely-powdered.

The loam was taken from a compost heap of rotted leaves. This was well dried out and screened through a 20-mesh sieve. Anything larger than 20 mesh was rejected.

The compressor was an oil-fired Chicago Pneumatic compressor delivering air at 90 pounds pressure.

The cement gun was the standard gun manufactured by the Cement Gun Co., Allentown, Pa.

During the application of gunite on November 21st and 22nd, 1922, the temperature hovered around the freezing-point, but since the finished work was subsequently warmed by salamanders, it is believed that no injury due to frost occurred.

REBOUND TESTS

Tests to find out what percentage of sand rebounded from the girder during the application of the gunite were made. The sand was collected on spread tarpaulins and measured. It was found that the coarse sand had the largest rebound and the fine sand the smallest. The percentages are:

coarse sand........	16%
mixed sand........	12%
fine sand....	4.7%

This is obviously not the total rebound but it is a close approximation. All of the rebounding material could not be collected since some of it had been thrown as far as 100 feet.

EARLY CRACKS

By January 8th, 1923, a number of cracks, most of them insignificant as to width or length, were observed. Of these only the larger are reported.

On panel C28 a crack 6 in. long and 3/32 in. wide developed at the lower side of the upper flange. Its depth, gauged with a piece of No. 34 copper wire, was ¼ in. On panel C31, a crack 4 in. long and less than 1/32 in. wide, was observed on the plane surface of the panel. On panel C32, which has no reinforcing metal, and to which two coats of gunite had been applied, a crack 15 in. long and 1/16 in. wide had developed, but this apparently passed through the outer coat only which was about 3/8 in. thick. A hair crack 10 in. long in the lower right-hand corner was also observed. On panel B16, a very narrow crack 4 in. long just below the upper flange and another about half a panel high in the flat area were observed. A diagonal crack was also found on panel A2, having a length about half the panel width. In panel C25 a crack almost the total height of the panel occurred. Speaking generally, the 1:5 mixtures showed less tendency to check than others. Most of the cracks observed were of a character not easily recognized by a casual observer.

During the latter part of April, '23, a number of openings were made in the gunite, exposing both the reinforcement and the steel girder web plates. The panels into which cuts were made were C25, C26, C27, C28, C29, C30, C31, C32, B18, B19, A3, A5, A7 and A11.

In breaking through at the large crack on panel C32 a pocket of unmatrixed sand was found. Some red ink had been introduced to find out if possible whether this crack extended as far as the steel. This could not be determined but there was no indication of rusting of the web. At the hair crack on the same panel water had penetrated as far as the steel and there were two patches of fresh rust on the web. The rusting had doubtless been facilitated by the loose material which was here found in contact with the steel. Where the gunite was chipped away on panel C31 in the corner made by the stiffener angle and the web, the steel was well preserved, the gunite having been well driven into the corners and around the curved fillet of the angle. Rust on the reinforcement before coating seemed to have been absorbed by the gunite. The fabric was extremely clean and absolutely free from rust. On all the other "C" tests examined the material was of a well-matrixed character, protecting perfectly the girder and the reinforcing mesh which was found to be in all cases 3/8 in. from the plate. The good quality of the gunite is evidenced by the fact that it was extremely hard to chip with a heavy hammer and sharp chisel. The bond between it and the steel was very good although there were no cases of "feather edge" fracture. In nearly every case where gunite was broken pebbles in the aggregate were fractured across.

A3 and A7 contained loam while A5 and A11 contained clay. The gunite in these panels was the same dense well-matrixed substance found in tests not containing impurities. There was, however, some rusting on the side of the reinforcement facing the girder and this was more pronounced with the clay than with the loam.

When test A7 was chipped on the north side of the girder, it was found that the reinforcement was very close to the girder web. The result of this was that behind the strands of the expanded metal there were air pockets. It is believed that the gunite did not have a chance to penetrate behind the strands sufficiently to make a compact mass. There did not appear to be any rusting on the back of the reinforcement, doubtless due to a rebound of neat cement which had adhered to the fabric. The girder web was very well protected also. When the reinforcement was cut away the paths of the individual strands could be easily picked out by a sort of serrated surface into which the grains of sand had worked themselves behind the strands.

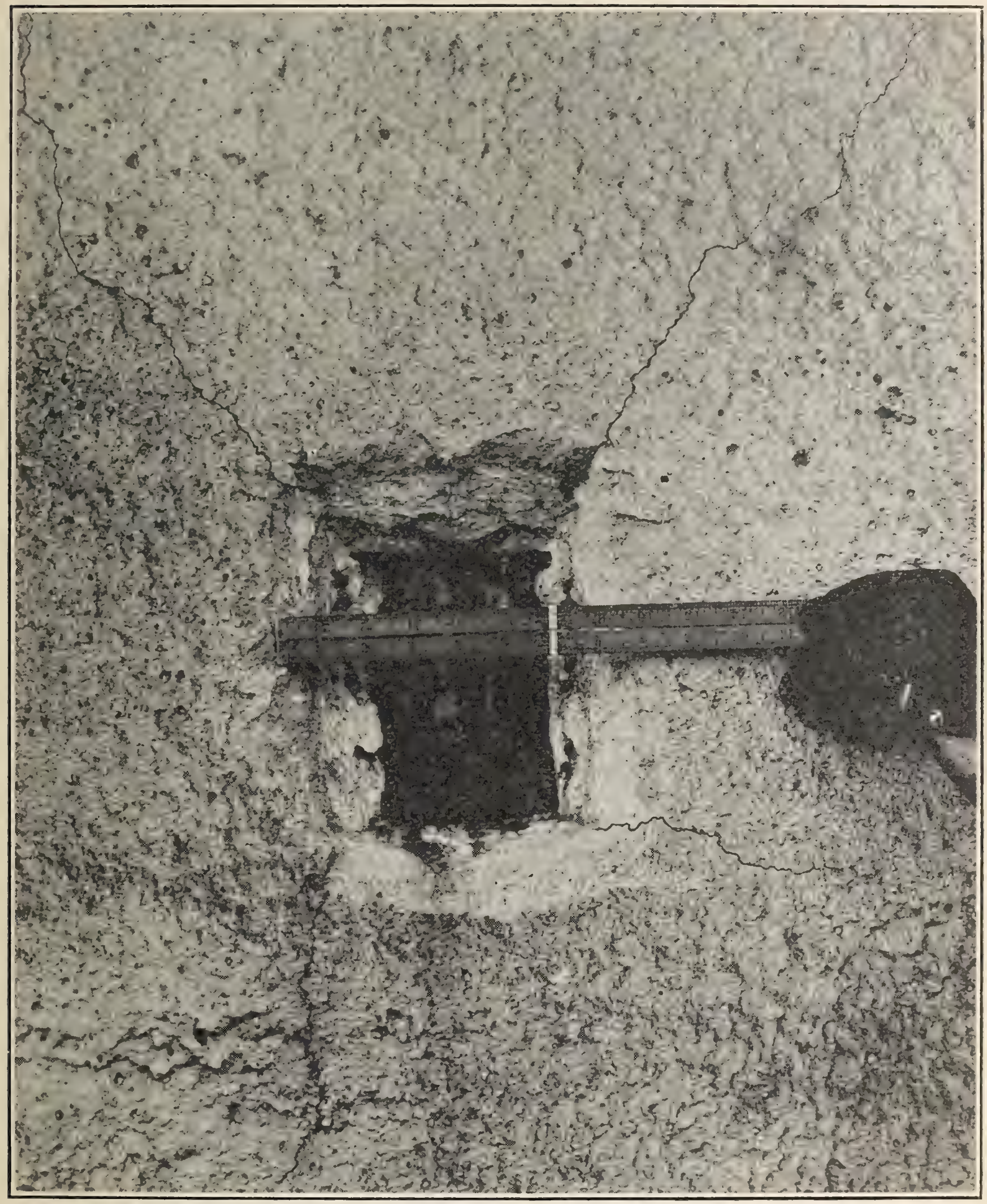

Fig. 3.—Cracks radiating from an opening chipped in Gunite.

The 3/8 in. steel rod holding the reinforcing was chiselled away on tests A3 and C30. There was no rusting visible either on the plate or on the back of the rod, the gunite having been well driven in behind the steel bar. Test B21 was chipped at a hair-line crack after a quantity of red ink had been introduced. It was found, on breaking the slab, that the crack had penetrated as far as the plane of the reinforcing mesh, but there was no indication that it had extended past it.

In December, 1923, a number of phenomena not previously observed were noticed. In practically every instance where an opening in the gunite had been made the previous spring, small cracks radiating therefrom were seen to have developed. This was particularly true where the layer of gunite was especially thick. In several instances vertical cracks corresponding to the edges of outstanding legs of stiffener angles had opened. When the material adjacent to such a crack was chipped away it was found that the gunite was less than ½ in. thick, and that the reinforcement was in close contact with the angle. Where cracks above stiffener angles had not developed, chipping showed the layer of covering to be thicker than ½ in. and the fabric less closely wrapped around the angle. In no instance, except on panel C32 which it will be recalled had no reinforcement, was it possible to follow a surface crack as far as the steel plate. In several cases, however, they were traced as far as the plane of the reinforcement. Nowhere was either the fabric or the plate in a condition other than perfect. The cracks were all of a width of .01 in. or thereabouts and in some cases due doubtless to the presence of soluble ingredients, these had completely sealed over. In all, twenty-three panels were uncovered to a greater or lesser extent.

SHRINKAGE ON A RIGID FRAME

Concrete, like wood, clay and some other materials, expands when it absorbs moisture and contracts when it is dried. Investigation has shown that some of the factors which influence the extent of this performance are quantity of aggregate, method of curing, quantity of mixing water and climatic conditions. It is known for example that a neat cement paste will "work" much more than a mortar and that a concrete will contract as much as .0005 per unit in hardening. If this tendency be resisted, as, for example, when an envelope of gunite is used to cover a plate girder, the concrete comes into a state of tension. This tension may easily be sufficient to rupture the shell. Probably the condition most favourable to cracking is where an area of gunite is not bonded to the plate beneath but is, outside of this area, together with its contained reinforcement, securely anchored to the metallic frame. This anchorage may be due to stiffener or flange angles, to rivet heads or to some favourable mechanical feature of the plate itself. The normal effect of shrinkage will be to create tension in the gunite but no stress in the reinforcement. When, however, the tension becomes great enough to rupture the gunite, tension must develop in the fabric at the region of failure and compression elsewhere. If the reinforcement be adequate in area, the stress therein will be moderate and the cracks microscopic. For this it is generally believed that one-third of one per cent. of metal is the minimum requirement. Obviously a high yield-point steel is preferred. A distributed fabric of adequate cross-section, therefore, while powerless to prevent cracks is very efficient in increasing their number and reducing their size. Where the shell of gunite is of unusual thickness, say 3 in., and where the fabric is quite close to the steel plate, it is evident that the influence of the reinforcement in controlling surface cracking is nearly non-existent. It was observed on the Leaside girders as stated above, that surface cracks which had developed to visible widths in thick shells of gunite could never be traced past the plane of the fabric. Another circumstance favouring the formation of visible cracks is the application of a surface coat of gunite which, for some reason or another, had failed to bond with the previous layer. Such a coat can easily be detected by tapping with a small steel hammer. Cracks have been observed, both at Leaside and at Hamilton, which pass through this surface coat only.

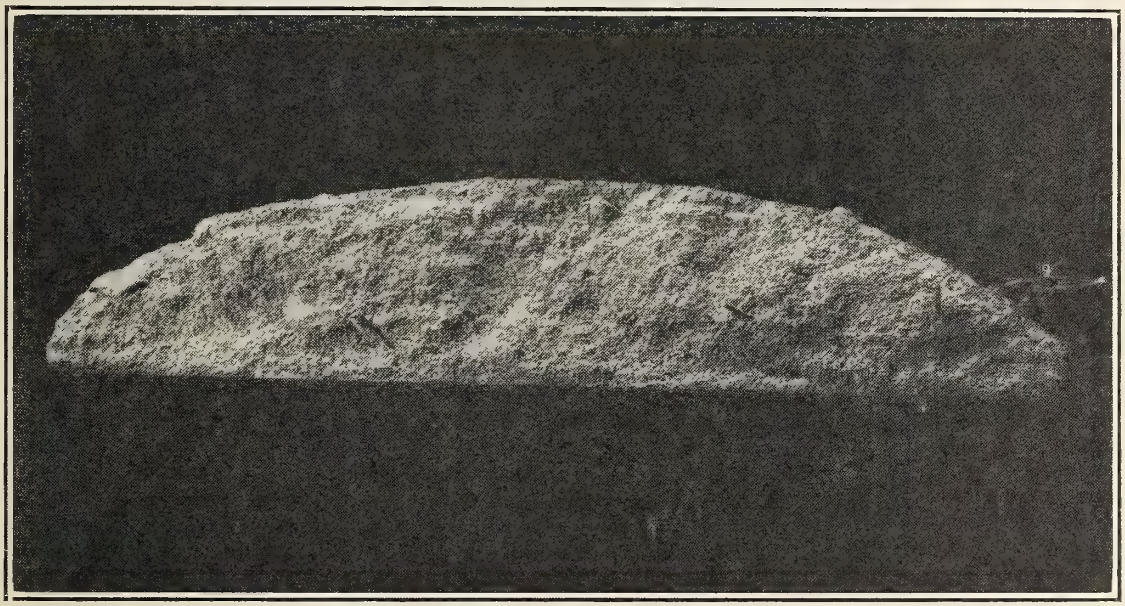

Fig. 4.— Section of Gunite Slab.

CORROSION

Neat Portland cement is known to be an effective preventive of rusting when applied to steel surfaces and on occasions has been employed as a paint for this purpose. To afford a satisfactory protection for steel reinforcement it is advisable that the metal be surrounded by a mortar rich in cement applied preferably wet, as this insures that all parts be coated. Gunite is a concrete essentially of this character, but since it is applied under pressure it possesses greater density than ordinary concrete. When the raw materials are forced under pressure against a hard metal surface there is considerable rebound—mostly of the sand. This means that the cement adheres to the surface as a matrix into which the particles of sand are subsequently driven. This matrix is a preventive of corrosion.

Figures 4 and 5 are photographs of fractured sections of a slab of gunite and a block of hand-placed concrete respectively, the mixture n each case being 1:3. The gunite is the denser of the two and being denser is more impervious to the entrance of water. An examination with a reading glass shows that the face of the slab, which was in contact with the steel, is a film of practically neat cement, any sand present being of an extremely fine character. The imperviousness of gunite and the layer of almost neat cement in contact with the metal surface are two circumstances which render corrosion difficult. If, however, cracks occur in the gunite, what may be expected?

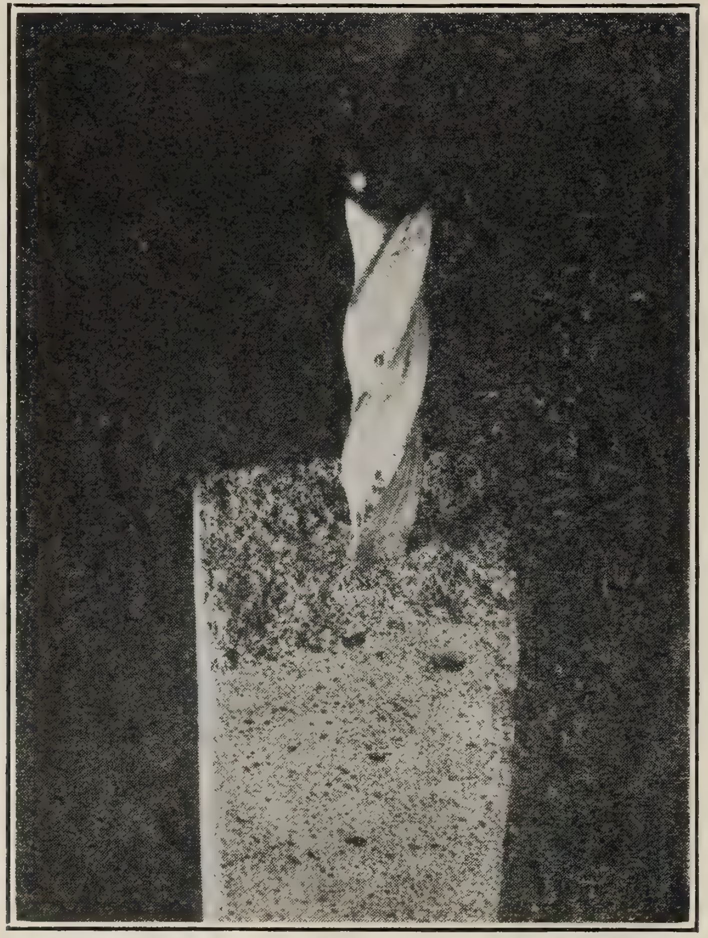

Fig. 5.—Showing spots of rust on Metal otherwise protected by Mortar.

Whether steel reinforcement in beams of reinforced concrete is in danger of rusting through the entrance of air and moisture at the cracks which inevitably occurs when such beams are subjected to flexure has been made the subject of much experimental investigation. Some years ago, at the Royal Testing Station at Berlin, it was shown that ordinary tension cracks occurring within the limits of permissible loading, do not allow atmposheric corroding influences to affect the steel. The entrance of moisture is to be feared only when the stress in the metal has passed the elastic limit and when in consequence the cracks are relatively wide. This occurred when stresses in excess of 35,000 lb. per sq. in. existed. The atmosphere was artificially moistened and supplied with CO_2 and free oxygen.

On two occasions, April, 1923, and January, 1924, examinations of the fissured gunite on the Hamilton bridges was made by chipping the material meanwhile following the cracks with red ink. Reinforcing fabric, consisting of 3 in. × 8 in. expanded metal, was well protected in all cases. In the first case, when the gunite was chipped down to the fabric, the ink was observed to creep along the strands, indicating an imperfect bond between gunite and steel. The crack was followed beyond the plane of the fabric to within a short distance of the steel plate when it was lost. In another case the crack was easily followed as far as the steel plate. There was rust on the web plate but it is thought that this had accumulated before the gunite had been applied. Several attempts to completely surround a block of gunite three inches square by chipping down to the surface of the steel plate were made. The "island" of gunite, however, invariably fell away just as soon as its last junction with the remaining body was severed. This indicated that the bond between the mortar and the steel had not been perfect. In still another case, where tapping with a hammer produced a distinctly hollow sound, the crack could be followed only through the surface coat which had not bonded with the layer beneath. In one instance the fractured concrete displayed clusters of magnetization lines characteristic of mortar or mud that has been frozen. The city engineer of Hamilton is of the opinion, however, that the gunite was not frozen in any case before it had a chance to set. Generally speaking the gunite at Hamilton was inferior to that on the Leaside girders. It contained pebbles of soft material and nodules of clay in places and was much more easily chipped than the gunite at Leaside. In no case on the Leaside girders had red ink, when applied, crept along the strands of metal, nor had cracks been followed past the plane of the fabric. In no case had striations or magnetization lines indicative of frost been observed there. Generally the bond between the gunite and the steel plate was distinctly better than on the Hamilton bridges. This may be the explanation of the deeper and wider cracks observed on the latter.

ABRASION TEST

This test was performed to compare the wear of gunite with that of mortar and concrete. Seven fragments of 1:3 gunite weighing 2557 grams, six fragments of 1:3 hand placed mortar weighing 2300 grams and six fragments of 1:2:3 concrete weighing 2744 grams were used. All samples were well seasoned. These fragments were placed in a Deval abrasion machine, together with ten pounds of Ottawa sand, and given 10,000 revolutions. The percentages of wear were as follows:

gunite	6.94%
mortar	15.65%
concrete	10.72%

After the test the number of pieces of gunite was seven, of mortar, ten, and of concrete, fifteen.

EXPANSION TESTS

These were undertaken to ascertain the extent to which gunite will expand when immersed in water for a number of days. To this end, pieces of gunite about 12 in. long, 6 in. wide, and 1½ in. thick were selected as specimens. These slabs were perforated at two points 8 in. apart for the reception of 3/8 in. round brass plugs, 4 in. long. These plugs were inserted in the holes and secured there by a cement of litharge and glycerine, the ends protruding 1¼ in. either way. The combination was then placed in a pocket of rubber tissue through perforations in which the extremities of the plugs protruded. The tissue, having been

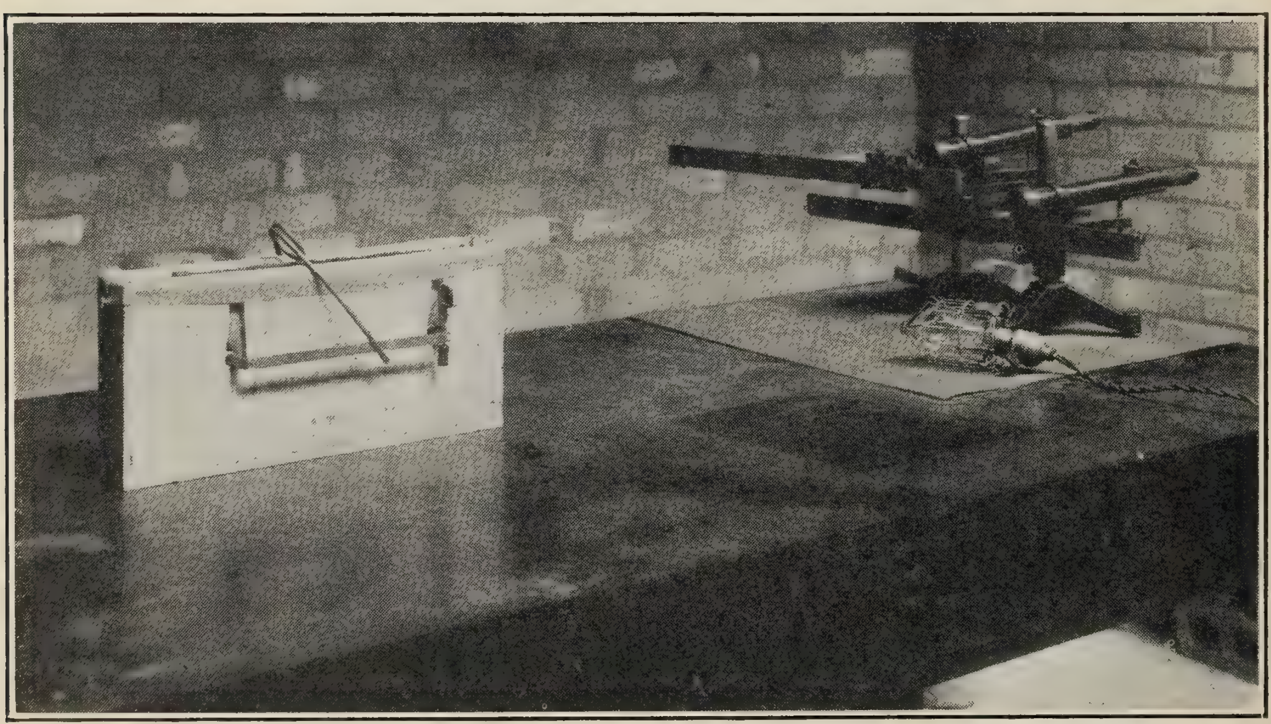

Fig. 6.—Device for measuring expansion of Gunite under water.

ligatured to the projecting plugs, there resulted a water-tight envelope. The whole was then placed in a rough pine box having four vertical slots at the bottoms of which the projecting plugs rested. To the extremities of these plugs the distance pieces of Martens extensometers were attached. The method consisted in mounting the specimen after thorough drying and recording the readings on the receiving scales. The rubber pocket was then filled with water and daily readings made for a period of ten days or until expansion had apparently pretty well ceased. These readings are taken by means of telescopes and admit of great accuracy. Temperature was maintained practically constant during the period of observation. Other disturbing elements were carefully eliminated.

The expansion per unit length between the dry and the saturated condition is given herewith. It is believed that had the time of observation been lengthened, small additional changes might have taken place:

1:4 gunite in 5 days' immersion lengthened .000185 per unit;
1:4 gunite in 17 days' immersion lengthened .000245 per unit;
1:3 gunite in 9 days' immersion lengthened .000212 per unit;
1:2½ gunite in 12 days' immersion lengthened .000195 per unit.

SMALL SIZE PANELS

The D series of plates were stiffened by 1½ in. × 1½ in. light weight T's which were rivetted to their edges. These plates were made up from old stock very much rusted in places and badly pitted by corrosion. Reinforcing fabric was applied to them and securely fastened on two opposite edges to the outstanding flanges of the T's which were drilled for the purpose. These plates were covered with gunite at the same time as the girders were coated and with the same mixtures. There were 8 plates in all and each plate was covered on both faces. These 8 plates were kept under cover at room temperature for three months. During that time it was observed that the gunite shrank away from two of the four flanges of the marginal T's on each specimen and that this shrinkage was along the edges to which the fabric was not attached. Practically no separation occurred between the gunite and the flanges of the marginal T's to which the fabric had been secured. These cracks varied from .015 in. to .04 in. and averaged perhaps .025 in. The widest separation was with the 1:2½ gunite. Subsequently these plates with their coverings were immersed in tanks where they remained for a period of 10 days, after which the tanks were drained and the specimens allowed to dry. This was continued for three cycles, after which they were taken out of doors and placed in a position exposed to the weather where they remained for 11 months. They were then taken inside and examined with the following results:

There were no cracks in the gunite although there were a few cases of surface "crazing".

The exposed edges of the stiffening T's were very much rusted. This rust had followed the metal inwards approximately ½ inch at the shrinkage cracks referred to above.

The fabric was in excellent condition except that the sheared edges of the expanded metal on the side toward the plates showed a very little discolouration due to rust. That the gunite may not have completely covered the remote sides of the strands is the probable explanation.

After the gunite had been completely removed it was possible to lift with a pen-knife, flakes of the original rust 1/32 in. or more in thickness. This rust had undergone no change during the months of its immersion and exposure and no additional corrosion had taken place.

Whenever an "island" of gunite was surrounded by careful chipping it separated from the plate. The separation may, however, have been the result of the impact of the hammer and chisel.

MICROMETER TESTS

These were made by means of a micrometer microscope, mounted on a tripod. This instrument had an accuracy of .0006 inch and by means of it the widths of a number of cracks which had developed in the gunite on the bridges at Hamilton were measured from time to time during the winter and spring of '22-'23 in order to find out what effect, if any, weather conditions had upon the material.

On December 21st, 1922, twenty-five points on different cracks were measured for width. Some of these were on the main girders; the rest were on the floor beams. The widths of the cracks varied from .004 inch to .04 inch. The sun was not shining although the atmosphere was quite clear. The temperature was 34° F.

On January 30th, 1923, when the temperature was 18° lower than on December 21st, the cracks at the same points were again measured, but no differences were observed. Apparently the steel substructure and the envelope of gunite had responded in the same way to the fall in temperature as indeed they might be expected to do unless the fall in temperature were a very rapid one.

A third measurement of the widths of the cracks was made on April 6th, following dull wet weather. The day was fine and the temperature was 48° F. Some of the cracks had not changed in width while the rest showed some contraction. These changes as given in percentages vary from 8% to 30% except one crack which contracted 65%. Evidently a humid atmosphere for a period of two days had resulted in a swelling of the concrete and a narrowing of the cracks.

CORROSION TESTS

A cabinet of beaver-board was constructed over an air outlet in the wall of a laboratory. The draft in this flue could be mechanically regulated so that passage of air through the cabinet was under complete control. The front of the cabinet was the door. Steam from a small gas-heated boiler was supplied as desired at the bottom of the cabinet. Racks were provided within, upon which specimens could be placed.

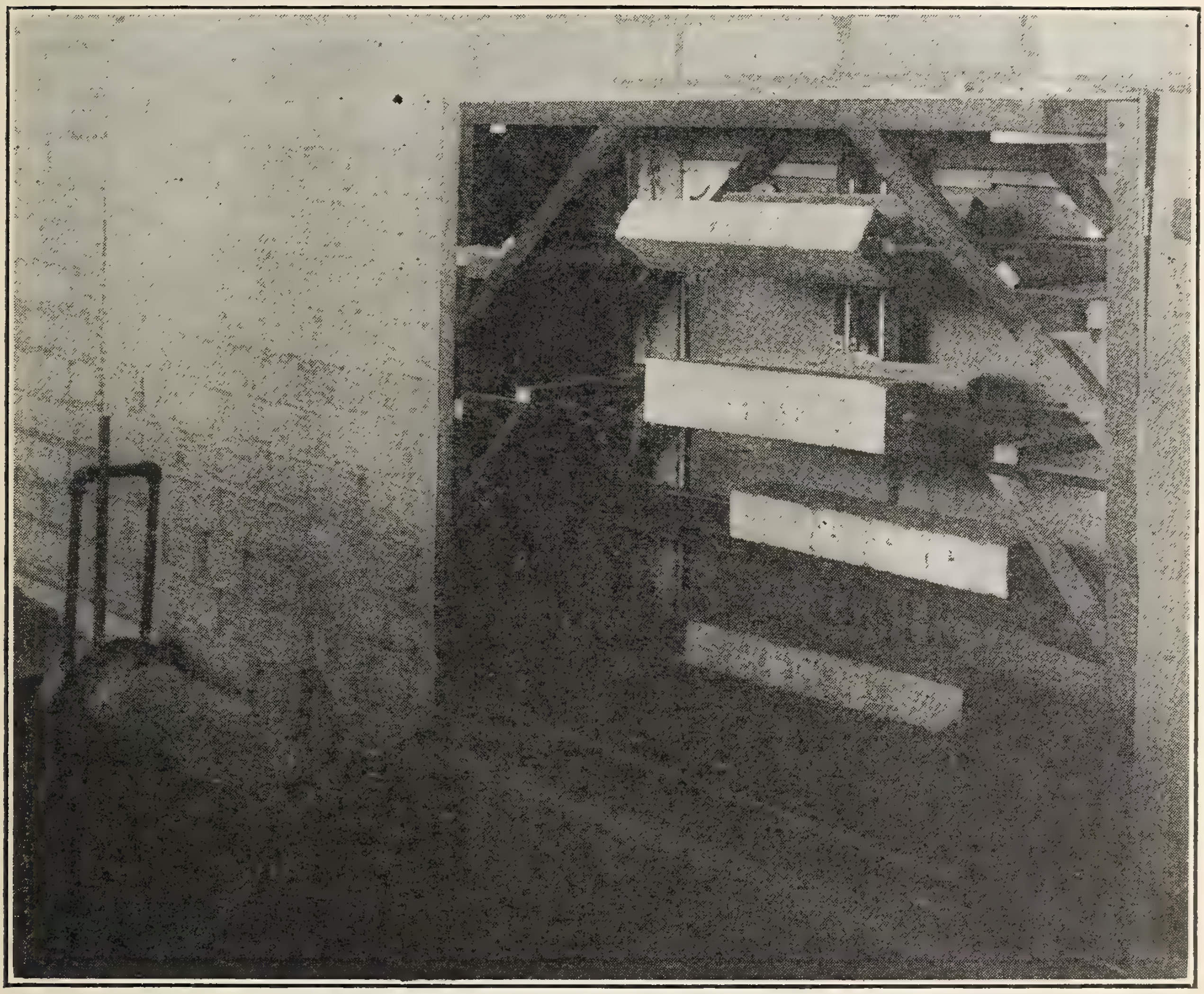

Fig. 7.—Cabinet in which Corrosion Tests were carried on.

A number of 1:3 mortar prisms, rectangular in form, hand-mixed and hand-placed, were made for corrosion tests. The sand was screened below 10 mesh. The specimens measured 2½ in. × 2½ in. × 12 in. long. Running axially through the centre of each was a soft steel core with projecting ends. After curing for two weeks they were stretched in a testing machine until cracks appeared in the mortar envelope. They were then subjected to a controlled humid atmosphere in the cabinet described above to ascertain whether corrosive influences would penetrate the cracks as far as the steel core. A wet and dry bulb thermometer was placed in the cabinet and the humidity therein determined from it. This was never less than 80 and in most cases it reached 100.

First Test—A number of the specimens were placed on shelves within the cabinet and alternately steamed and dried for three weeks, a slight draft meantime being maintained through the flue. The cracks in the specimens varied from .006 in. to .05 in. A block having a crack .04 inch wide was destroyed after the test and examined. The steel core a ½ in. square twisted bar, under the portion broken off, was observed to have spots of heavy rust. This was probably due to moisture filled cavities contiguous to the steel, where puddling had failed to bring the mortar in actual contact with the metal. These spots, however, were on the underside of the steel core. The steel exposed by the crack showed no rust but the ends of the cores had collected a heavy coat of rust and the metal parts of the wet and dry bulb thermometer showed considerable corrosion after three days. One of the small gunite covered plates was placed inside after its gunite covering had been removed. A very fine film or coat of neat cement still covered the metal. After three days the surface of the plate protected by the film was uncorroded while on the back of the plate corrosion had taken place.

Second Test—For this test the draft was completely shut out of the cupboard. The test lasted nine days with alternate steaming and drying. At the end of that time a specimen having a crack .02 in. in width was destroyed. The core, which was a 3/8 inch square bar, was not rusted at the crack but the uncovered ends of all cores were heavily rusted. Several small spots of rust were found on the underside of the core. This would seem to indicate insufficient puddling of the concrete.

Third Test—One of the small plates from which the gunite had been removed, and the steel core of the specimen destroyed after the first test, were steamed. The film of cement was intact on the plate and covered most of the surface of the rod. The core gradually developed spots of rust deepening in colour until at the end of the seventh steaming it was completely coated. The plate developed rust much less rapidly but when the 10th application of steam had been made, rust had formed over most of its surface, the coating of cement not availing to prevent its formation.

Fourth Test—For this test all the specimens left from the first test were used. The cracks had been widened by stretching the rods until a maximum of .08 in. was secured. The specimens were steamed twenty-five times, after which the specimen having a crack .08 in. wide was destroyed. Examination showed that there was no rusting of the steel exposed by the crack. There were three spots of rust on the underside of the steel core doubtless caused by the moisture cavities when the specimen was made.

Fifth Test—Four specimens which were not destroyed after the fourth test were used. The cracks were mechanically widened until a maximum width of .17 in. had been reached. The specimens were steamed twenty-two times after which the specimen having a crack .17 in. wide was destroyed. Examination showed no rusting to have occurred on the steel exposed by the crack. There was, however, rusting on the rod at the end of the mortar as if water had crept along the steel beneath the concrete. There were also spots of rust on the undersides of the steel core indicating insufficient puddling.

Sixth Test—Six specimens of similar form made of 1:3 hand-placed mortar and each having a burnished steel core of either square twisted or round steel rod, were exposed to a humid atmosphere in the steaming cabinet for one month after stretching. During that time the exposed ends of the metal rods rusted excessively. When the mortar was broken away rusting was observed in only three specimens and here the fractures had widths of .15 in. or more. On the bottom sides of the rods rust spots were frequently found. These were doubtless due to insufficient puddling. The results are as below:

Thickness of Covering	Width of Cracks—in.	Remarks
1 inch	.01 and .03	No rust at break
1 inch	.20	Rust on all four sides of rod at break
1 inch	.15, .15, .18	Rust on all four sides of rod at break
1 inch	.05, .23	Rust at larger opening only
1 inch	.03	No rust at break
¾ inch	.02	No rust at break

Seventh Test—These were two specimens of the same form as those reported above except that the box forms had neither top nor bottom and were filled with gunite from two opposite faces instead of from one only. The depth of covering to the burnished steel cores was one inch. They were stretched in a machine until cracks of substantial width appeared after which they were exposed for three months to the action of a humid atmosphere in the steaming cabinet. Twice daily for two hours the steam was admitted morning and afternoon. Pieces of bright metal suspended in the cabinet and the exposed ends of the core rods rusted excessively. At the end of three months the gunite was broken off and although the cracks varied from .06 in. to .09 in. not the slightest evidence of rust was observable on the rods either at the cracks or elsewhere. The burnished rods were quite as bright as when the gunite was applied.

EXPERIENCE WITH GUNITE ELSEWHERE

The experience of a number of representative American engineers with gunite as a protective coating for steel structures has been obtained by correspondence. While the reports are not in perfect agreement the consensus of opinion is that gunite affords a very substantial protection against corrosion of steel.

Satisfactory results have been obtained by the Grand Trunk Railway System from this method of protecting steel against locomotive exhaust.

The Missouri Pacific Railroad Company have two bridges coated with gunite. The coating was applied over steel girders in a highway viaduct crossing tracks. The gunite is over two years old, but so far there has been no apparent deterioration.

The New York Central Railroad at Columbus, Ohio, has used gunite as a protection for steel bridges for about ten years and has found it to be quite a satisfactory covering.

The Division of Engineering Construction of the Department of Public Service of the City of Columbus, Ohio, is satisfied that gunite is an efficient protection for metal exposed to the exhaust from locomotive stacks. Columbus has two viaducts over railroad tracks which were covered with gunite, one in 1913 and the other in 1914. It has been the practice to make careful inspection of the coating every year and in January, 1923, there were no signs of cracking or disintegration. The City used wire mesh reinforcing.

In 1921 the Delaware, Lackawanna and Western Railroad Company used gunite as a protection on the Chenange St. viaduct at Binghampton. In this instance the work has proved entirely satisfactory, no cracks having developed. Very little wear or abrasion has occurred due to locomotive blasts. The coating of gunite over reinforcement was nowhere less than ¾ in. in thickness. On flanges having button head rivets it was 1¼ in. thick. On bottom flanges with no button head rivets it was 1 in. thick, while on web plates of girders and brackets it was ½ in. thick. It is believed that the complete removal of all scale from the metal before application of gunite is very important.

The experience of the New York, New Haven and Hartford Railroad Company in respect to eleven bridges in the South Boston cut improvement is interesting. On these bridges in 1919 a 1½ in. coating was placed, using as reinforcement 2-inch Clinton wire cloth mesh, No. 12 gauge. In a number of cases the gunite has cracked and spalled so as to expose the reinforcement, but in these instances there is almost continuous switching on the tracks. Moreover, switchers were allowed to pass under the bridges within a few days after the gunite had been applied. Where switching is not so continuous the coating appears to be fairly well preserved. The clearances above track vary from 17 ft. to 19 ft. 9 in.

Respecting a highway bridge at Tiverton, R.I., coated in 1913, the same Company reports also. Portions of this structure were exposed to salt water spray. The coating was 1¼ in. thick, a mixture of 1 cement to 2½ sand placed on No. 16 gauge, 2-in. Clinton wire cloth. A few stiffener angles and sidewalk bracket angles now show rust through the thin layer of gunite which has cracked along the outer edge of the outstanding leg of the angle. It was found, on cutting back from the edge of the stiffener angle ¼ in., that the steel was thoroughly protected. The chief engineer's opinion is that experience has not been sufficiently extensive to determine whether gunite as a protection for steel on such low clearances as obtain in South Boston cut is the best material. Apparently it is not going to give the measure of protection that concrete jack arch construction between stringers affords. Gunite, in his experience, has been a better protection against salt water spray than against locomotive blast.

The Chicago, Burlington and Quincey Railroad Company have used this form of protection for about ten years and where properly applied it has not cracked. It has resisted locomotive exhaust very much better than poured concrete. Generally the protection has proved very satisfactory and it is the policy of the Company to continue its use.

The experience of the Northern Pacific Railway Company has been that wooden blast boards constitute the most effective means of protecting steel against blast. These, on the contrary, furnish little or no protection against gases which diffuse themselves through the structure and if the design be such that these gases become confined in pockets, the corrosive action is particularly destructive.

It is the belief of the Terminal Railroad Association of St. Louis that, if properly applied, gunite is an effective protection for steel. In their opinion the cause of the falling off of gunite is improper application.

The Kansas City Terminal Railway Company finds that poured concrete will not last when exposed to locomotive exhaust. This Company has now eleven structures protected by cement gun encasement, the first work having been done in 1913. A recent examination showed the coating to be standing up very well and to be in good condition. On one of the viaducts the overhead clearance is less than 17 feet. In other cases reinforcement protected by poured concrete was found in time to corrode, presumably owing to the penetration of locomotive gases.

During the years 1916 and 1917, the undersides of about seventeen bridges on the Boston and Albany Railroad in the City of Newton were gunited. Prior to this paint could not be kept on these structures due to the action of locomotive blast and the steel was rapidly deteriorating. The clearance was extremely low. Gunite was used but on account of the dense traffic the work had to be done at night so that the application was probably not what it should have been. It has, however, stood up exceedingly well. The gunite did not show any indication of failure until 1922, when there was cracking on practically all the bridges and some indication that the covering was spalling. It is the opinion of the chief engineer that the trouble was chiefly due to leaking from the top through the failure of the waterproofing. The water had percolated down behind the gunite and frozen there, thus breaking the material loose. It is also his experience that so far as resistance to the action of locomotive exhaust is concerned, there is no material superior to gunite except perhaps heavy plates of cast iron.

Gunite has been used by the Chicago, Milwaukee and St. Paul Railway Company for the purpose of protecting against the action of locomotive blast. In one case a portion of a structure was covered with gunite without reinforcement and the remainder with gunite with wire mesh. This has been on for a number of years and recent inspections of the work indicate that where proper reinforcement has been placed around the steel before applying the gunite and where the coating has been reasonably thick, it is successfully resisting the action of fumes from locomotives. It also indicates that where gunite is placed without reinforcement, and particularly where the steel work is subjected to deflection, the coating will eventually crack and fall away.

In May, 1920, the Pennsylvania System Central Region did some guniting and repairing on two bridges spanning their tracks at Oil City. Poured concrete had proved to be unsatisfactory in resisting locomotive blasts which were rather severe since the clearances are 16 feet and 18 feet respectively. These blasts and the vibrations due to traffic caused some of the gunite to break off before it had reached final set. The steel, however, was not exposed. No cracks have developed since and the blast of the locomotives is being resisted remarkably well. The same condition obtains on work done in 1919. Recent inspections show the material to be in good condition. No cracks are observed and the gunite is meeting the conditions better than concrete.

The Chicago, Rock Island and Pacific Railway Company did some guniting in 1913 and 1914. The engineering department reports that, from the service obtained during the past seven years, it appears to be a satisfactory protective coating. Some hair line cracks have formed but apparently, through a chemical action, they closed up in a very short time, and although no inspection has been made, it is believed that no action detrimental to the steel has resulted.

The bridge engineer for the Oregon State Highway Commission cites the interesting case of a bridge built over the Willamette River at Oregon City. The location of the bridge is in the immediate vicinity of large industrial plants which employ sulphurous acid as a reagent with the result that at times the atmosphere is more or less densely charged with sulphurous anhydride. Corrosive action in this atmosphere is therefore sensibly stimulated. Part of the bridge was gunited as a protection. The covering was 1½ in. thick and was shot against the steel which was wrapped with triangular mesh. On the arrival of hot weather quite a number of hair cracks developed. None of these are of a serious nature and probably do not extend very far below the surface. The majority of them are crzae cracks running in every direction with no regularity whatever. There are, however, a few well-defined radial cracks which are thought to have formed over radial reinforcing rods believed to lie too near the surface.

The Delaware and Hudson Company has used gunite to protect the steel of roundhouses and it has given better satisfaction than other methods of protection.

The New York Municipal Railway Corporation have used gunite on a portion of their elevated railroad structures and on the girders of their shops in Brooklyn. The work was done in 1918-1920 and up to the present is in perfect condition.

The Hudson and Manhattan Railroad Company have used gunite for air ducts in the boiler space of the power house, where it is exposed to hot ashes, boiler gases and steam, with entirely satisfactory results.

CONCLUSIONS

The following conclusions appear to be warranted in the light of observations covering two years on the Hamilton bridges and nineteen months on the Leaside girders:

In the application of gunite, coarse sand rebounds to a greater extent than fine sand.

Local pockets of unmatrixed sand resulting from inadequate mixing of ingredients in the machine are a cause of rusting on both plates and reinforcement.

Impurities such as clay and loam seem to favour rusting of the imbedded fabric.

The expansion of gunite between the dry and saturation points is about .0002 per lineal unit.

Reinforcing fabric is necessary where steel work is to be protected by gunite. It should be sufficient in amount to enable it to assume, in event of rupture of the gunite, the tensile stresses produced therein by shrinkage.

The reinforcement should be not closer to the plate than 3/8 in. One and one-half inches should be an adequate overall thickness for the covering layer.

Where the fabric is tightly wrapped over stiffeners and the covering thereon is less than ½ in. thick, vertical cracks corresponding to the outstanding leg of the angle tend to develop. These cracks sometimes extend through to the steel angles and in several cases rusting of the reinforcement has occurred. Vertical cracks obviously permit the entry of water more easily than horizontal cracks.

The application of a top or surface coat not adequately bonded to the first favours the formation of surface cracks.

Cracks in gunite at the Leaside girders can be traced only as far as the plane of the reinforcing fabric. The only exceptions to this are in the panel having no reinforcement and on the edges of stiffener angles.

Lean mixtures check less than the rich ones.

Frequently when gunite has been chipped from the centre of a panel, cracks radiating from the opening have been observed to develop in the course of a few weeks following the chipping. This is particularly noticeable where the shell of gunite is very thick. This shows the effect of lack of continuity in the covering envelope.

Widths of cracks in gunite respond more readily to humidity changes than to temperature changes.

A layer of gunite applied to heavily rusted steel plates resulted in the complete suspension of the rusting process when the plates, together with their covering, were alternately immersed and dried and exposed to severe weather conditions during a period of nearly 12 months. The imbedded fabric was always perfectly preserved.

A humid atmosphere failed to produce rust in three months on bright steel rods covered by one inch of gunite in which cracks .09 in. and narrower had been made. Rust, however, was produced by this same atmosphere in one month on bright steel rods covered by one inch of hand-placed mortar in which cracks .15 in. and wider had been made. Uncovered steel rusted excessively under these conditions.

Spots of rust on steel cores of hand-made specimens appear to be the result of insufficient puddling of mortar when the specimens were made. It is believed, however, that such rusting soon ceases under the covering shell.

Fabric which had been completely surrounded by gunite was always perfectly preserved against rust.

Where good materials and workmanship are assured, and the weather during application is favourable, gunite properly reinforced is believed to afford a satisfactory protection for steel structures against corrosion.

TORSIONAL STRENGTH OF STEEL I-SECTIONS

By C. R. Young, *Professor of Structural Engineering*, and
C. A. Hughes, *Instructor in Applied Mechanics*

Cases of torsion on steel I-sections are comparatively common in engineering practice. They arise in such members as circular girders supporting elevated tanks, in balcony girders for theatres, and in I-beams or plate girders loaded more heavily on one side than on the other, particularly where brackets are used for the support of applied loads. In Europe, bridge girders or trusses are sometimes constructed of a circular form in plan, although this type of construction is unknown in America, so far as the authors are aware.

While the methods of determining the torsional moment at a given cross-section of such beams and girders as have been mentioned above have been presented in several works, little has been written concerning the proportioning of a cross-section of I-form for torsion. Thus, while torsional moments are discussed in Hazlehurst's *Towers and Tanks for Water Works,* in Ketchum's *Structural Engineers' Handbook,* and in de Fontviolant's *Design of Circular Bridges,* these authors do not deal with the proportioning of the cross-section. By reason of the complication of the mathematics for even a simple rectangular cross-section, it is entirely impracticable to derive a theoretically exact formula for the critical stresses on an I-shaped cross-section due to torsion.

Bach, in experimenting on I-sections of lead, found that rupture occurred first at the mid-point of the side of the web and that the maximum stress there realized might be given by the formula:

$$q_m = \frac{T}{At} \qquad (1)$$

where T = torsional moment on the cross-section, A = total area of the cross-section, and t = thickness of the web. While this formula might apply in instances where the thickness of the web is greater than the average thickness of the flange, it would not apply where the reverse is the case.

WEBER'S APPROXIMATE THEORY

An able discussion of torsional stresses on sections of many forms was presented by Constantin Weber in *Zeitschrift des Vereines deutscher Ingenieure,* for August 12th, 1922. Based upon a theoretical study, expressions are given for the moment of resistance of an I-section and for the maximum stresses in web and flange due to torsion. These, however, are based upon the angular distortion in radians per unit of length of specimen, and as this distortion is not known to the designer, the formulae are not immediately useful for design purposes. Weber, however, points out what apparently was not clearly stated by Bach, namely, that where the average thickness of the flange is greater than the thickness of the web, the maximum torsional shearing stress will occur at the centre of the face of the flange and not at the centre of the side of the web.

Striking confirmation of the truth of Weber's observation on this matter was contained in the researches made for the British Advisory Committee for Aeronautics and described by G. I. Taylor and A. A. Griffith in volume 3 of the technical report of the committee for the year 1917-18. It had been pointed out by Prantl that the equations which represent the torsion of an elastic bar of any form of cross-section are identical with those which represent the displacement of a thin membrane stretched over a hole of the same shape as the cross-section of the bar, when the membrane is distorted slightly by a small uniform load acting over all its surface. Accordingly these investigators stretched a soap film over an I-shaped hole in an apparatus so designed that a small difference of pressure could be maintained between the two sides of the film. As the torsional shearing stress at any point of the cross-section of the assumed bar is proportional to the inclination of the soap film to the plane of its boundary at the corresponding point, it was easily possible to determine by precise measurements of deformation of the film the points of maximum shearing stress on a cross-section of the form under consideration. This experimentation showed conclusively that where the average thickness of the flange is greater than the thickness of the web, the shearing stress is greater at the centre of the face of the flange than at the centre of the side of the web. If, however, the radius of the fillets is small the shearing stress may be greater in them than anywhere else on the cross-section.

Useful experimental work was done on the torsional resistance of I-sections by E. G. Ritchie and reported in Gibson and Ritchie's *The Circular Arc Bow Girder.* The angular distortion of a series of I-sections was measured under known torques and by comparing this with the angular distortion of a round shaft of the same polar moment of inertia and of the same material, it was possible to ascertain the effective polar moment of inertia of the I-sections. This ratio proved to be very small indeed, varying between the limits of 0.83% and 3.44%. It is believed by the present authors, however, that these results do not form an absolutely logical basis for design. Mr. Ritchie's experiments show the ratio of the deformation of an I-section to a solid round section of the same polar moment of inertia for light loads, but they do not show the ratio of the elastic limit of the I-section to that of the corresponding round shaft. It is conceivable that the angular distortion ratio might be greater or less than the ratio of the torsional elastic limits. For that reason it was thought best by the present authors to ascertain, if possible, the ratio of the elastic limit of an I-section to that of a corresponding round shaft and thus ascertain the part of the tabular polar moment of inertia of an I-section that is really effective.

UNIVERSITY OF TORONTO TESTS

In order to provide an experimental basis for determining the torsional strength of I-sections and for proportioning such sections for given loads, a series of tests were undertaken by the authors in 1921 under the auspices of the School of Engineering Research of the University of Toronto. Since a basis of design was primarily sought it was thought well to try to determine particularly the elastic limit of the I-sections tested.

Eight I-beams ranging in depth from 3 to 6 in., 3 ft. in length, and with two weights for each depth were made the subject of the tests, as shown in Table 1. An effort was made to use the minimum and maximum weights for each depth, but the stock available in Toronto at the time did not permit of such selection in all cases, and consequently an intermediate weight had to be employed in the case of one 6-in. I-beam. The material was of ordinary structural grade with an average ultimate shearing strength of 45,000 lb. per sq. in., and an average shearing elastic limit of approximately one-half this amount.

The torsion was applied by means of the special grips described in bulletin No. 3, section 9, of the School of Engineering Research in connection with torsion tests on reinforced concrete rectangular sections. A 140,000 in.-lb. Olsen torsion machine was employed. Mirrors were attached to the flanges of the sections at 12 in. centres by means of sealing wax. For this reason the webs were in all instances placed horizontally at the outset of the test. Angular distortions were measured by means of telescopes and scales placed 30 in. from the specimen in the same manner as described in bulletin No. 3 for the tests of concrete sections already noted. The deformations were read at increments of loading of from 600 to 1000 in.-lb. up to the time when the mirrors dropped off the specimens due to pronounced distortion. The torque

TABLE 1—RATIO OF OBSERVED TO THEORETICAL (ROUND SHAFT THEORY) TORSIONAL STRENGTH OF I-BEAMS

(1)	(2)	(3)	(4)	(5)	(6)
Section of I-Beam (In.) (Lb.)	Polar Moment of Inertia J (In.^{4})	Observed Torque at Elastic Limit of Specimen (In.-Lb.)	Theoretical Torque (Round Shaft Theory) for E.L. Stress of 22,500 lb. per sq. in. at Extreme Fibre (In.-Lb.)	Ratio J'/J *i.e.*, of Col. 3 to Col. 4.	Effective Polar Moment of Inertia J' (In.^{4})
3× 5.7	2.96	3,000	35,200	0.085	0.252
3× 7.5	3.50	5,200	40,200	0.129	0.452
4× 7.7	6.77	5,400	63,500	0.085	0.575
4×10.5	8.10	9,600	74,200	0.129	1.045
5×10	13.30	9,000	102,700	0.088	1.170
5×14.75	16.90	18,800	127,100	0.148	2.500
6×12.5	23.70	8,000	155,500	0.052	1.235
6×14.75	26.10	11,000	170,000	0.065	1.697

was in most instances dropped to 200 in.-lb. after each increment of loading, and the elastic limit was determined by the appearance of a permanent set as well as by the drop of the beam and the form of the torque twist diagrams shown in Fig. 1.

TORQUE-TWIST RELATION

These diagrams show a very smooth relation between torque and degrees of twist in 12 in. of length up to certain points which have been designated as the elastic limits and fixed in amount by the figures attached thereto in Fig. 1. In all cases the higher weight for a beam, shown in dotted lines on the diagrams, indicates a lesser angular deformation for a given torque than does the corresponding lower weight. The torques at the elastic limit are in all instances greater also for the upper weight than for the lower one for a given beam. The influence of the thicker web and the wider flanges in contributing to rigidity is clearly manifest from the diagrams.

EFFECTIVE POLAR MOMENT OF INERTIA

As has also been noted, it is a matter of common knowledge that an I-section does not develop an effective polar moment of inertia of greater amount than a comparatively small fraction of the tabular polar moment of inertia. What this relation was in the case of the beams tested was determined by calculating the theoretical torque by the round shaft theory for an elastic limit stress of 22,500 lb. per sq. in. at the extreme fibre of the section under consideration. These torques are indicated in column 4 of Table 1. In column 5 of this Table the ratio of the observed torque at the elastic limit to the theoretical elastic limit torque on the basis of the round shaft theory is indicated. It is seen that this ratio ranges between 5.2% and 14.8%, and is, even in the lowest case, in excess of the corresponding ratios cited by Gibson and Ritchie in *The Circular Arc Bow Girder*. For purposes of ready use it is useful to regard the observed torques at the elastic limit as based upon the effective polar moment of inertia. The latter may be readily computed by multiplying the quantities in column 2 of Table 1 by the ratios given in column 5 of that Table. These effective polar moments of inertia are listed in column 6 of Table 1.

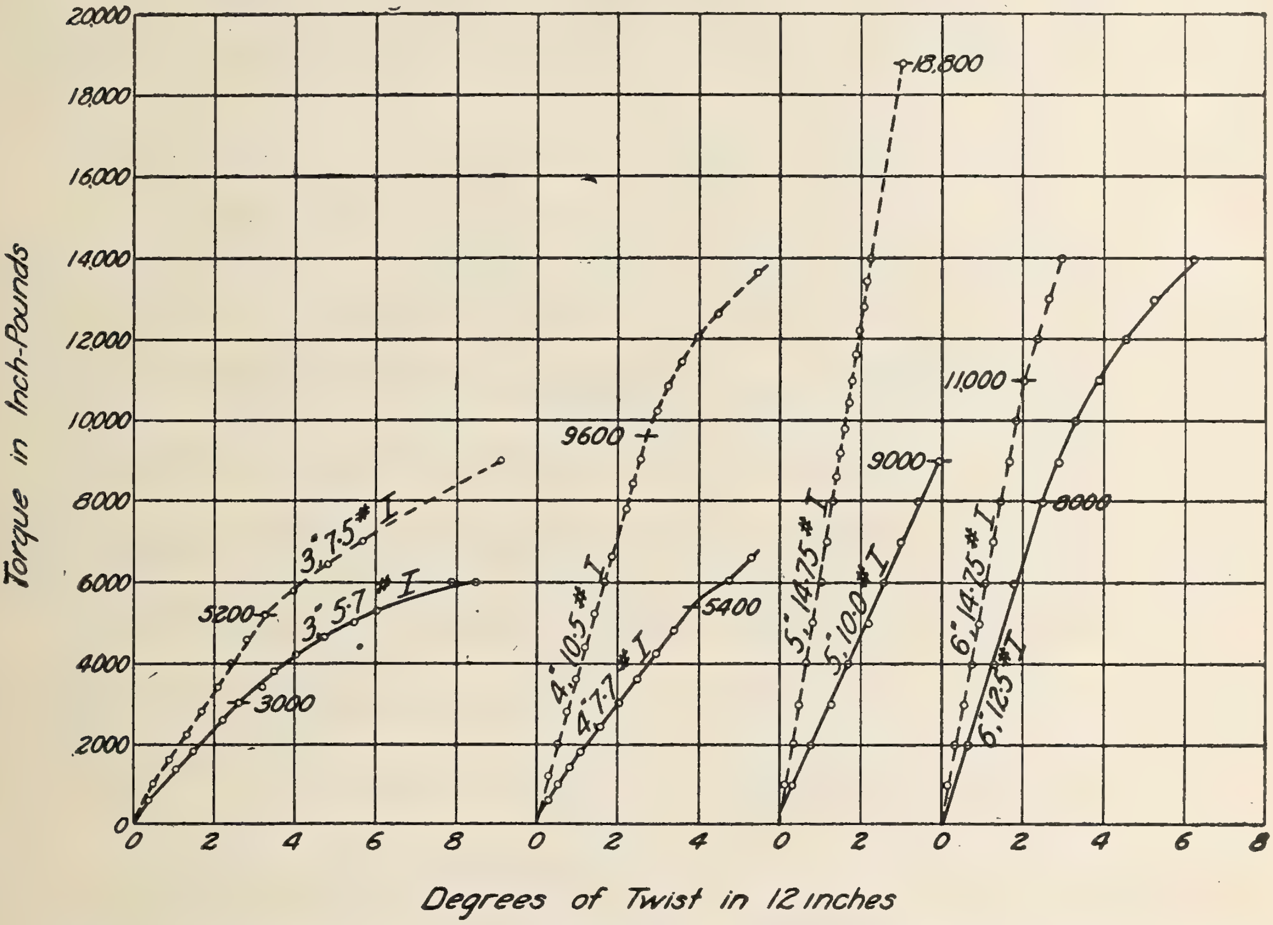

Fig. 1.—Torque-Twist Curves for I-Beams Stressed Beyond the Elastic Limit.

PLOW BEAM SECTIONS

Several years ago there were tested for torsion in the laboratories of the University of Toronto a series of plow beam sections of which Fig. 2 shows three forms. The elastic limit and the ultimate strength in torsion were determined in order to compare the behaviour of certain specially treated steels with a steel of standard structural grade. The ultimate torsional shearing resistance of these materials was not separately determined from small specimens, since in the particular investigation all that was sought by the company whose instructions were being followed was the comparative strengths of several alternative sections. However, the grade and heat treatment of the steels of which these sections were made had been recorded, and round specimens were made up afterwards at the request of the authors in order that the torsional results on the plow beam sections might be used for rough purposes of comparison with the torsional strengths of I-beams. Noting the ultimate torque developed by these plow beam sections, and having the apparent ultimate torsional shearing strength of the material of which they were composed, as disclosed by the tests on small specimens of like material, it was possible to make a comparison between the apparent ultimate torsional shearing stress in the plow beam sections, as computed by the round shaft theory, and the ultimate torsional

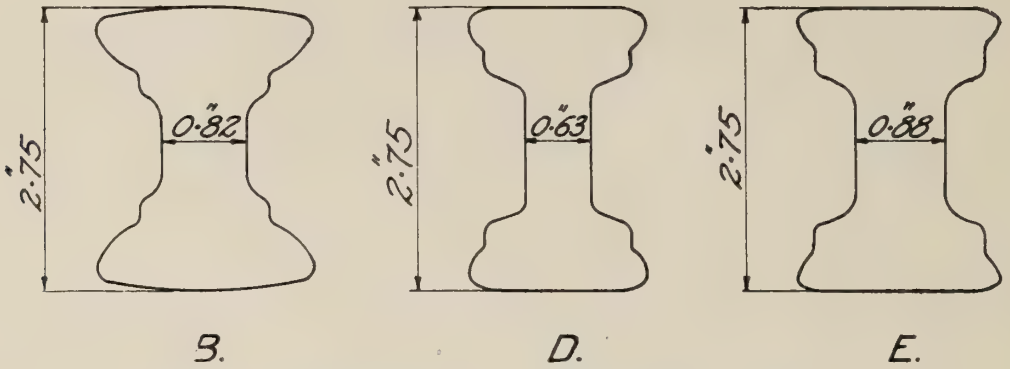

Fig. 2.—Sections of Plow Beams Tested.

TABLE 2—RATIO OF OBSERVED TO THEORETICAL (ROUND SHAFT THEORY) TORSIONAL STRENGTH OF PLOW-BEAM SECTIONS

(1)	(2)	(3)	(4)	(5)	(6)
Specimen Nos.	Polar Moment of Inertia J (In.^{4})	Apparent Ultimate Torsional Shearing Stress, by Round Shaft Theory (Lb./In.^{2})	Apparent Ultimate Torsional Shearing Stress on Solid Round Specimen of Material (Lb./In.^{2})	Ratio J'/J *i.e.*, of Col. (3) to (4)	Effective Polar Moment of Inertia J' (In.^{4})
B5, B6	3.529	40,150	122,000	0.33	1.165
D1, 3, 5, 6	3.040	49,590	155,750	0.328	0.997
E1, E2	3.487	64,650	104,600	0.618	2.133

shearing stress in a solid round specimen of the same material. Table 2 contains this relation, and in column 5 of this Table the ratio of the first to the second is set forth. From this, is seen that the effective polar moment of inertia of these plow beam sections apparently ranges from 32.8% to 61.8% of the tabular polar moment of inertia.

WORKING FORMULAE

It was thought well first to establish a formula for safe torsional resistance of an I-section by obtaining an expression for the effective polar moment of inertia. The values of this quantity have already been set forth in Tables 1 and 2 for the two classes of sections tested.

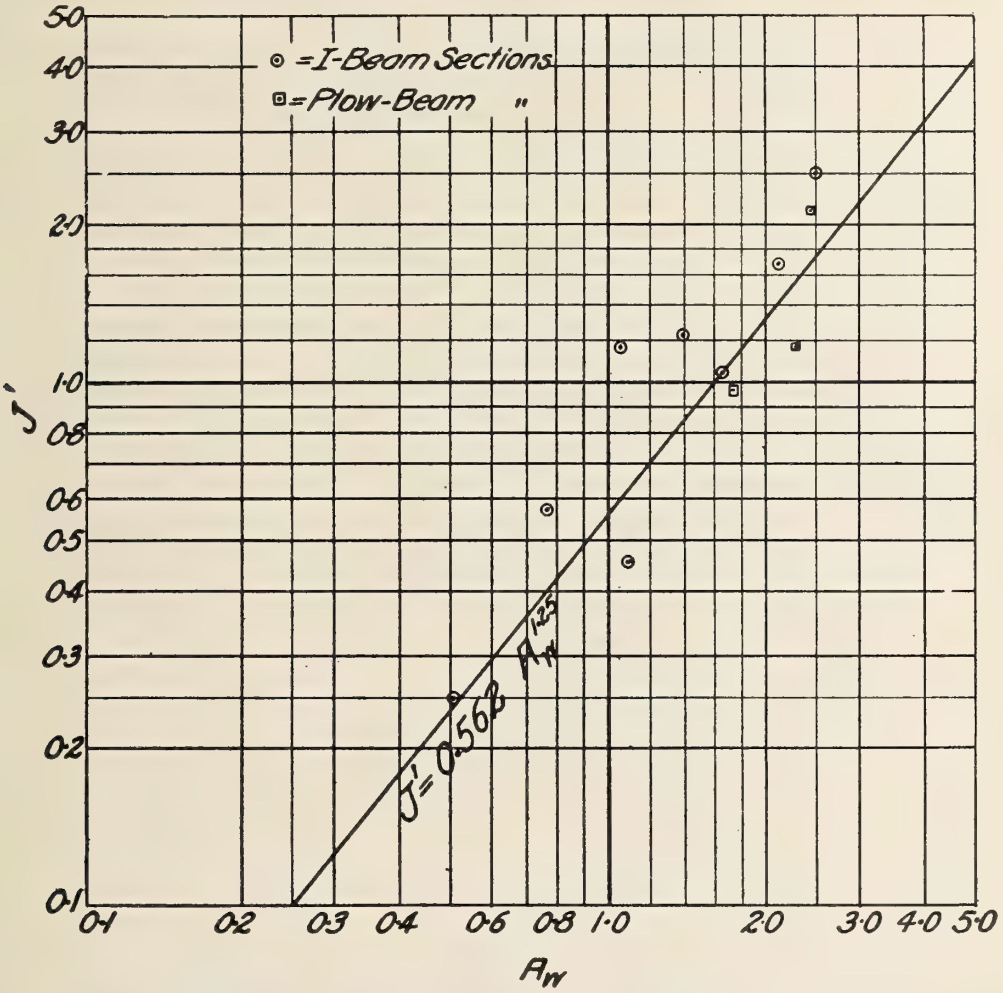

Fig. 3.—Average Effective Polar Moment of Inertia of I-Sections Based on Web Area Only.

Since the web is the primary part of an I-section with area and proportions that may easily be determined, and since previous investigations show that it plays a very important part in the torsional strength of an I-section, it was thought well to base the formula for effective polar moment of inertia upon the area of the web. To this end the values of the effective polar moment of inertia were plotted against the area of the web (see Table 3) for the respective sections in the logarithmic chart shown in Fig. 3. The equation of the line there laid down was found to be

TABLE 3—COMPARISON OF PROPOSED FORMULA FOR EFFECTIVE POLAR MOMENT OF INERTIA OF I-SECTIONS WITH OBSERVED VALUES

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Section (In.) (Lb.)	Polar Moment of Inertia J (In.^{4})	Effective Polar Moment of Inertia J' (In.^{4})	Area of Web, $A_w = dt$ (In.^{2})	Depth Ratio of Web d/t	Value of J' by Formula, $J' = 0.562 A_w{}^{1.25}$ (In.^{4})	Value of J' by Formula $J' = \left(0.40 + 0.017\frac{d}{t}\right) A_w{}^{1.25}$ (In.^{4})
3×5.7	2.96	0.252	0.51	17.70	0.242	0.302
3×7.5	3.50	0.452	1.08	8.60	0.620	0.601
4×7.7	6.77	0.575	0.76	21.10	0.400	0.540
4×10.5	8.10	1.045	1.64	10.00	1.035	1.045
5×10	13.30	1.170	1.05	23.80	0.600	0.858
5×14.75	16.90	2.500	2.50	10.12	1.750	1.785
6×12.5	23.70	1.235	1.38	26.10	0.840	1.260
6×14.75	26.10	1.697	2.10	17.50	1.410	1.750
B5, B6	3.529	1.165	2.26	3.36	1.550	1.260
D1, 3, 5, 6	3.040	0.997	1.73	4.36	1.120	0.945
E1, E2	3.487	2.133	2.42	3.12	1.700	1.368

$$J' = 0.562\, A_w^{1.25} \qquad (2)$$

The values given by this formula when applied to the cases of the different sections considered are illustrated in column 6 of Table 3.

On comparing these values with the experimental values as shown in column 3 of Table 3, it is seen that the discrepancies are in some cases large. It was thought that perhaps the ratio d/t, or the ratio of depth of beam to the web thickness, might influence the torsional resistance in some important way and when the values of the effective polar moment of inertia were related to these depth ratios given in column 5, it was found that the formula generally gave too low values for cases where ratio d/t is high, and vice versa. A correction coefficient, therefore, based upon the ratio d/t was devised and the formula indicated in Fig. 3 and in column 6 of Table 3 was amended to read:

$$J' = (0.40 + 0.017\, d/t)\, A_w^{1.25} \qquad (3)$$

The values of effective polar moment of inertia as given by this amended formula incorporated in column 7 of Table 3 are found to be reasonably close to the experimental values given in column 3. It is believed that this formula may be safely used, although it probably will be amended when the results of further investigations are known. Graphs of the formula for various values of d/t are shown in Fig. 4.

It might perhaps appear that with the increase of the values of the ratio d/t, the effective polar moment of inertia as given by the formula in column 6 of Table 3 would be too large rather than too small, as the longer and narrower a rectangle becomes, the less effective is the tabular polar moment of inertia. Thus, for a rectangle with a value d/t of 26.1 the ratio of effective to tabular polar moment of inertia is 0.57%, while for the case where d/t is 3.12 this ratio becomes 29.8%. However, as d/t increases, in the case of an I-section, the flanges have a greater and greater influence on the ratio of the effective to the tabular polar moment of inertia of the whole section. If the ratios of the experimental values of the effective polar moment of inertia of the whole section to the effective polar moment of inertia of the web only, as calculated by theory, be determined, they are found to run from 78 for the case of a typical section with a relatively deep, thin web, that is for the 5-in., 10-lb. I, to 2.8 for B5, B6, that is for a beam with a shallow, thick web. Thus, it is seen that the influence of flanges on the effective polar moment of inertia of an I-section is very great. The larger d/t is the greater is the part of the torsional strength contributed by the flanges, and the greater is the need for correcting a formula based upon web area only.

In order to predict the safe torsional shearing strength of an I-section, it is consequently only necessary to find the effective polar moment of inertia of the section. This being done by use of the formula in column, of Table 3 the ordinary round shaft theory may be applied on the hypothesis that the torsional stress at the extreme fibre is the maximum stress. The determination of the torsional strength of an I-section then becomes as simple as that of an ordinary round shaft.

MAXIMUM SHEARING STRESS

In the design of beams and girders subjected to torsion, it is generally necessary to provide for a combination of torsional shearing stress with ordinary shearing stress. The latter is readily determined by the usual methods, and the design of a beam subjected to torsion would be as easily carried out as the design of a beam not so subjected if the maximum stress in the web, or at any other critical point, due to torsion could be directly ascertained.

Weber points out in his paper, to which reference has already been made, that for I-sections where the depth of the section is more than four times the thickness of the web and the width of the flange is more than four times the average thickness thereof, the maximum torsional shearing stresses in web and flange are approximately as given in the following two formulae:

$$q_{mw} = Fat \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$q_{mf} = Fat' \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where q_{mw} = maximum torsional shearing stress in the web, q_{mf} = maximum torsional shearing stress in the flange, F = the shearing modulus of elasticity, a = angle of torsion per unit of length of specimen, t = thickness of the web, and t' = average thickness of a flange. If the unit torsion angle is known, it is then possible to determine with fair accuracy the maximum shearing stress in the section by an application of formula 4 or formula 5. It is evident that if the thickness of the web is greater than the average thickness of the flange, the maximum shearing stress will occur in the web, but if the thickness of the flange is greater than the thickness of the web, it will occur in the flange.

Application of formulae 4 and 5 for the determination of the probable maximum torsional shearing stresses in the I-beams tested was made with results that are given in column 5 of Table 4. Neglecting the the results for the first beam and the last two, it is seen that these stresses closely approximate the known shearing elastic limit of the material. The reason for the three considerable divergencies from the general run of the results is not clear, and it is believed that some experimental error may have vitiated these results. In the discussion which follows the results for the three beams mentioned will be given small weight as compared with the others.

Formulae 4 and 5 unfortunately cannot be applied for purposes of design as they stand, since the designer does not know what unit torsion angle to count upon as corresponding to a desired limit of stress in the most highly stressed portion of the cross-section. It, therefore, becomes necessary to devise an expression for the probable value of the torsion angle at the elastic limit, or for any other limit of stress prescribed. From fundamental considerations the torsion angle should within the elastic limit vary directly as the torque and inversely as the effective polar moment of inertia. Consequently one may reasonably write the relation $a=cT/J'$. The torsion angle a was determined for eight different cases, and since the torque T and the effective polar moment of inertia J' are known for the cases cited, it follows that $c=a\times J'/T$. The coefficient c could therefore be calculated for each of the eight beams tested for the elastic limit torque, or for any other torque. If now the value of a be inserted in formulae 4 and 5, they would become respectively

$$q_{mw}=k\,.\frac{T}{J'}\,.\,t \qquad (6)$$

$$q_{mf}=k\,.\frac{T}{J'}\,.\,t \qquad (7)$$

Table 4—Maximum Torsional Shearing Stresses in I-Sections Twisted to the Elastic Limit, as Calculated Directly from the Unit Torsion Angle by Formulae (4, 5)

$F=12{,}000{,}000$ lb. per sq. in.

(1)	(2)	(3)	(4)	(5)	(6)
Section (In.) (Lb.)	Unit Torsion Angle "a" at Elastic Limit (Radians)	Average Thickness of Flange t' (In.)	Thickness of web (In.)	Maximum Torsional Shearing Stress, Formulae (4, 5) (Lb./In.^{2})	Location of Point of Maximum Shearing Stress
3× 5.7	0.00378	0.260	0.170	11,800	Flange
3× 7.5	0.00481	0.260	0.349	20,200	Web
4× 7.7	0.00561	0.293	0.190	19,700	Flange
4×10.5	0.00400	0.293	0.400	19,200	Web
5×10	0.00575	0.327	0.210	22,600	Flange
5×14.75	0.00364	0.327	0.494	21,600	Web
6×12.5	0.00356	0.359	0.230	15,300	Flange
6×14.75	0.00291	0.359	0.343	12,500	Flange

where k equals Fc. Values of this coefficient k for the eight beams tested were calculated and have been listed in column 4 of Table 5, and have been plotted on the logarithmic chart given in Fig. 5. From this diagram a normal value for k may be determined which may be inserted in general formulae for the maximum torsional shearing stress. For reasons already pointed out the results for the first beam and the last two in Tables 4 and 5 are believed not to be so accurate as for the others, and consequently in Fig. 4 the three lower right-hand points have been marked with a ? as indicating their relative unreliability.

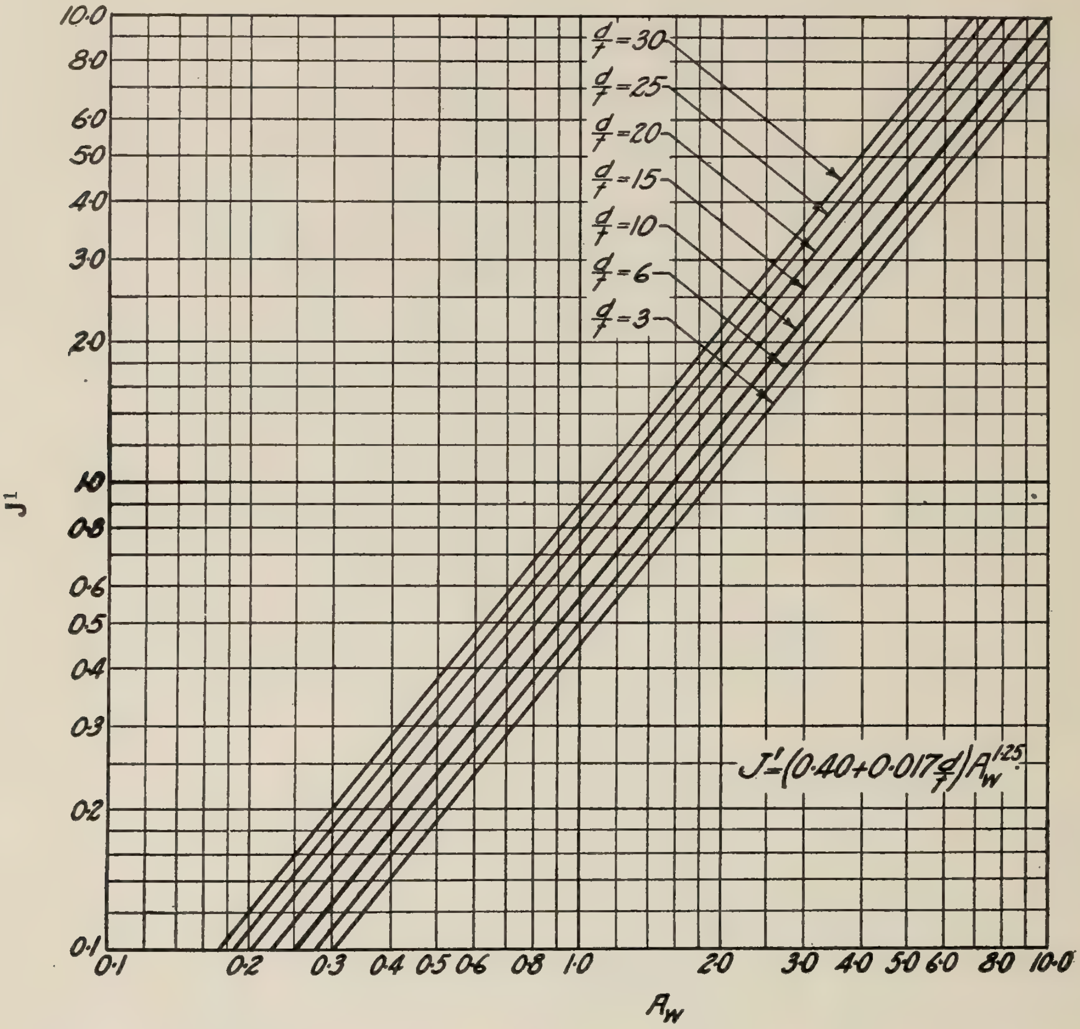

Fig. 4.—Effective Polar Moment of Inertia of I-Sections Related to Web Area and Ratio of Depth to Web Thickness.

From Fig. 4 the value of the coefficient k is found to be

$$k=2\left(\frac{d}{t}\right)^{0.45}$$

it evidently varying with the ratio of the depth of the beam to the thickness of the web. If this expression for k be inserted in formulae 6 and 7 they become respectively

$$q_{mw}=\frac{2Tt}{J'}\left(\frac{d}{t}\right)^{0.45} \quad\ldots\ldots\ldots\ldots(8)$$

$$q_{mf}=\frac{2Tt'}{J'}\left(\frac{d}{t}\right)^{0.45} \quad\ldots\ldots\ldots\ldots(9)$$

where d=depth of the beam and t=thickness of the web, the other quantities being as already defined. It is believed that these formulae afford a useful and safe means of predicting the maximum torsional shearing stresses in web and flange of a beam for a given torque. The values of the effective polar moment of inertia J' may be determined from Fig. 3.

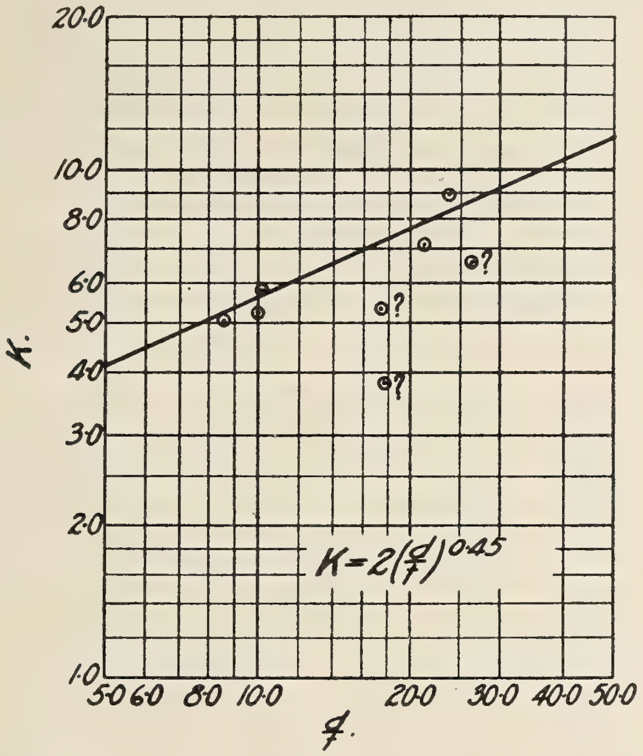

Fig. 5.—Value of Coefficient k in Terms of the Ratio of Depth to Web Thickness.

TABLE 5—MAXIMUM SHEARING STRESSES IN I-BEAMS LOADED TO ELASTIC LIMIT IN TORSION, AS CALCULATED BY RECOMMENDED GENERAL FORMULAE (8, 9)
$F=12{,}000{,}000$ lb. per sq. in.

(1)	(2)	(3)	(4)	(5)	(6)
Section of I-Beam (In.) (Lb.)	Effective Polar Moment of Inertia J' (In.^{4})	Torque T at Elastic Limit (In.-Lb.)	Calculated Value of $k=FaJ'/T$	Value of k from Graph, Fig. 4	Maximum Torsional Shearing Stress, Using values of k from Graph (Lb./In.^{2})
3× 5.7	0.252	3,000	3.810	7.2	22,300
3× 7.5	0.452	5,200	5.020	5.2	20,900
4× 7.7	0.575	5,400	7.175	7.8	21,500
4×10.5	1.045	9,600	5.230	5.6	20,600
5×10	1.170	9,000	8.980	8.2	20,600
5×14.75	2.500	18,800	5.810	5.6	20,800
6×12.5	1.235	8,000	6.590	8.6	20,000
6×14.75	1.697	11,000	5.380	7.2	16,800

BENDING IN FLANGES

In any I-section subjected to torsion there is incidentally set up a transverse bending moment in the flanges. In the case of beams subjected to heavy twisting, this lateral bending moment and the stresses resulting therefrom may be important. If a load applied at a point along the length of the beam produces a torsional moment T, then there may be considered as applied to each flange of the beam at this point a transverse load $Q=T/h$ where $T=$the torsional moment on the beam and $h=$the depth between centres of flanges. These transverse forces will be of equal magnitude, but of opposite sign, thus producing a couple exactly equal to the applied torsional moment.

CONCLUSIONS

(1) The elastic limit of the small I-beams tested in torsion varied from 5.2% to 14.8% of that of a round shaft of structural steel with the same tabular polar moment of inertia. For the thick-webbed plow beam sections this ratio varied from 32.8% to 61.8%. The effective polar moment of inertia of an I-section may, therefore, be taken as corresponding percentages of the tabular polar moment of inertia.

(2) The maximum torsional shearing stress occurs at the centre of the side of the web only in cases where the web has a greater thickness than the average thickness of the flange. In the reverse case the maximum stress occurs at the centre of the face of a flange.

(3) The effective polar moment of inertia of an I-section may be approximately expressed in terms of the area of the web, assuming the latter to be the web thickness multiplied by the depth of the section, and of the ratio of the depth of the section to the web thickness. The capacity of I-sections may, therefore, be computed by the ordinary round shaft theory, provided the effective polar moment of the inertia, rather than the tabular polar of inertia, is used.

(4) The maximum torsional shearing stress in an I-section varies as the angle of torsion per unit of length of specimen. It is possible to obtain a working expression for this unit angle of torsion by analysis of torsion tests, since it varies directly as the torsional moment and inversely as the effective polar moment of inertia. It is found also to vary to some extent in accordance with the ratio d/t.

(5) Where the torsional moment on an I-section is large, considerable lateral bending stresses arise in the flanges of the section, the force that may be considered as applied horizontally to each flange at the point of loading being equal to the torsional moment divided by the distance between the centres of the flanges.

A CONTRIBUTION TO THE KICK VERSUS RITTINGER DISPUTE[1]

By H. E. T. Haultain, *Professor of Mining Engineering*

The study of rock crushing or grinding in tube mills is difficult on account of the large size of the units employed in the field and the large number of variables entering into the problem. Three or four years ago, in the laboratory of the Department of Mining Engineering of the University of Toronto, a serious effort was commenced to study the matter under laboratory conditions. Experiments were carried on in a cylindrical tube mill 18 in. long and 12 in. in diameter, inside measurements, supported on ball bearings and connected with a dynamometer. Several hundred experiments were made, but it was early realized that, for proper interpretation of the results, more definite information was required in regard to the laws of crushing as interpreted by Kick and by Rittinger.[2] For the study of this phase of the work it was thought that a more simple apparatus, a more simple process of crushing was required, and that this could best be found in small carefully constructed rolls. This apparatus, shown in Fig. 1, was obtained by the financial help of the Honorary Advisory Council for Scientific and Industrial Research of Ottawa. Rolls 7 in. in diameter with 2¼-in. face were mounted on shafts supported by the highest grade of S. K. F. ball bearings, and were driven from a countershaft, also supported by S. K. F. bearings, which was connected with an integrating dynamometer. This apparatus proved to be more sensitive and reliable than had been expected and gave concordant results when working with quantities of rock as small as from 2 to 5 lb. The following figures show the range of error.

Feed, Pounds 4-6 Mesh Hollinger Quartz	Rate of Feed, 5 lb. in Seconds	Integrator Readings, Foot-pounds
5	116	3874
5	63	3952
5	40	3919
5	31	3945
	Average.	3922

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[1] Reprinted from Transactions of the American Institute of Mining and Metallurgical Engineers, 1923.

[2] *Kick's Theorem.*—"The energy required for producing analogous changes of configuration of geometrically similar bodies of equal technological state varies as the volumes or weights of those bodies."

Rittinger.—"The energy required for crushing is proportional to the new surface formed;" *i.e.*, for any given unit weight of rock is proportional to the reciprocal of the diameter of the average size.

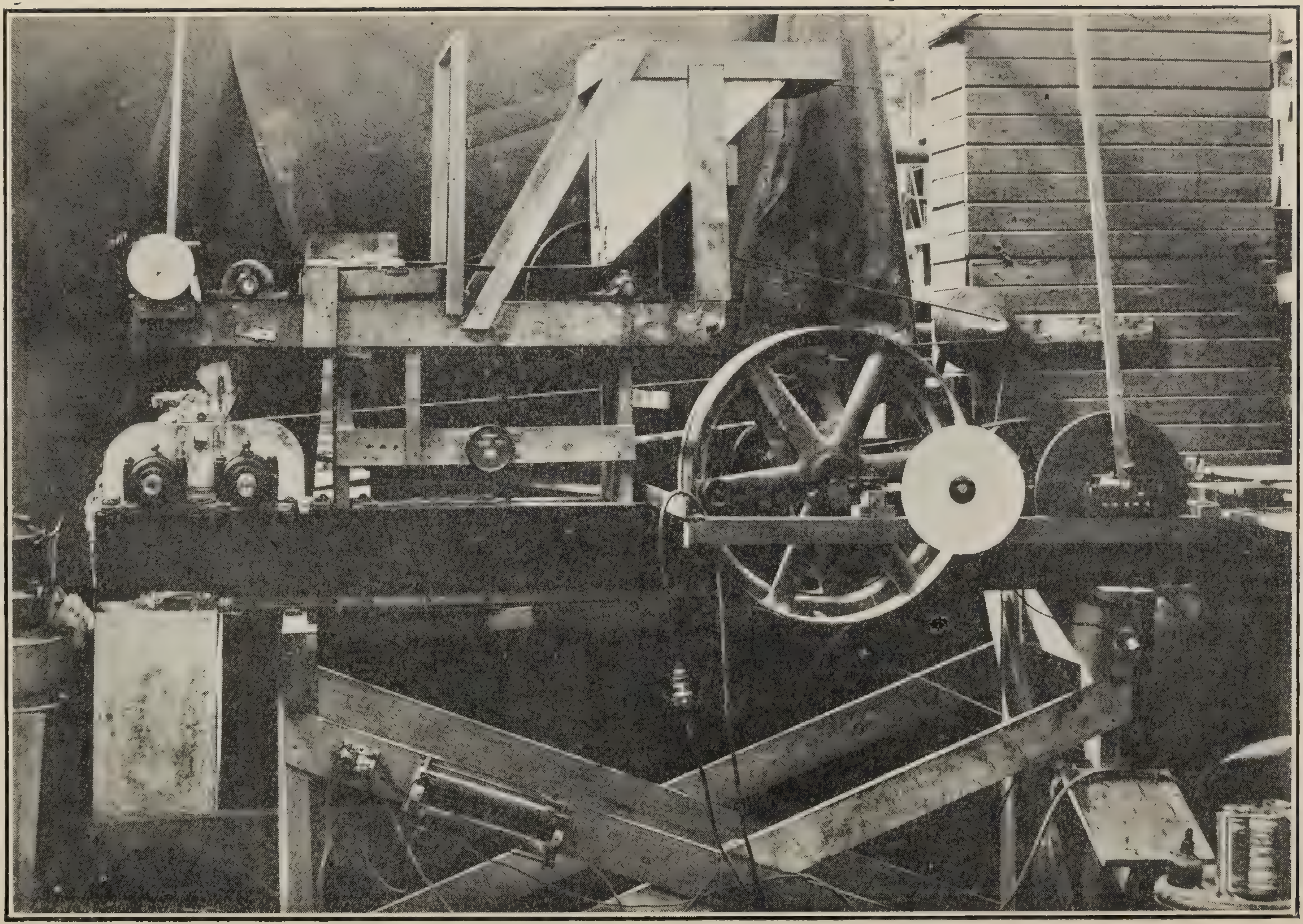

Fig. 1

With a different setting of the rolls, the rate of feed was kept practically the same.

Feed, Pounds 4-6 Mesh Hollinger Quartz	Rate of Feed, 1 lb. in Seconds	Integrator Readings, Per 1 lb. of Feed
1	11.0	754
2	10.0	754
5	9.4	744
10	10.3	733
	Average...........................	746

The process of crushing rock produces new rock surface and heat; also, to a minor extent, noise and electrical phenomena. The physicists have connected up the energy of new surface formation with the phenomena of surface tension and of latent heat. (The formation of steam from water is a surface-producing process pushed to the extreme.) From their calculations, it would appear that the energy actually absorbed in producing the new surface formed in the process of crushing is small compared with the total energy required in practice. Doubtless, the energy absorbed only in new surface formation is proportional to the new surface, as indicated in Rittinger's philosophy. Apparently most of the energy required in the practice of rock crushing appears as heat, but surface formation *per se* does not generate heat. The heat must be due to friction, friction occurring at the surface of the particles and friction occurring within the particles, the latter being the internal friction of distortion (molecular friction? hysteresis?). The surface friction will vary with the type of apparatus used and, doubtless, with the type of rock being crushed, and with the degree of crushing.[3]

The heat resulting from the internal friction of distortion (would that it might be safely referred to as molecular friction) that occurs prior to rupture may probably vary according to Kick's philosophy, but it would seem difficult to reasonably connect with either the Kick or the Rittinger theories the energy consumed in surface friction. The writer ventures to think that those responsible for lengthy arguments in favour of Kick or of Rittinger have not given due consideration to this phase of the problem. Apparently a very large number of experiments, rather than argument, will be necessary for a satisfactory conclusion.

Two series of experiments with the new apparatus are illustrated in Figs. 2 and 4. The rock was crushed in a small Dodge crusher, then carefully sized by Tyler screens in a Ferraris type of screening apparatus. In the Tyler series, the ratio of the diameters of the openings in adjacent screens is the square root of two, 1.414, and each feed product was screened between adjacent screens. For each size of feed product, the rolls were set with an opening equal to one-half the size of the smaller screen opening. For example, the product that passed through 8 mesh

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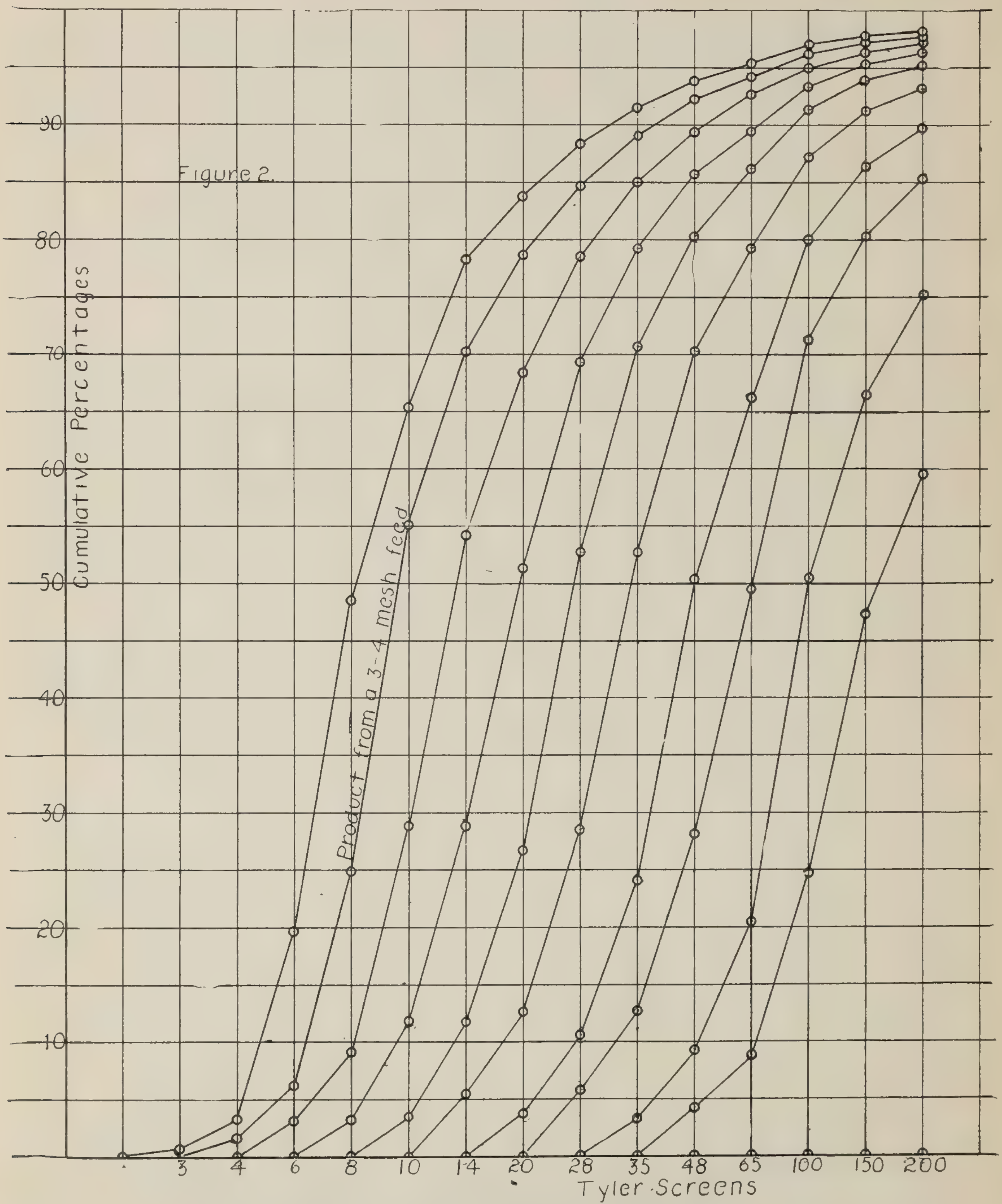
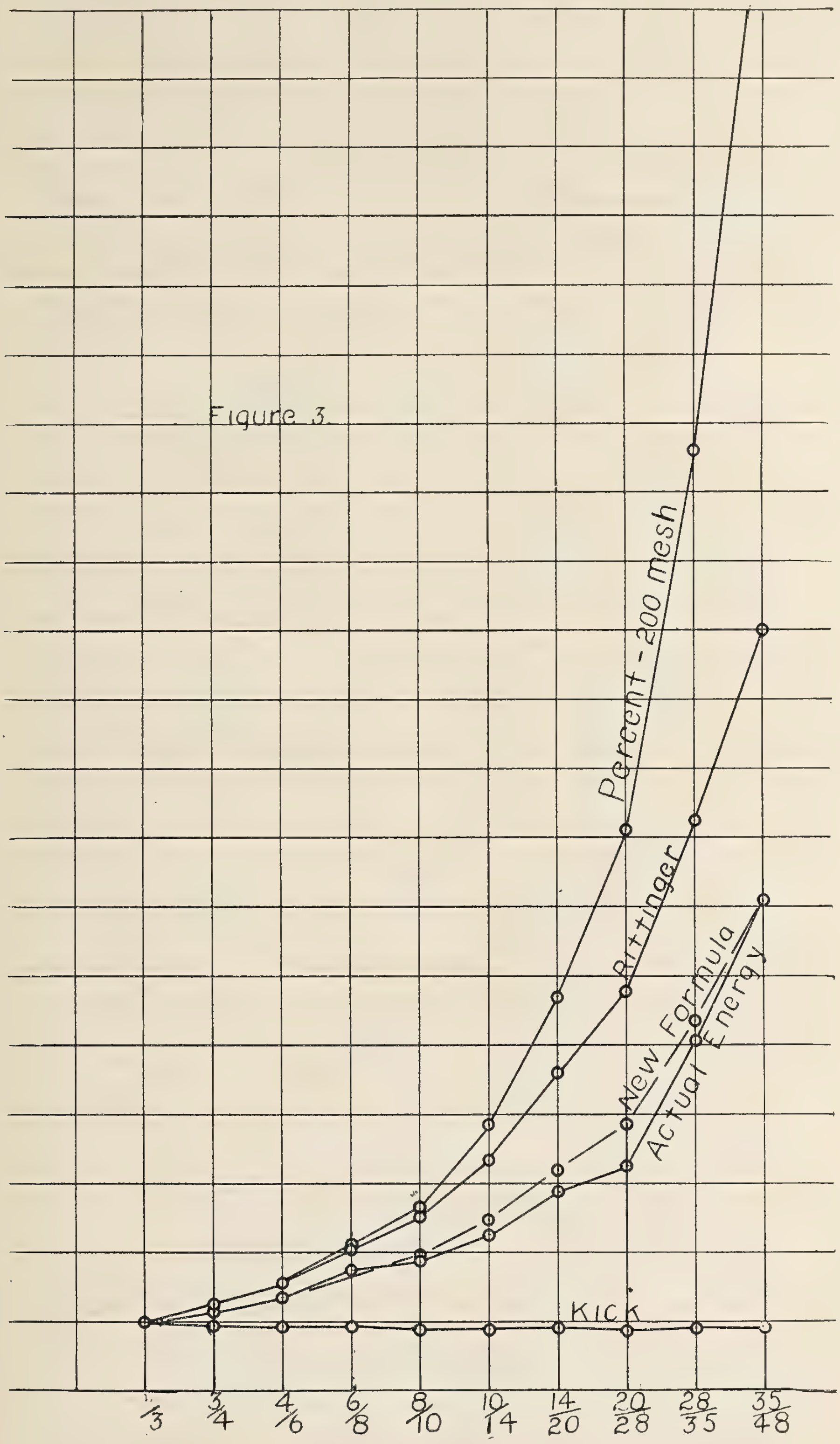
[3] There is no doubt that rocks behave very differently during crushing. Some rocks are tough, some are brittle. Professor Dyer and the writer have studied and visualized their behaviour by a series of photographs taken at the rate of 120 photographs per second, which are shown on the screen at the normal movie picture rate of 15 per second.



HOLLINGER QUARTZ SERIES

Fig. 2.—Cumulative percentage diagrams of screen analysis of products from ten crushing tests; in each case the ratio of average mesh of feed to spacing of the rolls was 2.4 to 1. In the Tyler series the coarsest screen designated by an ordinal number is 3 mesh. The next coarsest screen has an opening of .371 inches.

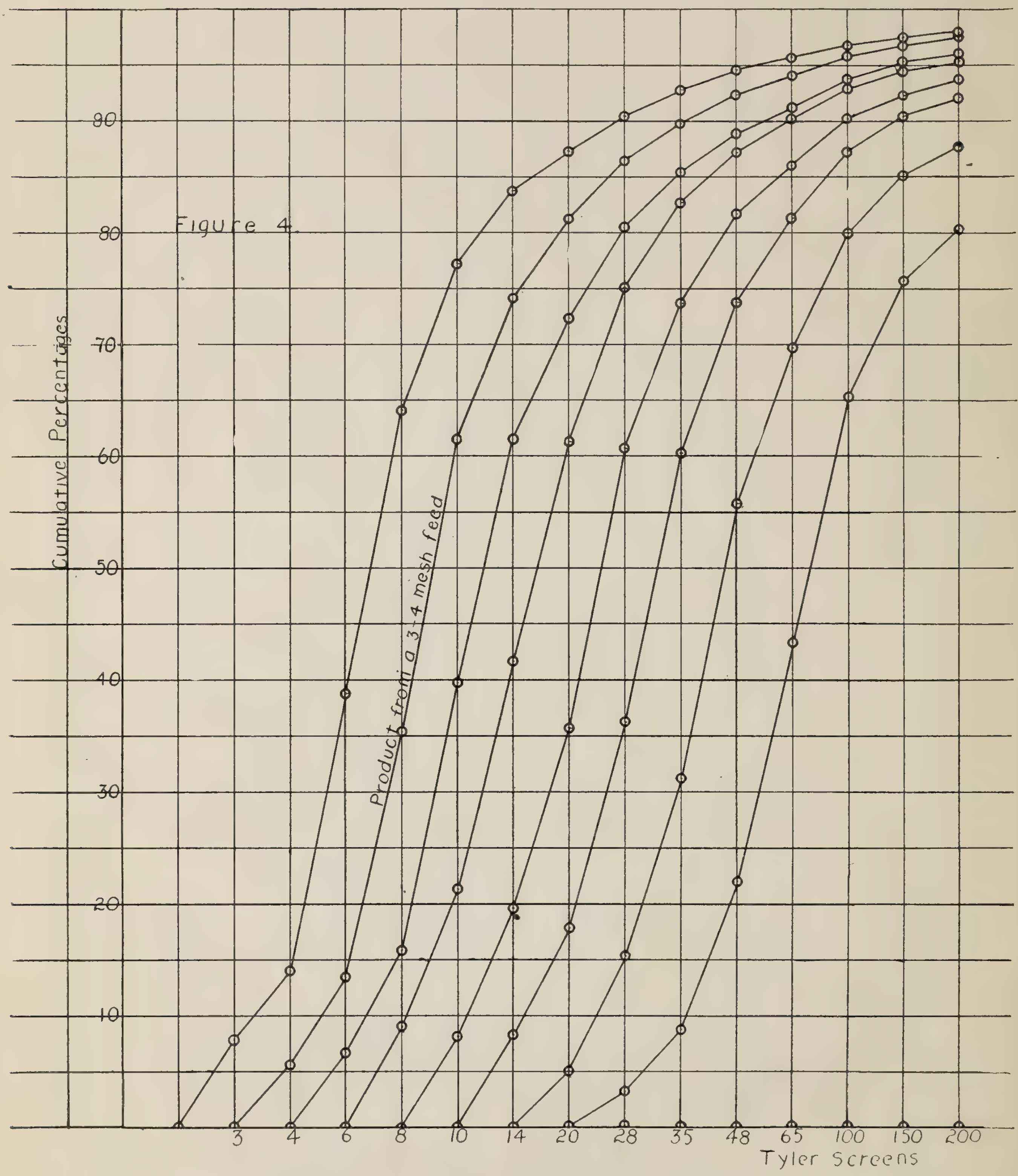
and stayed on 10 mesh was crushed by rolls set with an opening equivalent to 20 mesh; for a product between 28 and 35 mesh, the rolls' opening was 65 mesh. Thus for each product there was attempted an equal



HOLLINGER QUARTZ SERIES

Fig. 3.—Relative energy calculated by five methods from results obtained in crushing tests—percent of product through 200 mesh, energy calculated from screen analysis according to Rittinger's law, energy calculated from screen analysis according to new formula, energy recorded by integrating dynamometer, energy calculated from screen analysis according to Kick's law.

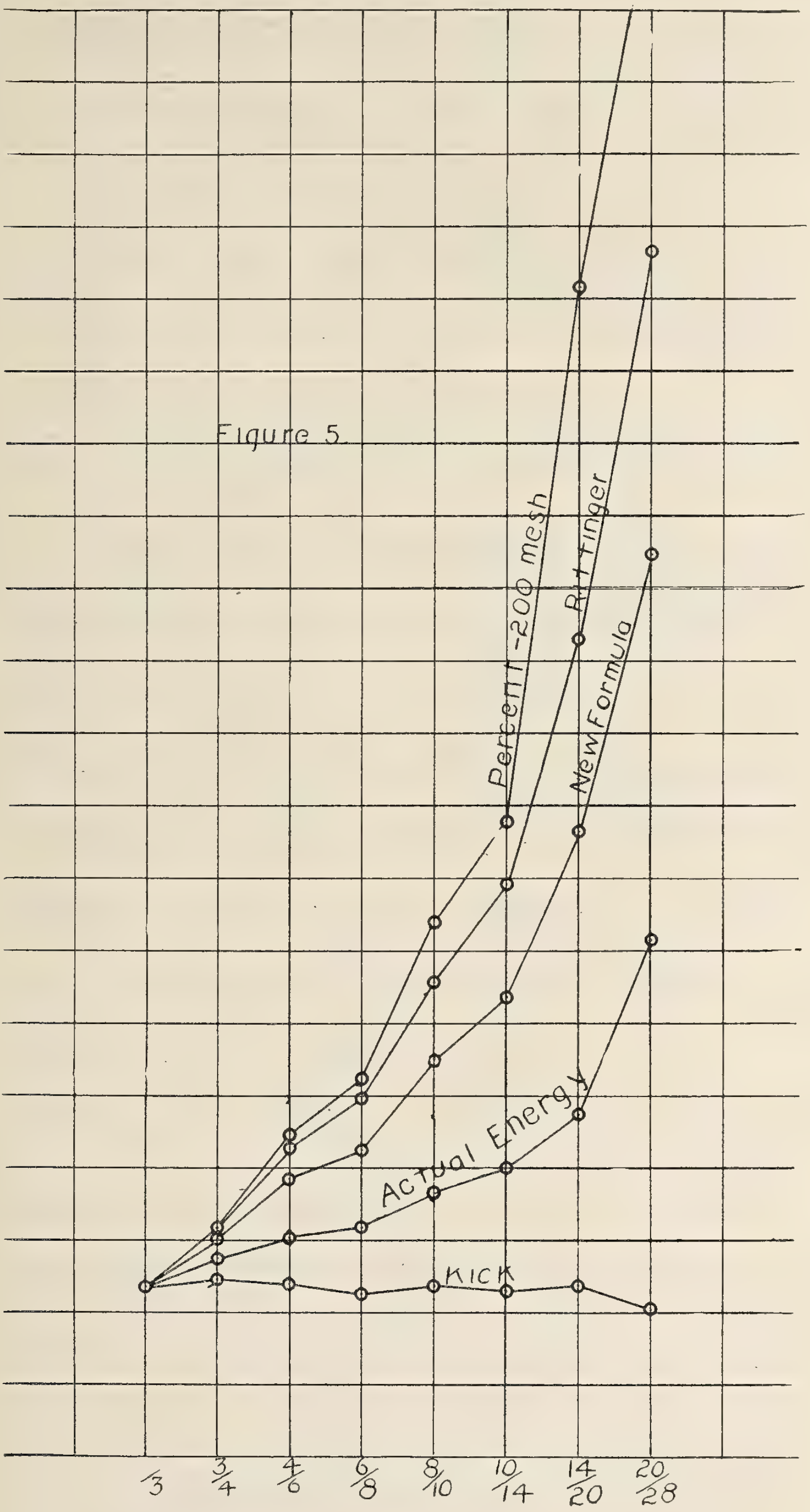
degree of crushing; "analogous changes in configuration." The crushed products were screen-sized with Tyler screens and are plotted as cumulative percentages. The similarity of the curves shows the degree of uniformity. Departure from complete uniformity may have been due to the difficulty of setting the rolls exactly the right distance apart.



CONIAGAS CONGLOMERATE SERIES

Fig. 4.—Cumulative percentage diagrams of screen analyses of products from eight crushing tests, in each case the ratio of average mesh of feed to spacing of the rolls was 2.4 to 1.

The energy actually required per pound of rock crushed, as shown by the dynamometer, is plotted in the graphs, Figs. 3 and 5. For comparison are shown the curves as calculated from the screen analyses,



CONIAGAS CONGLOMERATE SERIES

Fig. 5.—Relative energy calculated by five methods from results obtained in eight crushing tests—percent of product through 200 mesh, energy calculated from screen analyses according to Rittinger's law, energy calculated from screen analyses according to new formula, energy recorded by integrating dynamometer, energy calculated from screen analyses according to Kick's law.

according to Kick and to Rittinger, all calculated as starting from a common point for the first crushing in the series. Also for comparison are shown curves for the percentage of 200 mesh produced in each crushing. These are also calculated to start from a common point; that is, different scales are used, the object being simply to show the nature of the curve for comparison with the others.

A graduate student, J. G. McNiven, carrying out these experiments, developed a formula on a rational basis, which fitted very well with the results for Hollinger quartz, but not so well for Coniagas conglomerate, a compact, very fine-grained tough rock.

In each set of experiments both the Rittinger and the Kick figures are far from the actual results. With the Hollinger rock, the Rittinger figures are the nearer, while the Kick figures are somewhat nearer with the Coniagas rock. Coniagas rock took about twice as much energy as Hollinger quartz.

The sizing screens take no cognizance of shape. They are at best an imperfect measuring stick and as yet there is no measuring stick available for shape. In many of these experiments, particles were counted and weighed in the different sizes, showing a similarity between coarse and fine. A study of the shapes produced by various crushing methods has been commenced and promises to show marked differences.

NOTE—The figures used as multipliers in the Kick and Rittinger calculations are as follows:

Mesh	Kick	Rittinger	Mesh	Kick	Rittinger
2	3	2.23	28	11	35.8
3	4	3.15	35	12	50.5
4	5	4.46	48	13	71.5
6	6	6.34	65	14	101.0
8	7	8.92	100	15	142.8
10	8	12.65	150	16	204.2
14	9	17.85	200	17	286.0
20	10	25.1	−200	19	715.0

Attention should be drawn to the possible, or probable lack of "equal technological state" in different sized particles of even such a uniform rock as the Hollinger quartz, which was used in these experiments. In all rocks, there must be planes or zones of weakness due to strains of cooling or of pressure which may be classified as geological, or due to crystal structure or mineral aggregate which may be classified as mineralogical. Apparently a substance without any of these weaknesses breaks under compression with a conchoidal fracture, and in crushing with rolls tends to be very flaky. It may be that the reduction in the number of these weaknesses as the particles become smaller is the cause of the rise of the crushing energy curve. It may be that well-annealed glass would give a curve much nearer to Kick's.

The writer submits the following tentative conclusions:

Neither Kick nor Rittinger accurately connects up size of product with energy required to crush rock.

The mean of the two is probably more nearly correct than either in the simplest form of crushing; *viz.*, cracking with rolls.

The nature of the process and of the material prevents any simple formula applicable to all rocks, or all degrees of crushing.

The energy required for the actual process of crushing (disregarding losses in the machinery itself) is absorbed mainly in three ways: (1) Actual formation of new surface, that is, the final rupture; this is a small part of the total energy. (2) Internal friction accompanying the distortion prior to rupture. (3) Surface friction of particle on particle and particle on crushing surface.

This surface friction probably accounts for the greater part of the energy, and is the most important variable in practice. It varies with the rock, the type of crusher, and with the degree of crushing. It is probably least in rolls just cracking particles of a brittle material and greatest in an overloaded tube mill. It is high in a disk grinder and in the arrastre.

It is in variation in this surface friction that the main variation in efficiency occurs. This being a surface action it is probable that it follows closer to Rittinger than to Kick. If this be the case then the less efficient the crushing method the less will Kick's law apply.

This problem can best be studied with small laboratory apparatus on account of the large number of experiments required. With suitable apparatus, it will be possible to construct from laboratory experiments a characteristic curve for any given rock, which may be advantageously applied to the interpretation of field results.

It is hoped that with further development of methods and technique with small apparatus of the type indicated, it will be possible to predict with useful accuracy the power and size of plant required for any given rock crushing.

POSTSCRIPT

By H. E. T. HAULTAIN and J. G. MCNIVEN (*Research Assistant*)

Particular attention is drawn to the two charts of screen analyses in this paper. These charts show the screen analyses of the products from successive runs of the rolls. The distance apart of the rolls in successive runs varied by the square root of two, as did also the average size of the particles in the feed. The size of the feed was controlled by very careful screen sizing with Tyler screens and the spacing of the rolls was gauged as accurately as possible by micrometer. Theoretically this would result in "analogous changes in configuration" and the screen sizing curves would be similar and equi-distant on the chart. The approximations to these conditions is a criterion of the technique of the work. It was not possible, especially in the finer sizes, to gauge accurately the spacing of the rolls but the charts show remarkable concordance and the writers submit this as an evidence of the care and accuracy of their work.

The actual energy will depend upon the accuracy of the spacing of the rolls and an examination of the charts will show that the lack of smoothness in the actual energy curve corresponds with irregularities in the screen analysis curve, evidence that this is due to irregularities in the spacing of the rolls.

The writers have concluded, as a result of the experiments recorded and of many others along parallel lines, that the crushing characteristics will vary with the rock. The slow motion pictures by Haultain and Dyer of the actual crushing of rock in rolls adds further testimony to this conclusion. The energy curve represents not only the overcoming of cohesion but also represents friction within the mass of rock and the writers can see no hope of a general formula applicable to all rocks. Mr. McNiven made a special effort to arrive at an empirical formula and found that the energy varied from experiment to experiment in the Quartz series approximately as the cube root of two and using this as a basis suggested the following figures as multipliers instead of the Rittinger figures:

Mesh	Multiplier	Mesh	Multiplier
2	1.0	28	6.4
3	1.25	35	8.0
4	1.60	48	10.0
6	2.0	65	12.8
8	2.5	100	16.0
10	3.3	150	20.0
14	4.0	200	25.6
20	5.0	−200	64.0

The curve according to these figures is shown on the chart as the new formula and corresponds fairly closely with the actual energy in the case of quartz, but not in the case of conglomerate. He suggests that it may be quite possible to use either this series or Rittinger's series coupled with an empirical formula or a constant which constant will have to be worked out for each separate type of rock just as the constant *n* in Kutter's formula for the flow of water in open channels had to be worked out for each type of wall. It is quite probable that some rock will be found in which the Rittinger figures coincide with the actual energy figures.

The writers, however, emphasize that the energy of crushing will vary, not only with the rock, but also and perhaps still more, with the type of crushing device. In other words friction is by far the larger factor in the results. The writers of this paper, and other experimenters in the same field, have been trying to obtain figures or formula that would connect up the energy consumed in actual practice with the size of particle crushed and have used the term "efficiency of crushing" in perhaps a loose way. Efficiency of any apparatus or process should be concerned only with product which is useful. The efficiency of a steam power plant is gauged only by the actual kinetic energy produced; so also in calculating the relative efficiency of crushing machinery, theoretically we should consider only the desired product and that desired product is new surface.

Two additional charts are added showing the relative efficiency in the two series of experiments calculated according to Kick and according to Rittinger.

The writers are of the opinion that the actual efficiency of the most perfect crushing device is extremely low, by far the greater amount of energy appearing as heat, an undesired product.

The tables of screen analyses and energy figures accompanying the curves given for Hollinger quartz and Coniagas conglomerate are tabulated as follows:

HOLLINGER QUARTZ

Screen Analysis of products from ten crushing tests

35-48 mesh feed

Screen Size	%	Cumulative %
on 48 mesh	4.3	4.3
65	4.6	8.9
100	15.7	24.6
150	22.7	47.3
200	12.2	59.5
−200	40.6	100.1

28-35 mesh feed

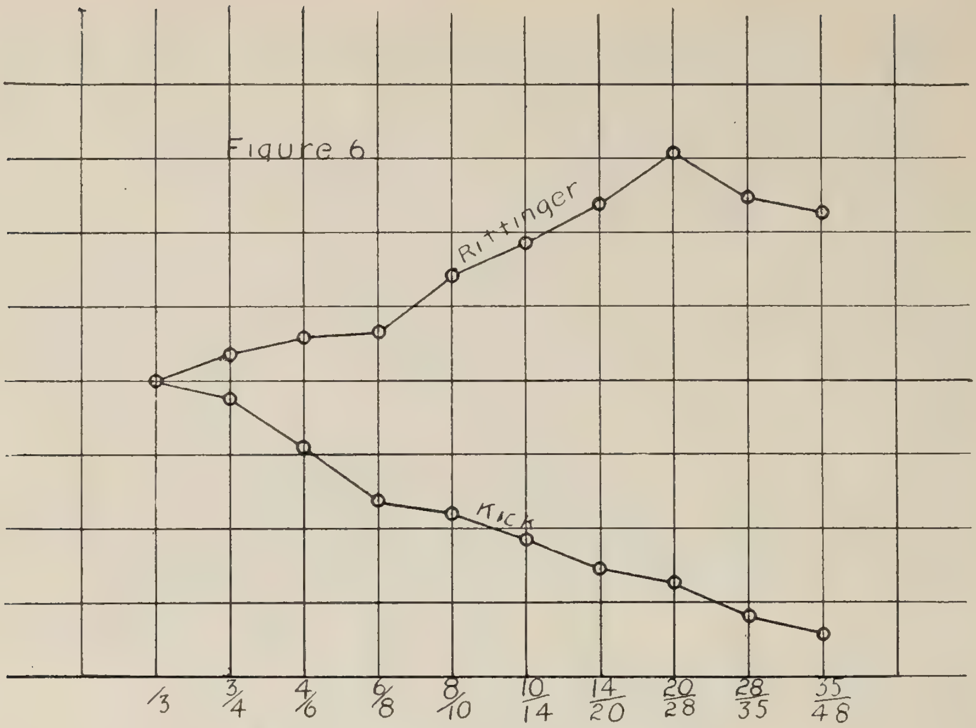
Screen Size	%	Cumulative %
on 35 mesh	3.3	3.3
48	5.9	9.2
65	11.3	20.5
100	29.8	50.3
150	16.2	66.5
200	8.6	75.1
—200	25.0	100.1

20-28 mesh feed

Screen Size	%	Cumulative %
on 28 mesh	5.8	5.8
35	6.9	12.7
48	15.4	28.1
65	21.4	49.5
100	21.7	71.2
150	9.1	80.3
200	5.0	85.3
−200	14.8	100.1

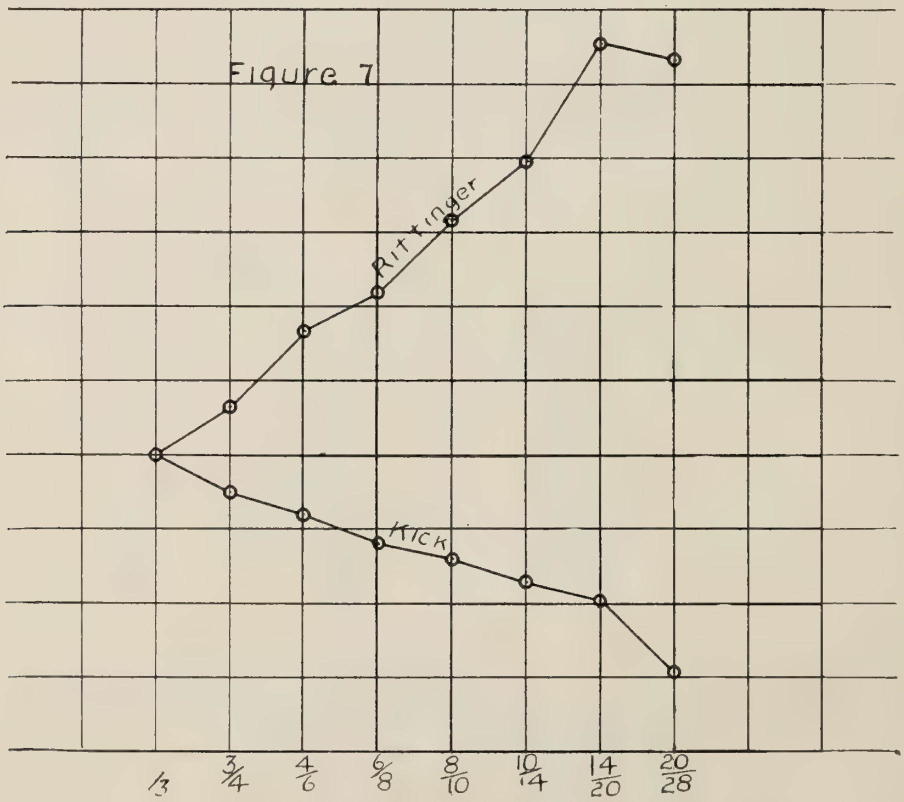
14-20 mesh feed

Screen Size	%	Cumulative %
on 20 mesh	3.8	3.8
28	6.8	10.6
35	13.5	24.1
48	26.2	50.3
65	15.8	66.1
100	13.9	80.0
150	6.4	86.4
200	3.3	89.7
−200	10.4	100.1



HOLLINGER QUARTZ SERIES

Fig. 6.—Efficiencies calculated according to Rittinger and according to Kick, from screen analyses, relative to actual energy recorded by the dynamometer. The scales of the chart are adjusted to show the curves with a common starting point for the coarsest size.



CONIAGAS CONGLOMERATE SERIES

Fig. 7.—Efficiencies calculated according to Rittinger and according to Kick. from screen analyses relative to actual energy recorded by the dynamometer. The scales of the chart are adjusted to show the curves with a common starting point for the coarsest size.

10-14 mesh feed

Screen Size	%	Cumulative %
on 14 mesh	5.5	5.5
20	7.1	12.6
28	15.9	28.5
35	24.2	52.7
48	17.6	70.3
65	8.9	79.2
100	8.2	87.4
150	3.8	91.2
200	2.0	93.2
−200	7.0	100.2

8-10 mesh feed

Screen Size	%	Cumulative %
on 10 mesh	3.5	3.5
14	8.3	11.8
20	15.0	26.8
28	26.0	52.8
35	17.8	70.6
48	9.8	80.4
65	5.8	86.2
100	5.2	91.4
150	2.6	94.0
200	1.6	95.2
−200	4.8	100.0

6-8 mesh feed

Screen Size	%	Cumulative %
on 8 mesh	3.1	3.1
10	8.8	11.9
14	17.0	28.9
20	22.4	51.3
28	18.0	69.3
35	9.9	79.2
48	6.4	85.6
65	3.9	89.5
100	3.9	93.4
150	1.9	95.3
200	1.0	96.3
−200	3.8	100.1

4-6 mesh feed

Screen Size	%	Cumulative %
on 6 mesh	3.1	3.1
8	6.0	9.1
10	19.8	28.9
14	25.2	54.1
20	14.3	68.4
28	10.1	78.5
35	6.5	85.0
48	4.4	89.4
65	3.2	92.6
100	2.4	95.0
150	1.4	96.4
200	.7	97.1
−200	2.8	99.9

3-4 mesh feed

Screen Size	%	Cumulative %
on 4 mesh	1.6	1.6
6	4.7	6.3
8	18.6	24.9
10	30.2	55.1
14	15.1	70.2
20	8.4	78.6
28	6.0	84.6
35	4.4	89.0
48	3.2	92.2
65	2.0	94.2
100	2.1	96.3
150	1.0	97.3
200	.5	97.8
−200	2.3	100.1

[1]2-3 mesh feed

Screen Size	%	Cumulative %
on 3 mesh	.7	.7
4	2.5	3.2
6	16.4	19.6
8	28.9	48.5
10	16.9	65.4
14	12.8	78.2
20	5.6	83.8
28	4.5	88.3
35	3.2	91.5
48	2.3	93.8
65	1.6	95.4
100	1.6	97.0
150	.8	97.8
200	.4	98.2
−200	1.8	100.0

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[1] The screen referred to here as 2 mesh has a screen opening of .371 inches.

TABLES OF ENERGY ABSORBED ON CRUSHING, CALCULATED BY VARIOUS METHODS

Size of Feed	Rittinger Surface Units in Product minus R.S.U. in Feed	Kick Energy Units in Product minus K.E.U. in Feed
35-48 mesh	32983	398
28-35 "	24590	398
20-28 "	17225	385
14-20 "	13761	401
10-14 "	9959	390
8-10 "	7467	393
6-8 "	6059	402
4-6 "	4656	403
3-4 "	3772	416
2-3 "	2983	440

Size of Feed	New Formula	Actual Energy
35-48 mesh	2701	3562
28-35 "	2043	2561
20-28 "	1475	1638
14-20 "	1222	1452
10-14 "	945	1144
8-10 "	751	930
6-8 "	640	878
4-6 "	521	683
3-4 "	444	579
2-3 "	383	503

Actual energy is the amount of energy (in foot pounds) absorbed in *crushing* one pound of the designated feed as recorded by the dynamometer.

PERCENTAGE OF VARIOUS FEEDS CRUSHED THROUGH 200 MESH

Feed	% Through 200 mesh	Feed	% Through 200 mesh
35-48	40.5	8-10	4.8
28-35	24.9	6-8	3.8
20-28	14.8	4-6	2.8
14-20	10.4	3-4	2.3
10-14	7.0	2-3	1.8

CONIAGAS CONGLOMERATE

SCREEN ANALYSES OF PRODUCTS OF EIGHT CRUSHING TESTS

20-28 mesh feed			14-20 mesh feed		
Screen Size	%	Cumulative %	Screen Size	%	Cumulative %
on 28 mesh	3.1	3.1	on 20 mesh	5.0	5.0
35	5.7	8.8	28	10.4	15.4
48	13.2	22.0	35	15.8	31.2
65	21.4	43.4	48	24.5	55.7
100	21.9	65.3	65	13.9	69.6
150	10.3	75.6	100	10.4	80.0
200	4.8	80.4	150	5.1	85.1
−200	19.6	100.0	200	2.6	87.7
			−200	12.2	99.9

10-14 mesh feed

Screen Size	%	Cumulative %
on 14 mesh	8.4	8.4
20	9.5	17.9
28	18.3	36.2
35	24.0	60.2
48	13.6	73.8
65	7.5	81.3
100	6.1	87.4
150	3.0	90.4
200	1.7	92.1
−200	7.8	99.9

8-10 mesh feed

Screen Size	%	Cumulative %
on 10 mesh	8.1	8.1
14	11.5	19.6
20	16.1	35.7
28	25.0	60.7
35	13.0	73.7
48	7.9	81.6
65	4.4	86.0
100	4.1	90.1
150	2.3	92.4
200	1.2	93.6
−200	6.6	100.2

6-8 mesh feed

Screen Size	%	Cumulative %
on 8 mesh	9.0	9.0
10	12.4	21.4
14	20.2	41.6
20	19.7	61.3
28	13.8	75.1
35	7.6	82.7
48	4.5	87.2
65	2.9	90.1
100	2.9	93.0
150	1.5	94.5
200	.8	95.3
−200	4.7	100.0

4-6 mesh feed

Screen Size	%	Cumulative %
on 6 mesh	6.6	6.6
8	9.3	15.9
10	23.8	39.7
14	21.8	61.5
20	10.9	72.4
28	8.1	80.5
35	4.9	85.4
48	3.5	88.9
65	2.4	91.3
100	2.6	93.9
150	1.4	95.3
200	.7	96.0
−200	4.1	100.1

3-4 mesh feed

Screen Size	%	Cumulative %
on 4 mesh	5.6	5.6
6	7.9	13.5
8	21.9	35.4
10	26.1	61.5
14	12.6	74.1
20	7.2	81.3
28	5.2	86.5
35	3.3	89.8
48	2.6	92.4
65	1.6	94.0
100	1.8	95.8
150	1.1	96.9
200	.6	97.5
−200	2.8	100.3

[1]2-3 mesh feed

Screen Size	%	Cumulative %
on 3 mesh	7.9	7.9
4	6.1	14.0
6	24.7	38.7
8	25.4	64.1
10	13.1	77.2
14	6.5	83.7
20	3.4	87.1
28	3.3	90.4
35	2.4	92.8
48	1.7	94.5
65	1.1	95.6
100	1.2	96.8
150	.7	97.5
200	.4	97.9
−200	2.1	100.0

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[1] The screen referred to here as 2 mesh has a screen opening of .371 inches.

Tables of Energy Absorbed in Crushing, Calculated by Various Methods

Size of Feed	Rittinger Surface Units in Product minus R.S.U. in Feed	Kick Energy Units in Product minus K.E.U. in Feed
20-28 mesh	20536	317
14-20 "	13934	370
10-14 "	9750	357
8-10 "	8102	369
6-8 "	6005	350
4-6 "	5282	379
3-4 "	3916	384
2-3 "	2890	364

Size of Feed	New Formula	Actual Energy
20-28 mesh	1806	3730
14-20 "	1256	2470
10-14 "	923	2067
8-10 "	798	1911
6-8 "	620	1645
4-6 "	561	1575
3-4 "	440	1435
2-3 "	341	1222

Percentages of Feed Crushed Through 200 mesh

Size of Feed	% through 200 mesh	Size of Feed	% through 200 mesh
20-28 mesh	19.6	6-8 mesh	4.7
14-20 "	12.2	4-6 "	4.0
10-14 "	7.8	3-4 "	2.8
8-10 "	6.6	2-3 "	2.1

The integrating dynamometer device has some unusual features. The rolls are driven by belts from a counter shaft which, in turn, is driven by a belt from a motor. The pulley *J* receiving the belt from the motor is mounted on S.K.F. ball bearings on the counter shaft. Alongside of this loose pulley is mounted a cast iron pulley *K*, of the same size, firmly attached to the counter shaft. Connecting these two pulleys are tangential springs. The extension of these springs is a measure of the torque between the two pulleys. This extension of the springs is conveyed to an indicating dial and the integrator by means of the thin steel tape passing over the pulleys as shown in Fig. 8. Pin *A* is fastened to the loose pulley driven by the belt *B* from the motor. A small pulley *C* is on the pulley that is attached to the counter shaft. Small pulley *D* serves to bring the tape to the central line of the shaft. *E* is a swivel from which the tape passes round a small horizontal pulley *F* to the indicator *G* and the integrator *H*. The indicator is simply a pointer on a barrel round which a string from the tape passes, and reads in degrees. During operation it vibrates and fluctuates so much that close readings cannot be taken and it only serves in a very general way. The integrator consists of two main parts, a smooth cast iron disc *I*, which is driven from the main counter shaft and the integrator itself *H*, the mechanism of which is driven by the disc *I*. The integrator is constructed like the hind axle of an automobile. The steel wheels 2″ in diameter take the place of the automobile driving wheels. Their shafts drive a differential which in turn is connected with a series of dials like a watt meter. This whole mechanism is suspended pendulum-wise so that it presses lightly against the cast iron disc *I*. When the integrator is adjusted centrally in regard to the disc the two small wheels revolve at equal rates in opposite directions and the differential and its train of indicating dials remain stationary. The integrator is displaced by the pull of the steel tape and this displacement is proportional to the torque of the main driving pulley. As the integrator is displaced from the centre of the disc one small wheel runs faster than the other and this difference, which is proportional to the displacement multiplied by the R.P.M. of the disc, is recorded on the dials, thus indicating energy transmitted, no matter how much the R.P.M. and the torque may vary. The integrator is calibrated from time to time by a Prony brake on the shaft of one of the rolls.

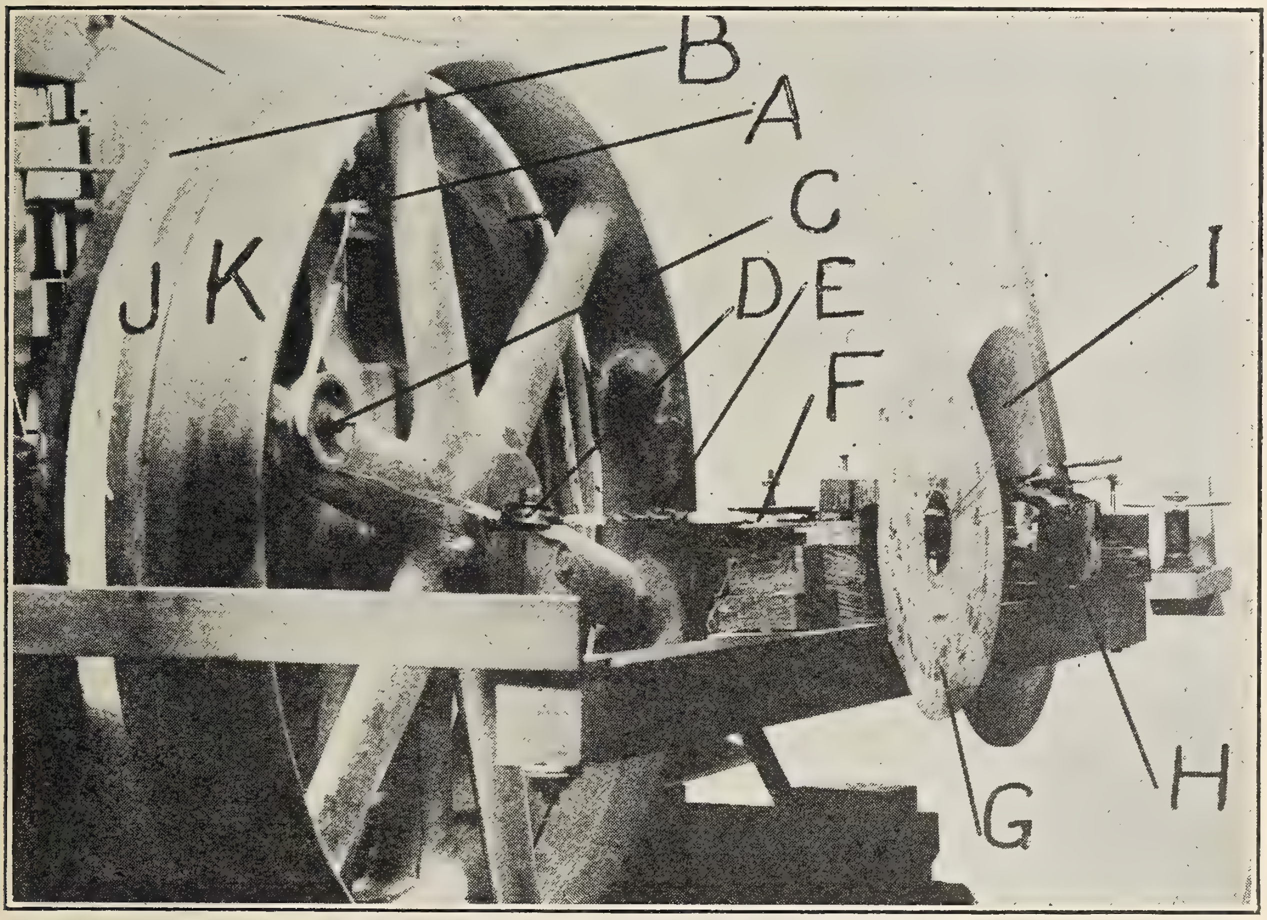

Fig. 8

The results of this mechanism have been one of the surprises of the work. It proves itself much more sensitive and reliable than was expected. One would expect that on account of the many points of friction that there would be considerable sluggishness and that it would be difficult to fix a definite zero point or get closely concordant results. The sensitiveness is apparently due to the fact that in operation the whole mechanism is constantly "alive"; it is in a constant state of vibration and oscillation. When the rolls are running empty at full speed, the integrator is set at its zero central position by a small adjusting screw. It then oscillates backwards and forwards slightly past the zero point and is never "dead".

The use of the two wheels and differential is an advantage in that the wheels always operate at a distance from the centre.

This device was designed by Professor Haultain but he has subsequently learnt that the general idea of this integrating dynamometer is an old one. The writers regret that they have not been able to find, in the literature, who originated it.

THE DETERMINATION OF LEAD AS SULPHATE

(Reprinted from *Canadian Chemistry and Metallurgy*, vol. 8, p. 62, March, 1924)

By J. W. Bain, *Professor of Chemical Engineering*, and
R. I. Wynne-Roberts, *Research Assistant*

In the course of an investigation of the variation in colour in the pigment known as Chrome Yellow[1], the authors found it necessary to determine as accurately as possible the lead content of this substance which is usually normal lead chromate, although some samples carry also varying percentages of lead sulphate.

A considerable amount of time was devoted to an effort to secure results of sufficient accuracy by the electrolytic method, which has been regarded with great favour in analytical practice. A number of determinations of lead in lead chromate carried out by the usual method in which the lead peroxide on the anode is weighed, yielded results varying by as much as 2%, after rejecting the earlier figures which might be open to criticism on the ground that the operators were at first not entirely familiar with the technique. One obvious defect lay in the fact that only 0.2 gram could be weighed out for each sample, the error being therefore multiplied by five when expressed as a percentage of the sample taken.

After a thorough trial of the electrolytic method, it was decided to investigate the possibilities of the determination as sulphate.

To investigate the accuracy which might be reached, the following experiments were carried out.

A sample of assay lead was analysed by the method described by Lunge[2], and was found to contain:

Sb	0.00510%
Cd	0.00078%
Cu	0.00082%
Fe	0.00091%
Zn	0.00082%
Ag	0.00048%
	0.00891%

The sample was accordingly 99.99% pure and was used in all the experiments as a standard.

LEAD ONLY PRESENT

1 gram of the assay lead was weighed out and dissolved in 25 c.c. of conc. HNO_3 and 10 c.c. water. After complete solution 5 c.c. of conc. H_2SO_4 were added and the evaporation was carried on till copious fumes of SO_3 were evolved. After cooling and diluting to 50 c.c. the precipitate was caught in a tared Gooch crucible and washed, first with dilute H_2SO_4 (1 : 4) and then with 25 c.c. of absolute alcohol in portions till free from sulphuric acid. The crucible was dried in an oven for 2.5 hours at 110° C. and weighed. The results are shown in Table I.

TABLE I

Sample	Pb used grams	Wt. of ppte. grams	Pb in ppte. grams	per cent.
1	1.0011	1.4692	1.0015	100.04
2	1.0009	1.4684	1.0010	100.01
3	1.0008	1.4688	1.0012	100.03

Several experiments were made to discover the optimum time for drying the sulphate, and it was found the 2.5 hours was most suitable. Heating the crucible to redness over a Meker burner as recommended by Mellor[3] gave results several per cent. too low.

LEAD IN THE PRESENCE OF CHROMIUM

A solution of the assay lead was prepared by dissolving 40 grams in 100 c.c. HNO_3 (1: 2) and diluting to 1 liter.

Approximately 25 c.c. of this standard solution were weighed into a 250 c.c. beaker and 0.4 gram of CrO_3 (85%) was added which gave a solution containing lead and chromium in ratio roughly equivalent to normal lead chromate. After adding 5 c.c. conc. H_2SO_4, evaporating to fumes and cooling, 150 c.c. of 95% alcohol was added, and after standing for half an hour the precipitate was caught on asbestos in a Gooch crucible washed with 25 c.c. dil. H_2SO_4 (1 : 4), then with 25 c.c. alcohol, dried for 2.5 hours at 110° C. and weighed. The following results were obtained:

Grm. soln.	Wt. $PbSO_4$	Wt. Pb found	Wt. Pb calc.	Per cent. recovered
25.800	1.4268	0.9744	0.9741	100.03
25.020	1.3835	0.9448	0.9447	100.02
26.000	1.4376	0.9818	0.9817	100.01

In the case of Chrome Yellow the method finally adopted for analysis was as follows: 1 gram was dissolved 50 c.c. dil. HNO_3 (3 : 2) in a 250 c.c. beaker, heating till solution was complete. After the addition of 5 c.c. conc. H_2SO_4 the succeeding operations were precisely similar to those described immediately above. Some results obtained on a sample of Chrome Yellow follow:

Wt. pigment	Wt. $PbSO_4$	Lead per cent.
1.0000	0.9423	64.35
1.0000	0.9421	64.34
1.0000	0.9425	64.37
1.0013	0.9435	64.39

For the estimation of chromium Vosburgh's[4] method gave very satisfactory results. Solutions were made up containing known quantities of Pb and CrO_3 to simulate the conditions when the pigment should be under analysis. It was found in a number of experiments that from 99.89 to 99.96% of the CrO_3 actually present was indicated with certainty by this method.

REFERENCES

[1] Trans. Roy. Soc. Can. [3] 17, (1923), III, 83; this Bull. p.

[2] Lunge and Keane—Technical Methods of Analysis—Vol. 2, Pt. I, p. 23.

[3] Mellor—Quantitative Inorganic Analysis.

[4] Jour. Am. Chem. Soc. (1922), 44, 2120.

A STUDY OF CHROME YELLOW

Reprinted from Transactions of the Royal Society of Canada,
[3], 17, III, 83, (1923)

By J. Watson Bain, *Professor of Chemical Engineering*

Chrome Yellow is an important and widely used pigment, manufactured in large quantities by the interaction of a lead salt with alkali bichromate. Thus when solutions of lead acetate and sodium bichromate are mixed, there settles to the bottom of the vessel a heavy yellow precipitate, which after washing and drying constitutes the commercial pigment. Its colour depends entirely upon the conditions under which it is precipitated, and a variety of shades of the same dominant hue may result. At the present time the information available is insufficient to permit the control of these conditions, and in consequence it is very difficult to produce shades which agree closely.

This investigation is an attempt to ascertain the factors upon which the variations in colour depend.

Chrome Yellow is assumed to be normal lead chromate $PbCrO_4$; it varies in colour from pale yellow to deep orange yellow, becoming darker on heating; it is cryptocrystalline; very insoluble in water; decomposed by strong acids or alkalies. There also exists a basic lead chromate, ($PbCrO_4$,PbO) known as Chrome Red, scarlet in colour and cryptocrystalline.

From the numerous prescriptions for the preparation of Chrome Yellow on a commercial scale, the following conditions given by Zerr and Rubencamp[1] may be quoted as typical. These authors state that the colour depends on:

(1) The concentration of the reacting substances.

(2) The temperature at which reaction takes place.

(3) The rate at which the reacting substances are brought together.

(4) The manner of washing the precipitate.

To produce the pale yellow variety, the solutions should be dilute, the temperature about 20°, the agitation vigorous and the whole operation should be conducted as rapidly as possible.

With these conclusions, Free[2], Ridley[3] and Morrell[4] agree. Milbauer and Kohn[5] have made a careful study of the lighter shades of commercial Chrome Yellow which are produced by the co-precipitation of lead chromate and sulphate, and have shown that solid solutions of these two salts can exist which would account for variations in shade: they do not discuss lead chromate alone. To account for the variations in colour which result in spite of the closest adherence to the above conditions, investigators in this field have advanced explanations based upon either (*a*) the presence of basic salts with the normal salt or (*b*) variations in particle size; or on both of these factors. After a careful search of the literature we have failed to find any satisfactory data in support of either of these contentions, and only general statements are to be found. It was therefore decided to undertake a study of this pigment as to its chemical composition and its state of sub-division.

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[1] Zerr and Rubencamp—Colour Manufacture, p. 130.

[2] Free—Jour. Phys. Chem. (1909), *13*, 114.

[3] Ridley—Chemical News, *125*, 12.

[4] Morrell—British Assoc. Advan. Science (1920), 110.

[5] Milbauer and Kohn—Chem. Zeit. (1922), *46*, 1145.

A preliminary investigation in which the reacting solutions were very rapidly and intimately mixed in equivalent quantities, showed clearly that a uniform product could not be produced in this way, although many experiments were made under varied conditions. The efforts to so modify the operating conditions as to avoid variation were therefore abandoned until the underlying causes were definitely appreciated, but these first attempts showed clearly that the size of the particles played an important rôle.

Precipitation of the Pigment

The usual method of precipitation, in which one of the reagents is poured into the other, suffers from the obvious defect that precipitation takes place in the presence of an excess of one reagent whose concentration is constantly changing during the operation. If, for example, the sodium bichromate solution be added to the lead acetate solution, the lead chromate would at first be formed in a solution containing a large excess of lead acetate, and the possibility of the formation of basic salts would thus be favoured. Accordingly, it was decided to mix as rapidly as possible small quantities of the reagents in solution in equivalent quantities, and the apparatus employed may be described briefly.

Over a beaker was suspended a funnel, through the stem of which passed a glass rod with paddles on its lower end at the bottom of the beaker to stir the solution. Solutions containing equivalent quantities of potassium dichromate and lead acetate were allowed to flow through glass stopcocks in equal volumes into the funnel where they mingled and ran down the stem and the stirring rod, being violently agitated as soon as they reached the liquid in the beaker. In this way the rate at which the reactants were mixed could be varied at will as well as the concentration of the solutions; to ensure temperature control the apparatus was placed in a thermostat.

After precipitation the pigment was sometimes stirred, or digested, or filtered at once; in any event it was thrown on a Buchner funnel and washed several times with tap water. It was then dried for two hours at 110° C.

Particle Size

The precipitated pigment was examined under the microscope with a magnification of 1200 diam. in all but a few cases. The microscopy of pigments has been admirably described by Green[1], and in accordance with his suggestions the following method of preparing a mount was developed.

On one end of a standard glass slide was placed a small quantity of the sample to which was added a drop of glycerine, and the two were mixed thoroughly with the end of a glass rod. The latter was then rolled in the viscous mass and the material adhering was transferred to the centre of the slide. Two drops of absolute alcohol were now added and incorporated in the pigment mass until a perfectly homogeneous mixture was obtained; the cover glass was then dropped on and gently pressed down to secure as complete and even dispersion as possible. It was found that all change in the size of the particles ceased as soon as the glycerine had been added, and the mixture might be left for some days.

By means of a camera lucida some hundred particles were drawn on paper, and their dimensions were measured with a millimetre scale. During the entire series of measurements the same conditions regarding lighting, magnification, position of drawing paper, etc., were kept constant to permit comparison of the results.

Measurement of Colour

When dealing with a pigment which exhibits variations in shade, it is obviously desirable to measure and record these shades in some permanent manner. A universal colorimeter made by the Eastman Kodak Company, Rochester, N.Y.,[2] was used and proved satisfactory. This is not a spectrophotometer, but depends upon the absorption of green, blue and red from the light of a standard lamp by three different wedges which may be so placed as to match any hue. By means of a neutral gray wedge the brightness may be varied, and the positions of the wedges are recorded on a scale, the most saturated colour being given by a reading of 100.

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[1] Green—Jour. Franklin Inst. (1921), p. 192, 637. Chem. Met. Eng. (1923), *28*, p. 28, 53.

[2] Mees—Jour. Indust. Eng. Chem. (1921), *13*, 729.

For pigments suspended in liquids a narrow glass cell was used which was practically opaque when filled. The dry pigments were compressed in a shallow circular glass dish of half an inch diameter; after being made even, the surface appeared homogeneous.

The change produced by alterations in the position of the minus green wedge were very much more definite and marked than in the case of the other two wedges. For this reason the readings of the minus green wedge and the particle size were selected as co-ordinates in Figs. 1 and 2.

The Chemical Composition of the Pigment

This question was investigated by four different methods which will be dealt with in succession.

1. *Chemical Analysis*

In the earlier experiments, the lead was determined electrolytically but the results were so variable that no definite conclusions could be based upon them. This was due almost entirely to the small quantities of the pigment which could be used for a determination, and not necessarily to the method itself. In a series of analyses of the pigment the percentages of lead varied from 63.6 to 64.3.

A study of the determination of the metal as $PbSO_4$ was therefore undertaken, and it was found that an accuracy of 0.03% of the lead present could be reached.

1 gram of the pigment was dissolved in a mixture of 30 cc. conc. nitric acid and 20 cc. of water in a 250 cc. beaker, covered with a watch glass, and heated till solution was complete. After cooling and adding 5 cc. conc. sulphuric acid, the solution was evaporated until white fumes appeared to ensure the removal of nitric acid. The solution was then cooled, 150 cc. of 95% alcohol was added, and after standing for half an hour, filtered through a tared Gooch crucible. The precipitate was washed with 25 cc. of dilute sulphuric acid (1:4) and then with 25 cc. of 95% alcohol, dried for 2½ hours at 110° and weighed.

The chromium was determined as follows: 0.4 gram pigment was placed in a wide-mouthed Erlenmeyer flask, covered with a watch glass, 50 cc. of dilute hydrochloric acid (1:9) was added and the flask was gently heated till solution was complete. The temperature during this operation must not exceed 60°. To the solution was now added 1 gram potassium iodide, and after standing five minutes, water to make up 400 cc. Sodium thiosulphate solution was then run in from a burette, starch solution being added near the end as usual. The thiosulphate solution was added in a slow steady stream until the first disappearance of the blue colour which was taken as the end point. Chemically pure potassium bichromate was used as a standard.

The following are the results of the analysis of four samples of a single lot of the pigment.

	LEAD			CHROMIUM		
Sample	Wt. Pigment grams	Wt. ppt. obtained grams	Lead percent	Wt. Pigment grams	cc. Thio.-sulphate solution	Chromium percent
1	1.0000	0.9423	64.39	0.4008	38.4	16.02
2	1.0000	0.9421	64.38	0.4004	38.4	16.03
3	1.0000	0.9425	64.41	0.4007	38.4	16.02
4	1.0013	0.9435	64.39	0.4008	38.4	16.02

The results of the chemical analysis of the various samples prepared will be given later, and it may suffice for the present to state that the evidence points very strongly indeed to the conclusion that the pigment is the normal lead chromate.

2. *Conductivity*

Lead chromate when precipitated as described above is light yellow at first and becomes darker in shade whether it remains in the mother liquor, or is washed and kept in water, or is dried. Formation of the normal chromate at first followed by hydrolysis yielding a basic salt might account for this phenomenon, but the chemical analysis showed that such a reaction could affect only a very slight part of the precipitate since the composition of the latter only altered very slightly if at all on standing. It occurred to us that if such a reaction were possible the supernatant liquid would become acid through the solution of chromium trioxide, and that such a change could be readily detected and followed by the conductivity method.

Samples of the freshly precipitated pale yellow pigment were placed in the usual U-shaped conductivity cell immersed in a thermostat at 25° C. No change in the conductivity of the supernatant liquid could be detected at intervals over a period of 15 hours. Samples of pale orange yellow pigment and of orange yellow pigment gave precisely the same values as the former, nor could any variation be observed after the samples had been under observation for many hours. The addition, however, of a mgrm. of chromium trioxide was immediately followed by a rise in the conductivity.

From these experiments it may be concluded that the pigments of various shades prepared as stated are identical in composition.

3. *Electrode Potential*

In the endeavour to secure independent evidence as to the chemical composition of the pigments, another method of investigation was tried. It was thought that valuable results might be secured by measuring the electrode potential of a cell made up of 5% lead-mercury amalgam, upon which rested a paste of the pigment under investigation, with N/20 potassium chromate solution saturated with pigment above. A normal calomel cell was connected through N potassium chloride solution, and the whole apparatus was kept at constant temperature in a thermostat. The assumption was that variation in electrode potential would indicate variation in chemical composition of the various pigments tested, but it was impossible to secure concordant results even with the same sample of the same pigment. A considerable amount of time was spent in searching for the cause of this behaviour, but without success. Diffusion of the potassium chromate could not be avoided by plugging the connecting tube with cotton wool, but this did not appear to be the serious difficulty. Former investigators of this electrode potential also encountered troubles and the subject requires further study.

4. *Centrifuging*

If a mixture of pale yellow and orange yellow pigments be suspended in water and centrifuged, two layers of distinctly different colour are formed. The variation in the colour of the precipitated pigments would be explained by assuming that varying proportions of the pale yellow and orange shades were mixed, and this possibility was investigated. None of our pigments, whether freshly precipitated, or old, kept moist, or dried, showed any sign of separation into two layers, while mixtures could be readily identified. This would indicate that the change in colour in a pigment is due to a change common to nearly all the particles, since otherwise layers of different colours would be formed on centrifuging.

Experimental Results

From the large number of numerical results which were obtained during this investigation only a few will be presented by way of illustration.

Table I shows the chemical composition of some of the samples and the particle size as precipitated; after a very short time the size of the particles became approximately uniform.

TABLE I

No.	CrO_3 %	PbO %	Particle Size μ	Shade
3	30.36	69.12	2.1	All orange yellow
4	30.42	69.10	2.0	
5	30.31	69.16	1.5 - 2.1	
6	30.27	69.22	2.1	
7	30.33	69.20	2.1	
8	30.43	69.13	1.4 - 2.1	
13	30.47	69.10	1.3 - 2.3	
14	30.45	69.14	1.2 - 3.0	
15	30.41	69.13	1.2 - 4.3	
21	30.43	69.11	1.2 - 2.5	
23	30.43	69.14	1.6 - 2.5	
26	30.45	69.11	1.6 - 1.8	

The change in particle size indicated in the fourth column of Table I commences as soon as the pigment is precipitated, and continues for some time until the maximum is reached. The factors governing this growth were not carefully studied, but at room temperature the period is rarely longer than one or two hours. The average particle size and corresponding colour were ascertained in thirty different preparations, and the results are shown graphically in Fig. 1. For the reason already given the position of the minus green wedge was taken as fairly representative of the variations of shade. By precipitating the pigment in a rectangular glass cell and measuring the colour at short intervals whilst removing simultaneously samples for particle measurement, the same interesting relationship becomes more apparent as may be seen from Fig. 2, the size increasing steadily with the lapse of time since the precipitation of the pigment.

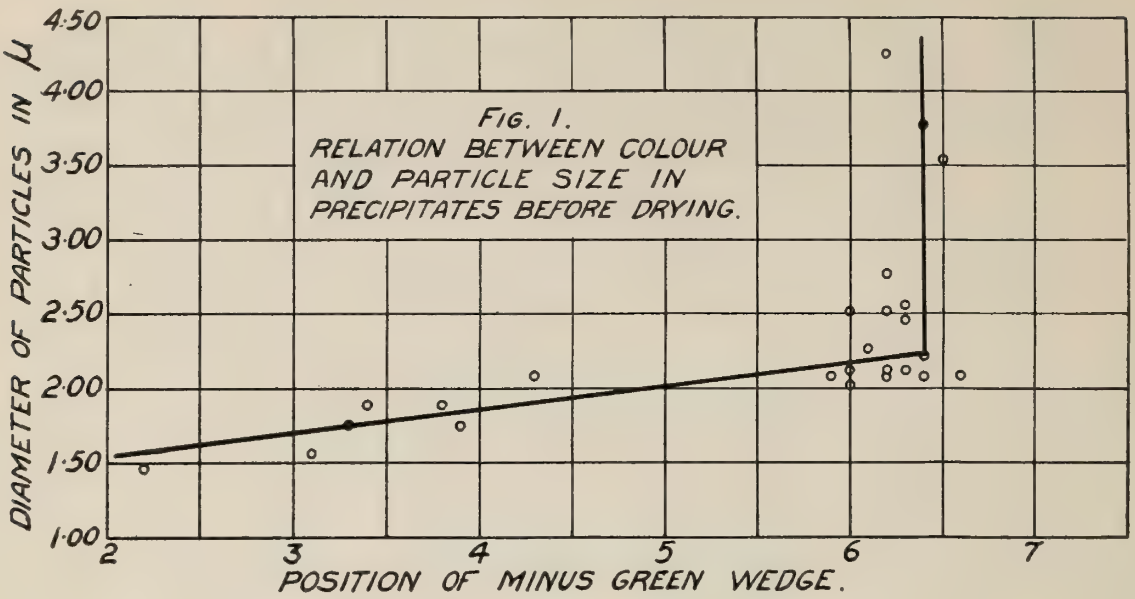

FIG. 1.
RELATION BETWEEN COLOUR AND PARTICLE SIZE IN PRECIPITATES BEFORE DRYING.

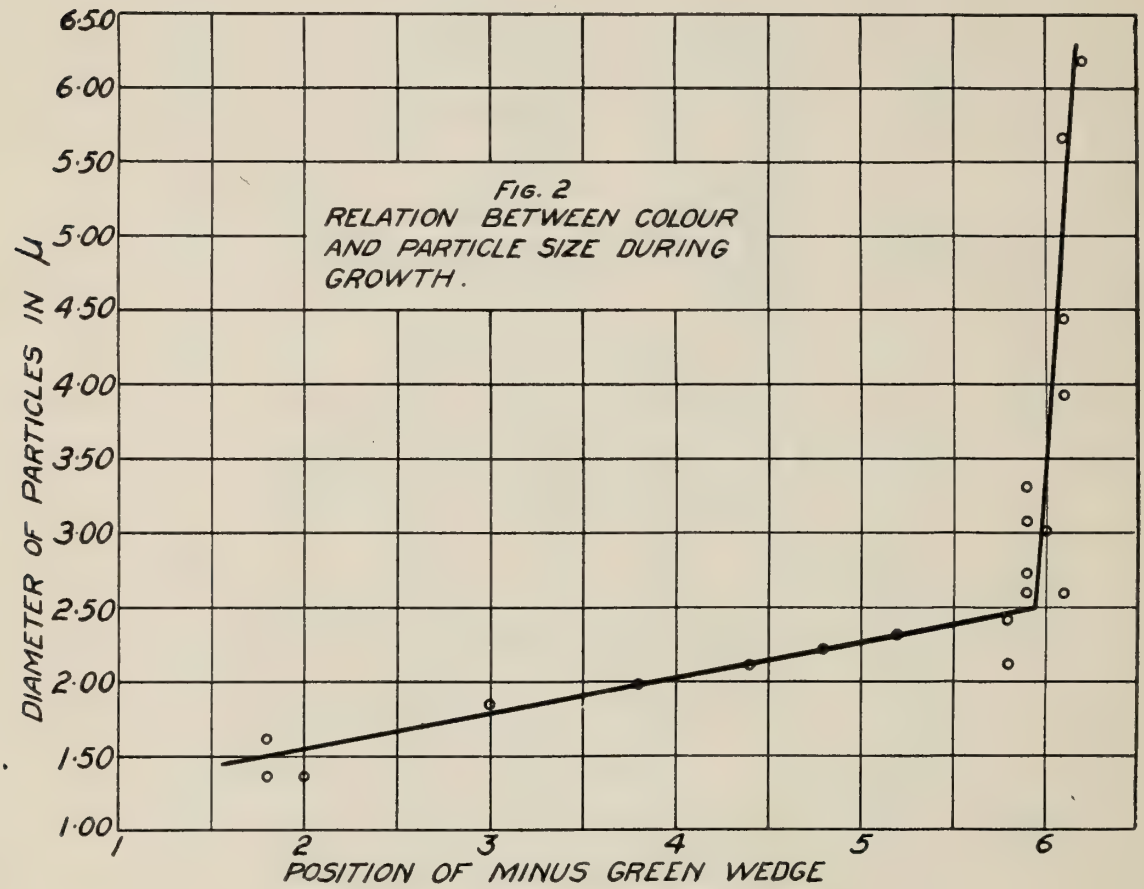

FIG. 2
RELATION BETWEEN COLOUR AND PARTICLE SIZE DURING GROWTH.

Summary

1. That the chemical composition is constant, although the colour and particle size may vary.

2. That the colour and particle size are related. An increase in particle size beyond that of the pale yellow variety results in a corresponding darkening of the colour up to and including a definite size of the crystal. Further increase in the size results in an almost constant value of the colour.

3. The size of the particle at the point at which the colour changes from pale yellow to orange is 2.2μ.

4. That except during precipitation and for a very brief interval thereafter the precipitate consists of one particle size.

5. That an excess of potassium dichromate assists the formation of the orange yellow pigment and an excess of lead acetate inhibits its production; the use of quantities demanded by the theoretical equation results in the orange yellow precipitate.

6. The pale yellow variety is present when precipitated in a state of very fine sub-division, almost colloidal, while the orange variety is formed by the aggregation of the finer particles.

7. That the filtration and washing of the pale yellow precipitate results generally in the formation of the orange yellow variety.

8. That this is another illustration of the change in colour in a solid of constant chemical composition which is accompanied by a change in particle size.

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THE MECHANISM OF CATALYSIS BY PLATINUM

(Reprinted from Transactions of Royal Society of Canada)

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and R. R. McLaughlin, *Research Assistant*

The very extensive literature dealing with the catalysis of reactions by means of platinum has been reviewed in detail in recent papers and books by Sabatier, Willstatter, Skita, Mellor, Woker, Bancroft, Henderson, Rideal and Taylor, and others. It will accordingly only here be necessary to indicate the general conclusions reached with respect to the mechanism of the catalysis, in order that the present condition of this question may be clearly understood, before presenting new experimental evidence in favour of another explanation.

Platinum has a marked capacity for absorbing considerable amounts of many gases.

Faraday (1) believed these gases to be condensed on the surface of the platinum and, as a consequence, that the reacting gases, being thus at much higher concentrations on the platinum surface, react with a very much higher velocity than in the absence of platinum.

De la Rive (2) and others believed that the platinum forms an unstable intermediate oxide PtO which alternately gives up its oxygen to the body oxidized, and is reformed from the oxygen gas.

Engler and Wohler (3) concluded that this intermediate oxide is PtO_2 or a hydrated form of this.

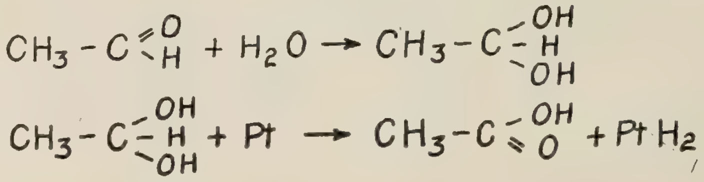
Berthelot (4) expressed the opinion that when platinum catalyzes the union of hydrogen and oxygen, intermediate hydrides $Pt_{30} H_2$ and $Pt_{30} H_3$ are formed.

These hypotheses of De la Rive, Engler and Wohler, and Berthelot postulate the existence of definite oxides or hydrides of platinum which act as oxygen and hydrogen carriers in reactions catalysed by platinum. The experimental work which is described in this paper shows, we believe, that the "intermediate" by which platinum brings about the catalysis of oxidation, or of reduction is not a definite oxide or hydride of platinum, indeed not a compound in the ordinary sense of the term at all, but is an adsorption complex of varying composition, depending on the conditions, particularly upon the temperature. The constitution of this complex will be considered presently. It may be pointed out here that the senior author (5) has already shown, that in the catalysis of hydrogenation by nickel, the analogous hypothesis of the existence of definite hydrides and oxides of nickel, as advocated by several investigators, is likewise untenable, and that the nickel functions by means of an adsorption complex, consisting of nickel carrying an interior content of oxygen, and a surface layer consisting of positively charged hydrogens and negatively charged hydroxyls thus:

$$\begin{matrix} Ni \\ (O)_x \end{matrix}\left|\begin{matrix} H^+ \\ OH^- \\ H^+ \\ OH^- \\ \vdots \end{matrix}\right.$$

The almost complete parallelism of the results obtained in our study of nickel and platinum catalysers will be apparent from the experimental data of this paper.

Recently Wieland (6) has shown that platinum, prepared and used with the complete exclusion of free oxygen, is able to effect the oxidation, in water solution, of acetaldehyde to acetic acid, of glucose to carbondioxide, of sulphur dioxide to sulphur trioxide. This action soon ceases, the platinum being now saturated with hydrogen. The addition, now, of a hydrogen acceptor, as quinone, methylene blue, or free oxygen, removes the hydrogen from the platinum, and the actions may then continue. That is, according to Wieland, these oxidations are really dehydrogenations by platinum of an addition complex of water with the compound undergoing oxidation. In the case of acetaldehyde Wieland expresses the mechanism as follows:



$$CH_3-C\begin{matrix}\nearrow O\\ \searrow H\end{matrix} + H_2O \rightarrow CH_3-C\begin{matrix}\nearrow OH\\ - H\\ \searrow OH\end{matrix}$$

$$CH_3-C\begin{matrix}\nearrow OH\\ - H\\ \searrow OH\end{matrix} + Pt \rightarrow CH_3-C\begin{matrix}\nearrow OH\\ \searrow O\end{matrix} + PtH_2$$

If quinone is present the further reaction occurs—Pt H_2+quinone ⟶ Pt+hydroquinone, and so the platinum is free to again dehydrogenate more water—acetaldehyde complex.

Doubtless in every instance there is some complex of water and dissolved substance formed. However it occurred to the senior author, on reviewing various other facts relating to a platinum catalyser, in conjunction with these very important observations of Wieland, that a high degree of probability attaches to the hypothesis that water forms a complex, not only with the body oxidized, but also with the catalyser, and that, of the two complexes, the latter, constituting a surface film on the platinum, is the more important, and is the real seat of the catalysis. There is a considerable body of evidence which not only points to this as the correct explanation, but which also indicates the probable nature of this active surface film.

Before reviewing this evidence it might be pointed out that it is quite natural that the emphasis should, at first, have been placed upon the complex of water and compound in solution, rather than upon the complex of water and catalyser. The extraordinary facts which have been disclosed since the time of Mrs. Fulhame (7) by Dixon (8), Traube (9), Schonbein (10), Brodie (11), Hoppe-Seyler (12), Bach (13), and others, with regard to oxidations by free oxygen, in the absence of a deliberately added catalyser (such as, to mention but one, the necessity for the presence of at least a trace of water in order that any action at all may occur), has led to many hypotheses to represent the mechanism of such oxidations. Since no catalyser was here involved, these mechanisms represented solely the relationship of water to the oxygen, and to the compound undergoing oxidation. It seems quite natural, then, that in the consideration of the mechanism of catalytic oxidation, hypotheses should first be advanced fixing the attention upon the water-compound complex rather than upon the water-catalyser complex. Following are some facts which point very strongly to the existence of a complex of water and catalyser, and which make it appear highly probable that it is this complex which is directly responsible for the catalysis:

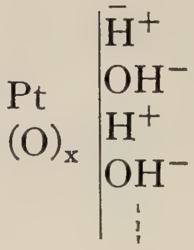
(1). Langmuir (14) observed that the activity of a platinum catalyst, for the combination of carbon monoxide and oxygen, is increased by first using the platinum to catalyse the union of hydrogen and oxygen.

(2). When an active platinum catalyser is gradually heated to a high temperature it loses water and simultaneously slowly loses its activity for catalysing the union of hydrogen and oxygen and carbon monoxide and oxygen.

(3). Platinum, active for catalysing oxidations by free oxygen, is also active for catalysing hydrogenations by free hydrogen.

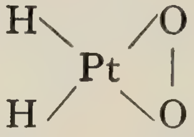
(4). Platinum, active for catalysing oxidation by free oxygen, is also active for catalysing hydrolyses as for instance the hydrolyses of sucrose and methyl acetate.

It is very difficult to understand this behaviour of platinum, particularly the catalysis of oxidations, hydrogenations and hydrolyses by the same platinum catalyser, without picturing hydrogens and hydroxyls in some sort of combination with platinum. Accordingly the tentative hypothesis was set up, that an active platinum catalyser consists of platinum with an interior content of oxygen, and a surface film consisting of dissociated water, that is of positive hydrogens and negative hydroxyls thus:



This complex is entirely analogous to the nickel complex already referred to. Here Pt represents a particle (not necessarily an atom) of platinum, the number of atoms bound together depending on the method of preparation—for instance, the conditions under which the platinum sol has been precipitated. In direct combination with this platinum particle, is oxygen, which can vary in amount. The particle with this content of oxygen has a surface consisting of charged hydrogens and hydroxyls, which can also vary in amount with variation in conditions, and which are held on the surface by some partial or secondary valencies of the platinum. The experimental data indicated certain expansions of this conception which will be described presently.

More recently Willstätter and Waldschmidt-Leitz (15) have arrived at the conclusion that platinum functions as a catalyser by means of a compound, platinum peroxide dihydride, to which they ascribe the constitution:



The existence of this compound, as an intermediate carrier of hydrogen and oxygen in catalyses, has been questioned by Hoffman (16). This important work of Willstätter and Hoffman will be considered presently. It should be pointed out here that Willstätter has shown that platinum, active for catalysing hydrogenations, must contain oxygen. Although we believe this statement to be true, we think that the experimental evidence furnished by Willstätter is open to adverse criticism. Willstätter concludes that platinum black, reduced at room temperature by free hydrogen for 30 hours, is entirely free of oxygen, and that, because such a platinum is inactive as a catalyser for hydrogenations at room temperature, oxygen in the platinum is necessary, in order that it may be a catalyser for this kind of action. Now we have not been able to prepare a platinum black free from oxygen by treatment with hydrogen for a much longer period than 30 hours at room temperature. We are inclined to think that Willstätter's platinum still had an inner content of oxygen, but that this was, relatively, so far removed from the surface layer, consisting of hydrogens alone, that his platinum acted practically in the same manner towards hydrogenation actions, as a catalyst entirely free from oxygen. This criticism will be much clearer when the mechanism of these actions is considered later in this paper.

Hoffman concludes from his experiments that the existence of Willstätter's platinum peroxide dihydride is highly improbable, and expresses the opinion that the next step in the investigation of the mechanism of catalysis must be a study of the nature of the water film on the platinum surface. This is the subject of our paper.

Experimental

The apparatus used was the same as that employed by the senior author (17) in the study of the mechanism of catalysis by nickel (page 178).

The platinum black, used in all the experiments described in this paper, was prepared by a modification of the method of Loew (18) as follows: Platinum foil was dissolved in *aqua regia* and the solution evaporated to dryness on a water bath. This platinum chloride was dissolved in water, using 30 c.c. water for each gram of platinum. To this, 15 c.c. of formaldehyde (40%) was added, and then a solution of 20 g. sodium hydroxide in 20 c.c. water. The whole was allowed to stand 48 hours, and then filtered and washed free from chlorine ions. We found that unless care is taken to keep the solution cool (10° C.) during the mixing and standing, resinous compounds readily form, probably by polymerization of the aldehyde, and that these act as protective colloids for the platinum sol which first forms, and thus prevent the precipitation of the platinum black. The resulting platinum black was mixed with moist asbestos, dried in air, sometimes at room temperature, sometimes in an air bath at 110° and sometimes at 150° C. The temperature of drying is indicated in the separate experiments. The gas in the apparatus between the two measuring burettes was, at the outset of each experiment, replaced by nitrogen, freed from oxygen by passing over hot copper gauze.

The purpose of the early experiments was to determine the behaviour of platinum black when treated separately, at room temperature, with hydrogen and oxygen, in order to see whether there was any indication of the formation of a surface film of dissociated hydrogen, or of dissociated water on the platinum.

Experiment 1.

About 2 g. platinum black, dried at room temperature, was reduced, at room temperature, by a measured volume of hydrogen. Following are the volume readings in separate experiments for successive passages over and back, the readings being taken with the same burette:

	360 c.c.	360 c.c.	356 c.c.	359 c.c.
	335	340	336	339
	330	337	330	335
	329	332	330	330
	329	331	328	324
	329	328	324	323
		328	321	321
		328	321	321
			321	321
Hydrogen reacted.........	31 c.c.	32 c.c.	35 c.c.	37 c.c.
Hydrogen equivalent of water formed...........	22.0 c.c.	15.0 c.c.	17.0 c.c.	13.6 c.c.

The following are typical results obtained in experiments of the same kind, carried out with platinum black previously dried at 100°, and heated at 100° during the passage of hydrogen:

	360 c.c.	360 c.c.	360 c.c.	360 c.c.
	330	345	341	343
	322	339	334	340
	320	335	328	339
	318	330	324	336
	318	326	325	337
	316	324	324	334
	317	324		334
	316	322		334
		322		
		322		
		322		
Hydrogen reacted.........	44 c.c.	38 c.c.	36 c.c.	26 c.c.
Hydrogen equivalent of water formed...........	33 c.c.	22 c.c.	23 c.c.	17 c.c.

Subsequent experiments showed, that the hydrogen equivalent of the oxygen contained in two grams of platinum black, prepared as described above, is greater than 400 c.c. It is evident, then, that when platinum black is treated with hydrogen, either at room temperature or at 100° C., the reducing action becomes so slow, that little or no hydrogen is used up in three or four passages back and forth, occupying altogether about 20 minutes. In reaching this condition only about one-tenth of the hydrogen necessary to combine with all the oxygen of the platinum black has reacted, and a considerable percentage of the hydrogen used up is not evolved as water, but remains in some form on the platinum. Notwithstanding that the platinum was in an atmosphere of hydrogen at the end of each experiment, and, as the results show, actually contained hydrogen in some form, its inner content of oxygen, amounting to about nine-tenths of the oxygen originally present in the platinum black, could react only very slowly with this hydrogen. The inference seems justified, that the hydrogen in the platinum is in some special condition, which affords excellent protection for the underlying large content of oxygen. Were the hydrogen simply dissolved in the platinum, it is difficult to see why the reduction action should become so slow. The view that the surface oxygen has been removed, and that the slowing up of the reduction is due to the necessity for the hydrogen to penetrate farther into the particles, is difficult to believe, in view of the rapid rate of diffusion of hydrogen. This, however, is a debatable point, and independent evidence must be furnished before the former view which we advance of a special protecting surface layer can be accepted. The following experiments furnish, we think, this evidence.

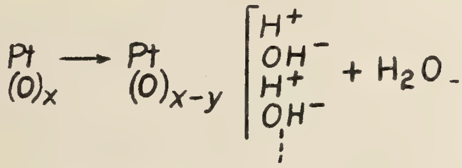
Experiment 2.

Freshly prepared platinum black was dried at 150° C. to constant weight. A measured volume of hydrogen was then passed over the platinum, heated to 150° C., until there was no further disappearance of hydrogen in three successive passages over and back between the two burettes. The diminution in volume, and the water formed were determined. The hydrogen in the apparatus between the burettes was displaced by nitrogen, and a measured volume of oxygen passed over the platinum heated to 150° C., until the oxygen ceased to disappear in three successive passages. This alternate treatment with hydrogen and oxygen was repeated several times. Following are the results in a typical experiment:

H disappeared	O disappeared	H equivalent of water formed	H left on the platinum
59 c.c.		54.5 c.c.	4.5 c.c.
	22 c.c.	7.4	−2.9
45		27.3	14.8
	20	7.1	7.7
48		29.5	26.2
	16	7.1	19.1
38		24.8	32.3
	14	3.5	28.8
36		25.4	39.4
	15	6.3	33.1
29		25.6	36.5
	13	3.5	33.0
34		24.3	42.7
	15	6.5	36.2
29		23.4	41.8
	15	4.1	37.7

After the last passage of oxygen the platinum stood in an atmosphere of oxygen over night. Upon heating to 150° C. on the following morning no further disappearance of oxygen occurred. That is, the amount of oxygen which disappeared in the first 25 minutes of the last treatment, was not increased by many hours further treatment with oxygen at room temperature, and at 150° C. Now at the end of this oxygen action the platinum contained about 40 c.c. hydrogen in some form. It seems impossible to accept the view that this hydrogen is simply dissolved in the platinum black, when it is borne in mind (1) that the platinum, in this condition, now contained oxygen equivalent to about 300 c.c. hydrogen, (2) that this platinum, containing this hydrogen, had been heated at 150° C. in an atmosphere of oxygen without any disappearance of oxygen, (3) that, as very numerous other experiments showed, a treatment of this platinum with free hydrogen, causes a disappearance of practically the same volume of hydrogen as disappeared in the hydrogen treatment just preceding (in this case about 30 c.c.) leaving the same amount of hydrogen on the platinum (in this case about 40 c.c.). On the other hand, a very satisfactory explanation of this behaviour, as well as of other facts which will be described presently in this paper, is afforded by the following hypothesis:

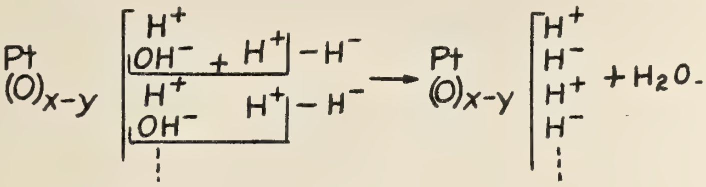
(1). On passing hydrogen over platinum black a relatively small part of the oxygen is removed as free water, which passes off, leaving first a film of dissociated water on the surface of the particles, thus:



$$\overset{\text{Pt}}{(O)_x} \longrightarrow \overset{\text{Pt}}{(O)_{x-y}} \left[\begin{array}{l} H^+ \\ OH^- \\ H^+ \\ OH^- \\ \vdots \end{array}\right. + H_2O \ldots\ldots\ldots (1)$$

This is a very rapid reaction.

(2). On continuing the passage of hydrogen this surface film is very rapidly transformed into a film of dissociated hydrogen, thus:

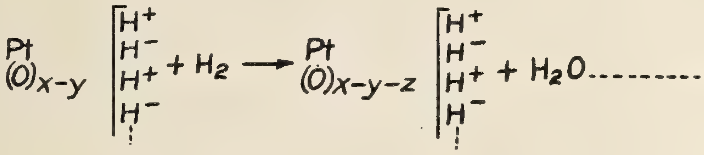


$$\overset{\text{Pt}}{(O)_{x-y}} \left[\begin{array}{l} H^+ \\ OH^- + H^+ \,|-H^- \\ H^+ \quad\ H^+ \,|-H^- \\ OH^- \\ \vdots \end{array}\right. \longrightarrow \overset{\text{Pt}}{(O)_{x-y}} \left[\begin{array}{l} H^+ \\ H^- \\ H^+ \\ H^- \\ \vdots \end{array}\right. + H_2O \ldots (2)$$

This is a very fast reaction.

(3). During the formation of the surface film of dissociated hydrogen according to reaction (2) the surface is momentarily free, here and there, from any covering film, and as a consequence the free hydrogen has access, to some extent, to the interior oxygen and removes some of it as water.

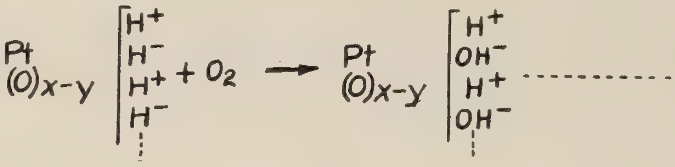
(4). Also, the film in the state in which it is present after reaction (2) reacts very slowly with the interior oxygen to form free water, thus:



$$\overset{\text{Pt}}{(O)_{x-y}} \left[\begin{array}{l} H^+ \\ H^- \\ H^+ \\ H^- \\ \vdots \end{array}\right. + H_2 \longrightarrow \overset{\text{Pt}}{(O)_{x-y-z}} \left[\begin{array}{l} H^+ \\ H^- \\ H^+ \\ H^- \\ \vdots \end{array}\right. + H_2O \ldots\ldots\ldots (4)$$

This occurs to some extent and, as a consequence, the platinum loses some of its interior content of oxygen. However, this reaction is very slow. This is in harmony with the fact, which we found, that the interior oxygen can be removed much faster by alternate treatments with hydrogen and oxygen than by treatment with hydrogen alone. That it occurs is clearly seen from the fact, that if platinum black, which has been treated for a considerable time with hydrogen, is allowed to stand over night in an atmosphere of nitrogen, and oxygen is then passed, very much less oxygen is used up than would be, were the oxygen immediately passed without standing. The surface hydrogen has reacted with the interior oxygen to produce the surface (consisting of positive hydrogens and negative hydroxyls), which would be produced by the immediate passage of oxygen. Following are typical examples: (1) a platinum black which used up 10 c.c. of oxygen when passed immediately required only 1 c.c. of oxygen after standing in nitrogen over night, and (2) a platinum black which required 6 c.c. of oxygen when passed immediately required only 1 c.c. of oxygen after standing over night. Moreover, this behaviour was also observed in the case of a platinum black which had lost a considerable proportion of its interior oxygen. This furnishes one more argument against the simple solution theory of catalysis. For, according to this theory, oxygen should, at this stage, be taken up in considerably larger amount.

(5). On now displacing the free hydrogen by nitrogen and passing oxygen the film of dissociated water is again formed, thus:



$$\begin{matrix}Pt\\(O)_{x-y}\end{matrix}\left[\begin{matrix}H^+\\H^-\\H^+\\H^-\\\vdots\end{matrix}\right. + O_2 \longrightarrow \begin{matrix}Pt\\(O)_{x-y}\end{matrix}\left[\begin{matrix}H^+\\OH^-\\H^+\\OH^-\\\vdots\end{matrix}\right. \text{ ----------- } (5)$$

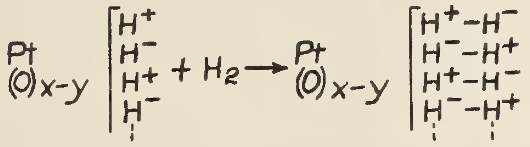
Upon examining the experimental results of the alternate treatment with hydrogen and oxygen it is seen, that the amounts of hydrogen remaining on the platinum after the passages of hydrogen, increase quickly at first, but that this rate of increase falls off until, finally, the hydrogen remaining becomes practically constant. The values are, in the experiments cited: 4.5, 14.8, 26.2, 32.3, 39.4, 36.5, 42.7, 41.8. In another case the values 13, 26, 36, 43 were obtained. That is, it takes a few alternate treatments with hydrogen and oxygen to form the surface film in its equilibrium condition.

When this condition has been reached the further successive separate treatments with hydrogen and oxygen result in using up an approximately constant amount of hydrogen and a constant amount of oxygen with the formation of a constant amount of water on passage of hydrogen. Even after removing a large proportion of the interior content of oxygen by long treatment with hydrogen, these amounts of hydrogen, oxygen and water, remain practically at the same equilibrium values. Thus the platinum from the experiment above was further treated with hydrogen for 10 hours at 150° C. and for 200 hours at room temperature when 99 c.c. oxygen was found to have been removed from the platinum. Upon now heating successively with oxygen and hydrogen the following results were obtained:

H disappeared	Oxygen disappeared	H equivalent of water formed
	10	2.7
35		27.3
	12	2.5
35		27.0
	10	6.0
33		

Thus the platinum surface even after prolonged treatment remained in approximately the same condition as that obtained by passage of hydrogen and oxygen half a dozen times. That the individual values of hydrogen and oxygen disappearing and water formed vary somewhat from the equilibrium values is to be expected owing to the possibility of further reactions of the interior oxygen. This will be discussed presently in this paper.

The above hypothesis postulates the following: (1). In the alternate treatments with hydrogen and oxygen, after the surface film has been formed, the hydrogen used up is equal to the hydrogen equivalent of the water formed. This relationship was sometimes obtained, but usually the hydrogen which disappeared was somewhat greater than the hydrogen equivalent of the water formed. This is probably due to the ability of the complex formed after reaction (2) to add on an equal volume of hydrogen, thus:



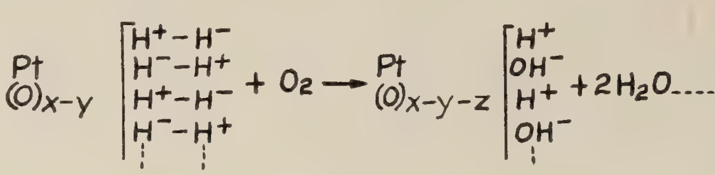
$$\overset{Pt}{(O)_{x-y}}\left[\begin{array}{l}H^+\\H^-\\H^+\\H^-\\ \vdots\end{array}\right. + H_2 \rightarrow \overset{Pt}{(O)_{x-y}}\left[\begin{array}{l}H^+-H^-\\H^--H^+\\H^+-H^-\\H^--H^+\\ \vdots \quad \vdots\end{array}\right. \quad \ldots\ldots\ldots\ldots (7)$$

In such a case the hydrogen disappearing is double the hydrogen equivalent of the water formed. In several experiments this ratio was obtained. For instance, in one experiment the following values were obtained:

H disappeared	H equivalent of water formed
40	20
40	22
50	28

Upon heating now with oxygen this extra layer of hydrogens is oxidized to water and the inner layer of hydrogens undergoes the transformation shown in reaction (6) below. Thus the larger the ratio of hydrogen disappearing to hydrogen equivalent of water formed, the greater the amount of water given off in the subsequent action with oxygen. This is in harmony with the measurements in the experiment just cited.

(2). The volume of oxygen disappearing in an oxygen treatment is one-half the volume of hydrogen which disappeared in the previous treatment with hydrogen. This relationship holds for the series of measurements given above. However, in some series, where reaction (6) has taken place, the results indicate that the two hydrogen layers react independently, one with the free oxygen and the other with the interior oxygen, thus:



$$\begin{matrix}Pt\\(O)_{x-y}\end{matrix}\left[\begin{matrix}H^+-H^-\\H^--H^+\\H^+-H^-\\H^--H^+\\ \vdots\quad\vdots\end{matrix}\right. + O_2 \longrightarrow \begin{matrix}Pt\\(O)_{x-y-z}\end{matrix}\left[\begin{matrix}H^+\\OH^-\\H^+\\OH^-\\ \vdots\end{matrix}\right. + 2H_2O \ldots\ldots(6)$$

That is, for every 4 vols. of hydrogen on the Pt one vol. of oxygen disappears and 2 vols. of H_2O are formed.

(3). According to reaction (5) no water is evolved upon passing oxygen. However, a small amount was always obtained due to the extra hydrogen absorbed according to reaction (6). This water, according to reaction (6), can rise to an amount having a hydrogen equivalent equal to one-half the hydrogen which disappeared in the previous treatment with hydrogen and this amount was sometimes obtained. To illustrate more definitely from the data, consider the series of measurements under experiment (2) above. Here the equilibrium value of the water evolved when hydrogen is passed has a H equivalent of about 24 c.c. This, according to the above hypothesis, was formed by 24 c.c. free hydrogen reacting as in reaction (2). Now it is seen that the volumes of hydrogen which disappeared, after the surface film had reached approximately equilibrium, were 29, 34, 29 instead of 24 c.c. The excess amounts of hydrogen, viz., 5 c.c., 10 c.c., 5 c.c., are the volumes of hydrogen which were taken up without the formation of water, according to reaction (6) above. On now treating with oxygen the value of the oxygen disappearing is greater than 12 c.c., by approximately the amount necessary to oxidize this excess hydrogen. Thus there are amounts of water given off on subsequent treatment with oxygen, corresponding to the amounts of excess hydrogen, viz., for excess hydrogen amounting to 5, 10, and 5 c.c. the amounts of water evolved on subsequent treatments with oxygen had hydrogen equivalents 3.5, 6.5 and 4.1.

Further evidence of the existence of these stable surface films, and that in hydrogenations and oxidations, catalysed by platinum black, the seat of these actions lies in these films, is shown by the following series of experiments. Platinum black was prepared as before and dried to constant weight in air at 150° C. This was then reduced in hydrogen at room temperature till no further hydrogen disappeared in three successive passages back and forth between the burettes. Nitrogen was then passed to expel free hydrogen, and the temperature of the platinum was raised to 150° and the passage of nitrogen continued in order to drive over all free water into the absorption vessels. The platinum was then cooled to room temperature, and a measured volume of oxygen passed, and the same procedure adopted as in the previous passage of hydrogen. This alternate passage of hydrogen and oxygen was continued until the hydrogen remaining on the platinum after the passage of hydrogen became constant. The two final sets of values obtained were:

H disappeared	O disappeared	H equivalent of water formed
	12	0.8
26		27.2
	12	2.2
26		27.0

Four experiments with mixtures of hydrogen and oxygen were now carried out with the platinum in the two conditions, viz., after treatment with oxygen, when the catalyser was in the condition

$$\begin{array}{c|l} & H^+ \\ Pt & OH^- \\ (O)_x & H^+ \\ & OH^- \\ & \vdots \end{array}$$

and after treatment with hydrogen when it was in the condition

$$\begin{array}{c|l} & H^+ \\ Pt & H^- \\ (O)_x & H^+ \\ & H^- \\ & \vdots \end{array}$$

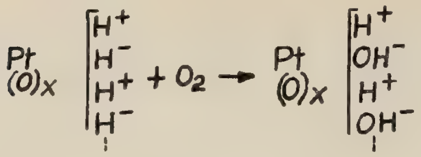
using hydrogen in excess in two experiments, and using oxygen in excess in the other two. The gas mixture was, in each case, passed over the platinum at room temperature until there was no further change in volume on passage of the gas back and forth. Nitrogen was passed to expel the gas mixture, and the platinum heated to 150° C., while the passage of nitrogen was continued till the free water had all passed into the absorbers. The platinum was then cooled to room temperature and the next gas mixture passed.

Mixture (1). 170 c.c. nitrogen, 150 c.c. oxygen, 40 c.c. hydrogen. This was passed over the above-mentioned platinum black which had just been treated with hydrogen. Gas analysis gave the following results:

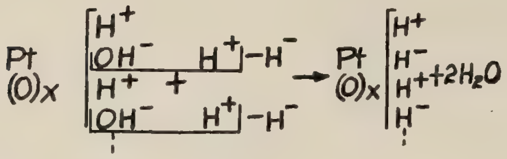
Diminution in volume	73 c.c.
Hydrogen equivalent of water formed	33 c.c.
Hydrogen used up	40 c.c.
Oxygen used up	33 c.c.

In these experiments with hydrogen and oxygen mixtures two reactions take place:

(a)



(b)



In the above experiment the oxygen and hydrogen react alternately, commencing with oxygen, until 40 c.c. of hydrogen and 20 c.c. of oxygen have combined. The platinum black is now in the original condition shown on the right of reaction (*b*) and the left of reaction (*a*). But there is an excess of oxygen present. Hence the final state of the platinum will be that shown on the right of reaction (*a*). Now the prior treatment of the platinum black with hydrogen and oxygen separately showed, as pointed out above, that, in the equilibrium condition, 26 c.c. hydrogen and 12 c.c. oxygen are used up in successive treatments. We would expect then to find a further disappearance of 12 c.c. of oxygen, making a total oxygen loss of 32 c.c. Actually, 33 c.c. disappeared.

Mixture (2). 208 c.c. nitrogen, 25 c.c. oxygen, 125 c.c. hydrogen. The platinum black had just been treated with hydrogen and the hydrogen displaced with nitrogen.

Diminution of volume	77 c.c.
Hydrogen equivalent of water formed	46 c.c.
Hydrogen used up	52 c.c.
Oxygen used up	25 c.c.

In this experiment 25 c.c. of oxygen and 50 c.c. of hydrogen react according to reactions (*a*) and (*b*), leaving the platinum black in the condition it was in at the outset; and as hydrogen is in excess in the experiment the platinum remains in this condition. Actually 52 c.c. of hydrogen disappeared instead of 50 c.c.

Mixture (3). 210 c.c. nitrogen, 25 c.c. oxygen, 128 c.c. hydrogen. The platinum black had just been treated with oxygen and the oxygen in the apparatus displaced by nitrogen.

Diminution of volume	101 c.c.
Hydrogen equivalent of water formed	75 c.c.
Hydrogen used up	76 c.c.
Oxygen used up	25 c.c.

In this experiment 50 c.c. hydrogen and 25 c.c. of oxygen reacted according to reactions (*b*) and (*a*) leaving the platinum in the condition it was in at the outset, viz., as shown on the left of equation (*b*). Hydrogen is in excess in the experiment, and we have seen that it requires 26 c.c. of hydrogen to carry out reaction (*b*) completely, thus requiring a disappearance of 76 c.c. hydrogen, which was the volume of hydrogen which actually disappeared in the experiment.

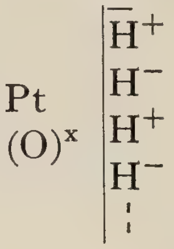
Mixture (4). 180 c.c. nitrogen, 155 c.c. oxygen, 25 c.c. hydrogen. The platinum black had just been treated with oxygen and the oxygen in the apparatus displaced by nitrogen.

Diminution of volume	38 c.c.
Hydrogen equivalent of water formed	24.8 c.c.
Oxygen disappeared	13 c.c.
Hydrogen disappeared	25 c.c.

In this experiment 25 c.c. of hydrogen and 12.5 c.c. of oxygen combined by alternate action according to reactions (*b*) and (*a*), leaving the platinum in the original condition shown on the left of reaction (*b*). Now oxygen was in excess in the experiment. Hence the platinum remains in this condition, and the oxygen and hydrogen would, according to this explanation, disappear in the amounts 25 c.c. of hydrogen and 12.5 c.c. of oxygen. Actually 13 c.c. of oxygen was observed to disappear.

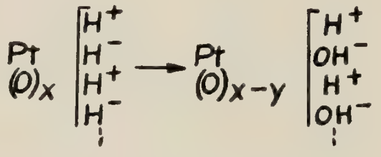
It should be borne in mind that, in all these experiments with mixed gases, the platinum had an inner content of oxygen whose hydrogen equivalent was between 150 and 200 c.c. Once more the conclusion seems unavoidable, that the reactions occur in surface films, having a remarkable capacity for protecting the underlying inner content of oxygen. The hypothesis here presented seems to represent adequately these facts.

An objection might be raised to the above experimental procedure, viz., when reactions, as some of the above, have been carried out at room temperature and the platinum has then been heated in nitrogen at 150° to expel all free water, free hydrogen may have been given off and if so the measurements and conclusions would be vitiated. Several experiments were performed to test this point according to the following procedure. Platinum black, which had been treated several times with hydrogen and oxygen separately at room temperature and heated in nitrogen at 150° between the successive passages of gas, was finally treated with hydrogen at room temperature. The free hydrogen was swept out at room temperature by the passage of nitrogen. The platinum was then heated to 150°, and 350 c.c. nitrogen passed and the gas collected. This was analysed for hydrogen and oxygen. Neither hydrogen nor oxygen could be found. However, a small amount of water was always obtained, whose hydrogen equivalent was in one case 2.7 c.c. and in another 5.1 c.c. That is, platinum black after treatment with hydrogen, when in the condition



$$\begin{array}{c} \mathrm{Pt} \\ (\mathrm{O})^x \end{array} \left| \begin{array}{l} \mathrm{H}^+ \\ \mathrm{H}^- \\ \mathrm{H}^+ \\ \mathrm{H}^- \\ \vdots \end{array} \right.$$

reacts to a slight extent, when heated in an atmosphere of nitrogen to 150°, by a small amount of the surface hydrogen combining with some interior oxygen, thus:



$$\begin{array}{c} \mathrm{Pt} \\ (\mathrm{O})_x \end{array} \left| \begin{array}{l} \mathrm{H}^+ \\ \mathrm{H}^- \\ \mathrm{H}^+ \\ \mathrm{H}^- \\ \vdots \end{array} \right. \longrightarrow \begin{array}{c} \mathrm{Pt} \\ (\mathrm{O})_{x-y} \end{array} \left| \begin{array}{l} \mathrm{H}^+ \\ \mathrm{OH}^- \\ \mathrm{H}^+ \\ \mathrm{OH}^- \\ \vdots \end{array} \right.$$

The composition of this adsorption complex depends on the temperature, *i.e.*, it can hold less dissociated water as the temperature rises. So that at 150° some of this is given off.

The theory presented in this paper appears to offer a satisfactory explanation of some recent experimental observations of Hoffman, Willstätter and Skita, as well as of the experimental basis for a recent patent.

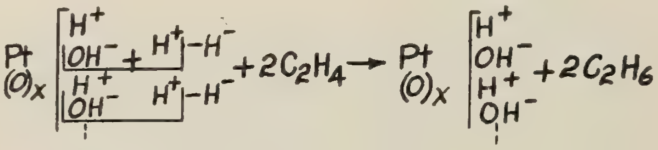
Willstätter (19) believes that a peroxide hydride is the active agent in platinum catalysis and that the loss of activity of the catalyst is due to loss of oxygen, thus preventing the intermediate formation of this compound. Hoffman (20) finds no indication of the existence of this compound, argues against this as the explanation, and suggests that a study of the surface film should be the next step in the investigation of the subject. We believe, from our experiments, that any satisfactory representation of the mechanism must embrace the fact that there are two kinds of oxygen involved, viz., oxygen in the interior and oxygen in the surface film, and also the fact that this surface film is remarkably stable and affords excellent protection for the underlying oxygen. Any theory which pictures the whole of the platinum in the same condition in a definite compound is for this reason unsatisfactory. Accordingly we think that Willstätter's hypothesis is not acceptable. This argument against the existence of definite intermediate compounds as the cause of the catalysis in the case of platinum, was also advanced by the senior author in the case of nickel catalysis (21). If, instead of definite oxides or hydrides or peroxide hydrides of platinum and nickel, we picture relatively stable adsorption complexes of the constitutions represented in this paper and in the nickel paper, then the behaviour of platinum and nickel catalysts seems to follow. Hoffman believes that the previous treatment of platinum with oxygen simply removes contaminations (Veruneinigen) and thus frees the platinum surface for action. We judge from our experiments that instead of freeing the platinum surface, it actually produces the film necessary to carry out the normal catalysis.

It seems that the vital point of this question of mechanism of platinum catalysis lies in the recognition, that when hydrogen acts on platinum black, a protective film is rapidly formed containing both hydrogen and oxygen, which is transformed rapidly into a film consisting of hydrogen alone, and that this latter film is quickly changed on treatment with oxygen into a film containing both oxygen and hydrogen, and further that these films can take part in reactions stoichiometrically with very little interference with the underlying interior content of oxygen. The senior author suggests that one of these films, containing both hydrogen and oxygen, consists of dissociated water in the form of charged hydrogens and hydroxyls, while the other, containing only hydrogen, consists of positively and negatively charged hydrogens. These are believed to be adsorption complexes consisting of varying amounts of dissociated water and dissociated hydrogen, depending on the temperature and conditions of preparation, and not definite compounds.

If the preceding considerations are valid, it is obvious that measurements which have appeared in the literature of the extent of hydrogen and oxygen adsorption by platinum and nickel have very little meaning, except in those cases where the whole history of the platinum and nickel has also been recorded.

Skita (22), in a recent paper, deals with the necessity for the presence of oxygen in platinum in order that the catalysis of hydrogenation may occur as pointed out by Willstätter, and considers the latter author's hypothesis of the existence of a platinum peroxide hydride. Skita prepared platinum catalysers, taking great care to exclude free oxygen, and found that these could catalyse hydrogenations, in the absence of free oxygen, just as effectively as when free oxygen is present, both in the preparation of the catalyser, and in the subsequent hydrogenations. Skita concludes that external oxygen is not necessary for the production of an active catalyst and that the hypothesis of Willstätter, that a peroxide hydride of platinum is formed, is not valid.

However, a patent (D.R.P. 301364) recently obtained by Berlin-Anhaltischen Maschinenbau-Akt-Gesselschaft seems at first sight to offer evidence in opposition to the conclusion of Skita. According to this patent, a small amount of free oxygen is used along with the free hydrogen in hydrogenations, catalysed by platinum, in order to maintain the platinum in an active condition. At first sight these views seem to be in opposition. However, the theory presented above offers a satisfactory explanation of the apparent conflict, as follows: When platinum black is prepared in water solution, alike whether in the presence of or absence of free oxygen, a complex is obtained containing a large content of oxygen. Whether the dry platinum black is a definite oxide of platinum, or a mixture of oxides of platinum, or an adsorption complex of platinum and oxygen, has no bearing on the constitution of the normal catalyser subsequently made from it. All that concerns us here is that this platinum black has a content of combined oxygen. On treating this platinum black with hydrogen the reaction, already discussed in detail in this paper, occurs, whereby an adsorption complex of the constitution already considered is formed. Thus whether free oxygen is present or absent in the preparation of the platinum catalyst and in the subsequent hydrogenation where this catalyst is used, is of no importance, for the platinum catalyst always possesses an inner content of oxygen, even in the case where the surface film consists solely of hydrogens. However, in hydrogenations this inner content of oxygen is slowly used up, thus:



$$\begin{matrix}Pt\\(O)_x\end{matrix}\left[\begin{matrix}H^+ & & \\ OH^- + & H^+ & -H^- \\ H^+ & & \\ OH^- & H^+ & -H^-\end{matrix}\right. + 2C_2H_4 \rightarrow \begin{matrix}Pt\\(O)_x\end{matrix}\left[\begin{matrix}H^+\\ OH^-\\ H^+\\ OH^-\end{matrix}\right. + 2C_2H_6$$

Although there is a strong tendency for the water to be readsorbed, and the original adsorption complex to re-form, yet momentarily the surface layer is entirely removed, and free hydrogen, present in excess, has access to the interior content of oxygen and partially removes it in order to re-establish the surface adsorption film. That this occurs is confirmed by the formation of considerable free water in the catalysis of hydrogenation of ethylene. This would eventually result in the complete removal of the interior oxygen, and finally the platinum would exist with only adsorbed hydrogens on the surface, having lost its entire oxygen, both interior and surface oxygen. Hydrogenation by platinum and nickel in this condition occurs by a mechanism quite different from the normal mechanism represented in the equation above, and requires a much higher temperature (see also Nickel paper, page 16). The use of a small amount of free oxygen along with the hydrogen, in hydrogenation actions, supplies the oxygen otherwise lost in this manner and thus the activity of the catalyser is maintained. Thus, the observations of Skita are not in opposition to the conclusions which would naturally be reached from a consideration of the patent.

Summary

A general review of the present condition of the problem of the mechanism of platinum catalysis is given.

Experimental data is given which shows:

(1) that when platinum black is reduced by hydrogen a relatively stable surface film is formed containing both hydrogen and oxygen which rapidly passes into a film containing only hydrogen;
(2) that both of these films have a marked capacity for protecting the underlying inner content of oxygen;
(3) that treatment of the platinum black separately with hydrogen and oxygen several times results in the production of a constant condition of the platinum black whereby approximately constant amounts of hydrogen and oxygen are used up and constant amount of water formed;
(4) that this constant condition is maintained after the inner content of oxygen has been greatly reduced;
(5) that this constant amount of oxygen taken up is not increased by standing or changed by heating to 150° C.;
(6) that both the surface films resume their composition and stability after treatment with hydrogen and oxygen mixtures, *i.e.*, the same constant amounts of hydrogen and oxygen are used up and same amount of water is formed on separately treating with hydrogen and oxygen, after these gas mixtures have been passed;
(7) that the simple solution theory of hydrogen and oxygen in the platinum is not tenable owing to these facts;
(8) that the theories of definite hydrides, oxides or peroxide hydride of platinum are not in harmony with these facts.

An hypothesis has been advanced to represent the mechanism of platinum catalysis. According to this hypothesis one condition of the platinum, viz., after treatment with oxygen, consists of platinum with an interior content of oxygen and an adsorption surface film of dissociated water consisting of charged hydrogens and hydroxyls, and the other condition, viz., after treatment with hydrogen, consists of

platinum with an interior content of oxygen and an adsorption surface film of dissociated hydrogen consisting of positively and negatively charged hydrogens. These charged atoms and groups in the adsorption films react stoichiometrically with hydrogen and oxygen and the catalyses of oxidation, reduction and hydrolysis reactions occur through reactions with these films. The mechanism of the first two classes of reactions is represented.

The catalysis of hydrolysis by platinum as well as by nickel will be discussed in a separate paper.

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(19) See No. 15.
(20) See No. 16.
(21) See No. 5.
(22) Skita, Ber., p. 139 (1922).

THE ASSAY OF ZINC ORES[1]

Use of Powdered Magnesium for Removing Copper and Lead

By E. G. R. ARDAGH, *Associate Professor of Chemical Engineering*,
and G. R. BONGARD

The standard zinc sulphate and potassium ferrocyanide solutions, as well as the conditions for titration, were those described in detail in the preceding article.

The reason for substituting metallic magnesium for the metallic aluminium recommended by earlier investigators lies in the fact that, although the presence of aluminium salts does not affect the blank determination, it does very seriously affect the figures obtained when zinc is present, as the following experiments very clearly show:

(1) *Effect of Aluminium Chloride on Titration of Zinc by Potassium Ferrocyanide.*

Effect on blank. Conditions same as in earlier determination of blank as described in the preceding paper

$AlCl_3$ Grams	$K_4Fe(CN)_6$ Required c.c.
3	0.20
5	0.20

Effect of adding aluminium chloride to 20 c.c. standard zinc sulphate

$AlCl_3$ Grams	$K_4Fe(CN)_6$ Required c.c.
3	43.05
5	45.30
5	45.95
None	39.45

(2) *Effect of Magnesium Chloride on Titration of Zinc by Potassium Ferrocyanide.*

Effect on blank. 12 grams $MgCl_2.6H_2O$ added

$K_4Fe(CN)_6$ Required c.c.
0.20
0.15
0.15

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[1] Received July 25, 1923.

Effect on titration of zinc. 20 c.c. standard zinc sulphate

$MgCl_2.6H_2O$ Grams	$K_4Fe(CN)_6$ Required c.c.
1	39.50
5	39.40
10	39.40
30	39.45
30	39.40

It is evident that even large quantities of magnesium chloride do not interfere in the potassium ferrocyanide titration.

If, then, metallic aluminium is used to remove copper, lead, etc., the aluminium itself must be removed before the zinc present can be titrated by potassium ferrocyanide as this titration is usually carried out. On the contrary, if metallic magnesium is used to precipitate copper, lead, etc., it need not be removed prior to the titration of the zinc.

(3) *Precipitation of Copper and Lead by Magnesium Followed by Titration with Potassium Ferrocyanide.* To 20 c.c. standard zinc sulphate were added 1 gram $CuSO_4.5H_2O$ (approximately 0.25 gram copper), 0.5 gram lead chloride (approximately 0.375 gram lead), and 3 c.c. concentrated hydrochloric acid. The whole was diluted to about 75 c.c. and heated to boiling. Magnesium powder was then added cautiously to the covered beaker, little by little, until the copper and lead had been completely precipitated. Small additions of the hydrochloric acid to the almost boiling solution were also made from time to time as required. This took about 5 minutes. The solution was then neutralized with ammonium hydroxide and, after adding 5 c.c. concentrated hydrochloric acid and 10 grams ammonium chloride, diluted to 250 c.c. It was then heated to 70° C. and titrated without filtering. The figures show that the precipitated copper and lead carry down an appreciable amount of zinc.

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
38.65	0.1960	98.0

On standing 5 minutes longer with vigorous stirring, more zinc entered the solution.

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
39.20	0.1988	99.4

The end point obtained in this way was somewhat blurred by the small particles of metal and possibly by a little copper having gone into solution.

(4) This experiment was a repetition of (3), but the precipitated metals were washed by decantation, then on the filter with warm hydrochloric acid (5 c.c. concentrated hydrochloric acid diluted to 100 c.c.).

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
36.95	0.1874	93.7

The metallic precipitate was boiled with 10 c.c. concentrated hydrochloric acid for 5 minutes. Any copper left undissolved was now in a very finely divided state. The solution was diluted to about 75 c.c., treated as before with magnesium powder, the copper and lead filtered off, and the zinc titrated as above.

	$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
	2.05	0.0104	5.2
TOTAL	39.00	0.1978	98.9

It is evidently a difficult task to prevent loss of zinc in the precipitated copper and lead.

(5) *Repetition of* (4). The precipitated copper and lead were washed three times by decantation with the dilute hydrochloric acid. They were then boiled with 8 c.c. concentrated hydrochloric acid for 5 minutes. Some copper was not dissolved. The copper and lead were reprecipitated, washed several times by decantation and then twice on the filter with the dilute hydrochloric acid. The combined filtrates were titrated as before.

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
39.30	0.1992	99.6

Evidently magnesium can be used satisfactorily to remove copper and lead, but it is necessary to filter off the precipitated metals, boil them with hydrochloric acid, and reprecipitate with magnesium in order to separate the zinc completely. The powdered magnesium must be as free as possible from iron. If traces of iron are present, no more magnesium should be used than absolutely necessary. One must also guard against traces of copper going into solution as a result of oxidation or through the use of too concentrated hydrochloric acid.

(6) *Effect of Lead on the Titration of Zinc by Potassium Ferrocyanide.* Seaman[7],* claims that lead interferes; Lenher and Meloche[8] declare it is without influence. It seemed to the writers that these contradictory

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*The reference numbers refer to the bibliography at the end of the preceding paper.

statements might have resulted from working with different concentrations of hydrochloric acid. To test this the following series was run on 20 c.c. standard zinc sulphate containing 2 grams lead chloride (approximately 1.5 grams lead) and varying amounts of hydrochloric acid in 250 c.c.

Concd. HCl c.c.	$K_4Fe(CN)_6$ Required c.c.
1	45.10
3	44.90
6	39.60
7	39.45
10	39.40

These experiments make it evident that when the amount of concentrated hydrochloric acid falls below 7 c.c. in 250 c.c. of solution, high results for zinc will be obtained when lead is present, but by increasing the concentrated hydrochloric acid present to from 7 to 10 c.c. in 250 c.c. of solution, correct results can be obtained for zinc even when large amounts of lead are present.

CONCLUSIONS

(1) Aluminium does not affect the blank in the usual potassium ferrocyanide titration, but results obtained for zinc in the presence of aluminium are high under the usual conditions of carrying out the titration.

(2) Magnesium chloride, even in large quantities, does not interfere with the titration of zinc by potassium ferrocyanide.

(3) Powdered magnesium may be used with advantage to precipitate copper and lead in hydrochloric acid solution. In order to separate the zinc completely, however, the precipitated metals must be filtered off, boiled with concentrated hydrochloric acid, and reprecipitated with magnesium. The magnesium should be as free from iron as possible, and one must also guard against any copper going back into solution.

(4) Correct results for zinc by titration with potassium ferrocyanide can be secured in the presence of lead, provided the hydrochloric acid present amounts to from 7 to 10 c.c. of concentrated hydrochloric acid in 250 c.c. of solution.

SIMPLIFICATION OF THE SEPARATION OF ZINC FROM IRON AND ALUMINIUM[1]

By E. G. R. Ardagh, *Associate Professor of Chemical Engineering,*
and G. R. Bongard

Zinc can be separated completely from iron, and also from aluminium, by ammonium hydroxide and ammonium chloride in one precipitation. Two-tenths of a gram of each metal, or even more, can be handled with celerity and ease.

Iron and aluminium hydroxide are precipitated in a compact state by evaporating their solutions to a very small volume, adding more than sufficient ammonium chloride to saturate the solution, and then adding concentrated ammonium hydroxide in considerable excess. By using gentle suction the precipitate is easy to wash.

The determination of zinc in ores always necessitates its separation from iron. This is usually accomplished (1) by repeated precipitation of the iron by ammonia, (2) by ammonium carbonate, (3) as basic acetate, or (4) by precipitation of zinc as zinc sulphide in weakly acidified solution. Not only are all these processes, as usually carried out, time-consuming, but in methods (3) and (4) the conditions must be maintained within quite narrow limits to insure success.

It is the purpose of this article to show that perfect separations can be made quickly and conveniently with ammonia and ammonium salts, even when relatively large quantities of the two metals are present at the same time.

The determination of zinc in ores in North America is invariably carried out volumetrically by the ferrocyanide method. The method was first proposed by Galletti,[2] * and the modification in use to-day is very well described in Low's[4] "Technical Methods of Ore Analysis." Schaffner's method[1] never attained any vogue on this continent.

Of all the authorities on the determination of zinc, A. H. Low is the only one who could be found who maintains that this metal can be completely separated from iron (when the latter is present in more than very small amount) by ammonia and ammonium chloride by a single precipitation. All other writers declare a double precipitation to be necessary. Low states that zinc can be completely removed from precipitated ferric hydroxide by washing with a solution containing 100 grams ammonium chloride and 50 cc. concentrated ammonium hydroxide

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[1] Received July 25, 1923.

*Numbers in text refer to bibliography at end of article.

per liter, but he does not give any confirmatory figures, nor state how much iron was present when he carried out the experiments that led to his conclusion.

At one time Kansas, Missouri, Illinois, and Indiana smelters could command a supply of ore averaging 0.5 to 1.5 per cent iron. While, it is true, conditions have changed, ample provision has been made for much higher iron content by employing in the great majority of experiments 0.2 gram iron and 0.2 gram zinc, corresponding to 40 per cent. of each metal when 0.5 gram is taken for assay. Ingalls[12] states that low-grade calamines assaying 40 per cent ferric oxide (28 per cent. iron) have been smelted, but that with this exception no ores containing over 20 per cent. iron are acceptable to zinc smelters. A European smelter would rarely care, says Ingalls, to have more than 10 per cent. iron in his charge. In New Jersey the ore smelted at the present time contains 2 to 3 per cent. iron, while the same authority says that in Kansas and Missouri probably few smelters carry more than 2.5 per cent. iron in their charges.

STANDARD SOLUTIONS AND DETERMINATION OF THE BLANK

The standard ferrocyanide solution contained about 22 grams potassium ferrocyanide and 8 to 10 grams sodium sulphite per liter.[5] It was checked every 2 weeks against pure $ZnSO_4.7H_2O$ solution containing 44.000 grams of the pure salt per liter. One cubic centimeter of this solution contained 0.0100 gram of zinc. The indicator used contained 15 grams $UO(NO_3)_2$ in 100 cc. of water.[10]

To test the influence of varying the concentration of the ammonium chloride on the blank, a series of trials was made in which the other factors were kept constant.

TRIAL	NH_4Cl Added Grams	$K_4Fe(CN)_6$ Required c.c.
1	5	0.15
2	10	0.15
3	20	0.15
4	30	0.15
5	40	0.15

It is evident that the ammonium chloride can vary over a wide range, so that one can safely use large quantities of ammonium hydroxide and ammonium chloride if necessary in the separation of iron, neutralizing the ammonium hydroxide in the filtrate with hydrochloric acid.

DETERMINATION OF $K_4Fe(CN)_6$ REQUIRED FOR STANDARD ZINC SOLUTION

To a 400-cc. beaker were added 20 cc. of the zinc sulphate solution containing 0.2000 gram zinc from a pipet, 5 cc. concentrated hydrochloric acid (specific gravity 1.19), 10 grams ammonium chloride, and water to 250 cc. The temperature was maintained between 60° and 70° C. during titration. The potassium ferrocyanide was added slowly, not faster than 10 cc. per minute, with constant stirring. Too rapid addition may give a false end point, so that too little potassium ferrocyanide is used. When the titration is carried out as described the precipitate at first produced is greenish or bluish white. After most of the potassium ferrocyanide required has been added, the precipitate becomes quite flocculent and settles fairly quickly. About 0.5 cc. before this end point the solution turns a creamy white, and the particles of the precipitate become much smaller and will settle only after 2 or 3 hours' standing. The change in size of the particles is more readily noticed if the solution is allowed to stand for half a minute without stirring. The potassium ferrocyanide should now be added drop by drop, testing after each. By using a glass tube for stirring three to four drops can be removed for each test. On arriving at the end point, one should wait 2 or 3 minutes to see if the immediately preceding one or two tests develop a colour. From the reading decided on as being correct, the blank is naturally to be deducted.

TRIAL	$K_4Fe(CN)_6$ Required c.c.
1	39.50
2	39.70
3	39.55
4	39.55
5	39.65
Average	39.60
Blank	0.15
Net $K_4Fe(CN)_6$ required	39.45

Therefore, 20 c.c. $ZnSO_4$ solution require 39.45 c.c. $K_4Fe(CN)_6$ or 1 cc. $K_4Fe(CN)_4$ indicates 0.005070 g. Zn.

SEPARATION OF IRON FROM ZINC BY NH_4OH AND NH_4Cl

A solution of ferric sulphate containing very close to 0.01 gram iron per cubic centimeter was made up by stirring powdered $Fe_2(SO_4)_3 . 9H_2O$ into warm water faintly acidified with sulphuric acid. The following series of experiments was performed with 20 c.c. of this solution and 20 cc. of the standard zinc sulphate—*i.e.*, 0.2 gram of each metal. Precipitation was carried out in a volume of about 150 to 200 cc., the solution being heated to boiling. The solution was filtered hot, using suction, and washed as described in each case. The filtrate was neutralized with concentrated hydrochloric acid and 5 cc. in excess added. Five grams ammonium chloride were then added, the solution made up to 250 cc. and titrated at 60° to 70° C. as before.

(1) Twenty-five cubic centimeters 5*N* ammonium hydroxide and 5 grams ammonium chloride were added:

TRIAL	$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.	Zinc Retained by $Fe(OH)_3$ Grams	Zinc Present Per cent.
1	35.40	0.1794	89.7	0.0206	10.3
2	36.10	0.1830	91.5	0.0170	8.5

The precipitate, which was washed three times with hot water, was very bulky, filling a 9-cm. paper about three-quarters full.

These two trials make it clear that even when very considerable quantities of ammonium chloride and free ammonium hydroxide are present, a single precipitation does not by any means remove all the zinc from the ferric hydroxide.

Experiments on the separation of copper and nickel from iron carried out by the senior author[13] proved astonishingly successful when the solutions were evaporated to 2 or 3 cc., thus making it possible to saturate them with ammonium chloride and make them very strongly ammoniacal without being compelled to employ extravagantly large quantities of these two reagents.

(2) Accordingly, 20 cc. standard zinc sulphate and 20 cc. ferric sulphate were evaporated, with addition of two or three drops of concentrated hydrochloric acid to about 2 cc. Five grams ammonium chloride and 10 c.c. concentrated ammonium hydroxide (specific gravity 0.90) were added, the mass was thoroughly worked up with a stirring rod, and 40 cc. of cold water were added. The solution was filtered cold and washed four times with cold water. The filtrate was titrated under the usual conditions.

$K_4Fe(CN)_6$ Required cc.	Zinc Separated Grams	Zinc Present Per cent.	Zinc Retained by $Fe(OH)_3$ Per cent.	Zinc Present Per cent.
37.35	0.1896	94.8	0.0104	5.2

Evidently, washing with water alone does not completely remove the zinc from the precipitate. The ferric hydroxide was quite dense, occupying only a fraction of the space required in Experiment 1. Low[11] has also observed this phenomenon, and refers to it in his chapter on the determination of zinc.

(3) Forty cubic centimeters of mixed sulphates and two drops of concentrated hydrochloric acid were evaporated to about 1 c.c., worked up with 5 grams ammonium chloride and 5 cc. concentrated ammonium hydroxide, diluted with 25 c.c. cold water, filtered, and washed with 50 cc. of cold solution containing 5 grams ammonium chloride and 5 cc. concentrated ammonium hydroxide per 100 cc. of solution.

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.	Zinc Retained by $Fe(OH)_3$ Grams	Zinc Present Per cent.
39.25	0.1994	99.7	0.0006	0.3

In order to obtain a confirmatory check on the zinc retained by the precipitate, the ferric hydroxide was dissolved in concentrated hydrochloric acid, precipitated with concentrated ammonium hydroxide, filtered, and washed with ammonium chloride-hydroxide solution.

$K_4Fe(CN)_6$ required, 0.10 c.c.
Zinc present, 0.25 per cent. (0.0005 gram)

The precipitate produced on addition of the potassium ferrocyanide was just discernible.

(4) This was carried out exactly as in (3), except that the precipitate was washed with 100 cc. of cold solution containing 5 grams ammonium hydroxide.

TRIAL	$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.	$K_4Fe(CN)$ Required for Zinc Retained by $Fe(OH)_3$ c.c.	Zinc Retained by $Fe(OH)_3$ Grams	Zinc Present Per cent.
1	39.35	0.1996	99.8	0.10	0.0005	0.25
2	39.40	0.1998	99.9	0.05	0.0002	0.1
3	39.35	0.1996	99.8	0.05	0.0002	0.1
4	39.45	0.2000	100.0	0.00	0.0000	0.0
5	39.40	0.1998	99.9	0.05	0.0002	0.1
6	39.50	0.2002	100.1	0.00	0.0000	0.0
7	39.35	0.1996	99.8	0.00	<0.0005	<0.25

Before titrating the filtrate containing this trace of zinc in Trial 2, hydrogen sulphide was bubbled through the ammoniacal solution. No precipitate could be detected.

In Trials 4, 5, 6, and 7, the precipitate was washed eight to ten times with the ammoniacal wash solution.

On testing the precipitate from Trials 4 and 6 for zinc in the usual way with potassium ferrocyanide, no precipitate could be detected, even on standing 1 hour. In Trial 5 a faint precipitate was observable.

Titration to determine the zinc retained by the washed ferric hydroxide in Trial 7 gave the same figure as for the blank—viz., 0.15 cc. potassium ferrocyanide. There was produced a faint precipitate of zinc ferrocyanide; comparing this nephelometrically with the precipitate produced by 0.5 mg. zinc, it was found to be distinctly smaller.

These seven trials prove conclusively that as much as 0.2 gram zinc can be separated from 0.2 gram iron by a single precipitation. The error, if any, lies within the error inseparable from the titration itself.

The precipitate of ferric hydroxide must, however, be thoroughly washed with ammonium chloride-hydroxide solution. The dense character of the ferric hydroxide renders this operation easy and rapid if a gentle suction is applied.

(5) These separations were carried out exactly as those recorded under (4), except that the precipitate was filtered hot and washed with hot ammonium chloride-hydroxide solution of the concentration already given.

TRIAL	$K_4Fe(CN_6)$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
1	39.40	0.1998	99.9
2	39.30	0.1996	99.8

When tested for zinc with potassium ferrocyanide, the washed ferric hydroxide gave a faint white precipitate in each case after standing 15 minutes. This indicates only a small fraction of a milligram of zinc.

Evidently the method works satisfactorily both hot and cold.

(6) The following separations were carried out exactly as those recorded under (4), using in each trial 100 cc. of wash solution containing 5 grams ammonium chloride and 5 c.c. concentrated ammonium hydroxide.

TRIAL	Zinc Taken Grams	Iron Taken Grams	$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Taken Per cent.
1	0.2000	0.3	39.55	0.2002	100.1
2	0.4000	0.2	78.95	0.4000	100.0
3	0.2000	0.1	39.50	0.2000	100.0
4	0.2000	0.4	39.35	0.1996	99.8

In Trial 1 the wash solution was hot; in Trials 2, 3, and 4 it was cold. The washed ferric hydroxide in Trial 2 contained enough zinc to give a faint precipitate of zinc ferrocyanide on 2 hours' standing. The precipitate was much smaller than one produced by 0.5 mg. of zinc. Zinc could not be detected in the washed ferric hydroxide from Trial 3, but in Trial 4 a faint test was obtained.

It is evident that 0.2 gram of zinc can be completely separated from as much as 0.4 gram of iron in one precipitation by the foregoing simple procedure.

When such a large quantity as 0.4 gram iron was present, the most satisfactory technique was found to be the following:

To the zinc-iron solution in a beaker or casserole add a few drops of concentrated hydrochloric acid and evaporate to about 5 cc., taking care to avoid local baking. Then take the vessel in the hand and swirl the contents while continuing the evaporation over a small flame, until the volume is reduced to about 2 or 3 cc. While still warm add 5 grams ammonium chloride and work up the mass with a stirring rod. Add 10 cc. concentrated ammonium hydroxide, break up lumps, add about 25 cc. cold water, and filter on a 7-cm. paper in an 8-cm. Büchner funnel. Wash back the bulk of the precipitate into the beaker or casserole, using the ammoniacal wash (5 grams NH_4Cl+5 cc. concentrated NH_4OH made up to 100 cc.) and transfer again to the filter. Wash six to eight times on the filter, consuming about 100 cc. of the ammoniacal wash liquid in all. Neutralize the ammoniacal solution with concentrated hydrochloric acid, adding about 5 cc. in excess, dilute to about 250 cc., and titrate at from 60° to 70° C. in the manner already described.

A powerful suction packs the precipitate on the paper too tightly, and hence should be avoided. A suction equivalent to about 75 mm. of mercury works very satisfactorily, and with it the whole operation of filtering and washing should not require more than about 5 minutes. When handling smaller quantities of iron, say 0.1 to 0.2 gram, an ordinary funnel is quite satisfactory, but with larger quantities the precipitate packs in the apex of the paper.

SEPARATION OF ZINC FROM ALUMINIUM

Upon separating zinc from aluminium, a small amount of aluminium passes into the filtrate owing to the slight solubility of aluminium hydroxide in solutions strongly alkaline with ammonia, but not enough dissolves to interfere with the subsequent titration of zinc. A test determination gave 4 mg. aluminium in the filtrate.

A solution containing very nearly 0.01 gram aluminium per cubic centimeter was made from $Al_2(SO_4)_3 . 18H_2O$, a few drops of sulphuric acid being first added to the water.

(7) Twenty cubic centimeters of standard zinc sulphate and 20 cc. of the aluminium sulphate were treated exactly as in Experiment 4.

TRIAL	$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.	$K_4Fe(CN)_6$ Required for Zinc Retained by $Al(OH)_3$ c.c.	Zinc Retained Grams	Zinc Present Per cent.
1	39.25	0.1990	99.5	0.15	0.0008	0.4
2	39.40	0.1998	99.9	0.00	0.0000	0.0
3	39.40	0.1998	99.9			

The washed aluminium hydroxide was tested for zinc exactly as described in Experiment 3.

The aluminium hydroxide formed in the concentrated ammonium hydroxide saturated with ammonium chloride, like the ferric hydroxide formed under the same conditions, is much denser than that produced in the usual way. In Trial 2 zinc could not be detected in the washed aluminium hydroxide. In Trial 3 a faint precipitate of zinc ferrocyanide appeared.

Zinc can evidently be separated from aluminium without difficulty in one precipitation, when as much as 0.2 gram of each metal is present, by ammonium hydroxide and ammonium chloride. The precipitate of aluminium hydroxide appears to require somewhat more washing than the ferric hydroxide.

(8) To compare Demorest's method[9] with that of the writers, they proceeded exactly as in Experiment 4, except that they used 5 grams powdered ammonium carbonate in place of 5 cc. concentrated ammonium hydroxide.

The wash solution also contained 5 grams ammonium carbonate per 100 cc. in place of 5 cc. concentrated ammonium hydroxide. Using 20 cc. of standard zinc sulphate and 20 cc. aluminium sulphate.

$K_4Fe(CN)_6$ Required c.c.	Zinc Separated Grams	Zinc Present Per cent.
38.45	0.1950	97.5

Care was taken to prevent loss through effervescence when adding hydrochloric acid to the filtrate. It is evident that ammonium carbonate, though effective, does not give quite so satisfactory a separation as concentrated ammonium hydroxide.

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AN INVESTIGATION OF DISCOLOURATION IN CORN OIL SOAP AND OF THE POSSIBILITY OF PREVENTING ITS DEVELOPMENT

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Introduction

In recent years the soap industry has been compelled to look about for new sources of fats. These it has found in fish and whale oils, and still more abundantly in certain vegetable fats and oils from the Tropics, more especially cocoanut and palm oils. Olive oil foots, of course, have been used for many years. This means that the north is going to become more and more dependent upon imported oils, principally those produced in the Tropics, for soap making, and importation spells dependence. Dependence is not desirable, and in this instance we can lessen it, to some extent at least, by utilizing in this industry certain of the semi-drying oils that are products of the temperate zone. The practical difficulty of making soaps from these semi-drying oils, cottonseed oil, corn oil, etc., even when they constitute only a part of the soap stock employed, is that the soap containing such deteriorates on storage, becoming covered in time with brown spots which occasionally even become soft and rancid. It was with the object of discovering the causes of these brown discolourations and of finding out some means of preventing their formation and development that this research was undertaken.

Nature of the Objectionable Discolouration

These characteristic dirty brown discolourations occur in irregularly scattered areas on the surface of the cake of soap. They become visible at first as nuclei, darkest in the centre and shading off into the remainder of the surface of the cake. Frequently they make their first appearance on the corners or edges where the surface exposed relative to the mass of the neighbouring soap is greatest. As the areas become larger they gradually merge into one another until the whole surface of the cake takes on a dirty brown colour, in appearance totally unsuitable for marketing purposes. In the samples of commercial soap examined the discolouration never penetrated more than 1/16″ to 1/8″ into the cake.

SOURCE, MANUFACTURE AND CHARACTERISTICS OF CORN OIL

The only source of maize or corn oil is the starch and glucose industry. The shelled corn, after softening by steeping, is put through attrition mills in which the kernels are broken up, but the germ left uninjured. The germ, containing about 35% of oil, is then separated from the remainder of the kernel in germ separators in which it floats and is skimmed off, and is then dried in revolving driers and crushed in steam heated presses to remove the oil. The crude oil is dark coloured, being heavily charged with albuminous matter.[9] On refining by dropping the foots it yields a bright yellow oil possessing a characteristic odour and taste.[10]

The fresh oil contains only a small amount of free fatty acids. The content of unsaponifiable matter is high, ranging according to various authorities from 1.35% to 2.86%.[10, 11, 12, 13] This unsaponifiable matter consists of lecithin and sitosterol.[14] The specific gravity ranges from 0.924 to 0.926 according to different observers, and the iodine number from 121 to 125. The saponification number is 189.7. The viscosity, rape oil being taken as 100, is 70.4 to 70.65.[15]

The composition of corn oil was recently determined by the Bureau of Chemistry, U.S. Department of Agriculture, and found to contain the glycerides of:

	%
Oleic acid	45.4
Linolic acid	40.9
Palmitic	7.7
Stearic	3.5
Arachidic	0.4
Lignoceric	0.2
Unsaponifiable matter	1.7
	99.8

MAIZE OIL AS A SOAP STOCK

Lewkowitsch[1] states "such maize oil as cannot be used for edible purposes is used for making soft soap for which it is eminently suitable. For hard soaps of acceptable quality maize oil cannot be used." Of these statements the latter probably refers to the characteristic discolouration of soap made from this oil.

Refined corn oil saponifies with difficulty and only after continued boiling. Saponification is best begun with caustic soda of about 15 Bé., and, once started, goes rapidly to completion. Saponification proceeds more rapidly with crude oil, due to the presence of free fatty acids, the soap from which appears to catalyze the reaction. To quote Lewkowitsch again[1]: "Unless carefully refined and thoroughly saponified, soap made from corn oil is subject to rapid deterioration through discolouration and rancidity". With regard to this assertion, we were unable to prove that thoroughness in the refining or bleaching influenced the rate of reaction to any marked degree, but owing to the methods which were used in preparing the soaps certain other effects, which will be enumerated later, may have more than counterbalanced the beneficent results of careful bleaching and refining.

The great affinity of sodium oleate for water (soluble in 10 parts of water) causes soaps of this class containing a large proportion of this substance to "sweat", and so great is this tendency that any considerable increase in humidity is shown by the accumulation of a film of moisture on the surface of the soap. As the reaction under investigation is a surface one, this phenomenon of "sweating" in these soaps may have some connection with it. It is known that NaOH under certain conditions does act as an oxidizing catalyst, and the discolouration does seem to occur with increased speed once started.[9]

GENERAL PLAN OF ATTACK

Naturally the first half of our problem consisted in discovering the cause or causes of the discolouration. The possibilities that suggested themselves to us as worthy of investigation are the following:

(1) Influence of colour of the oil.
(2) Influence of the unsaponifiable matter.
(3) Action of light.
(4) Action of oxygen.
(5) Influence of moisture.
(6) Influence of unsaturated glycerides.

(1) INFLUENCE OF COLOUR OF THE OIL ON THE SOAP. REMOVAL OF COLOUR AND MUCILAGINOUS MATTER

Our first hypothesis on commencing the investigation was that the discolouration of the soap was due to the colour of and mucilaginous matter in the oil, and it was thought that if, by bleaching or decolourizing, the oil could be clarified to such an extent that the colour is almost entirely removed, we could produce a soap that would remain white and sweet indefinitely.

"Dropping the Foots"

Dropping the foots, as carried out commercially, removes a large proportion of the colour. We decided to adopt this method as the first step in clarifying and bleaching, to be supplemented later on by bleaching with fuller's earth and various brands of activated carbon.

The method of Dropping the Foots used in our investigation and found to be most satisfactory with our oil is the following: The oil is heated up to 140° F. and placed under stirring paddle rotated sufficiently rapidly to insure thorough mixing, but not so rapidly as to interfere with the formation of a coarse granular precipitate. Caustic soda solution (18° Bé), to which 10% NaCl by weight has been added, is run into the oil, slowly and in a very fine stream, while warming the oil up to, but not above, 150° F. Higher temperatures prevent the formation of a good-settling precipitate. Sufficient caustic solution is used to give about 5% excess NaOH and the stirring continued until the precipitate coagulates. This operation generally requires about 10 minutes, after which the oil is allowed to stand a sufficient time for the precipitate to settle. In some cases the precipitate would be down in 20 minutes, while in others up to 4 hours were required.

After settling, the clear oil is decanted from the "foots" and washed several times by running in water at about hand temperature, the oil being stirred during the process at a speed not sufficient to produce an emulsion. From such emulsions the water settles only with difficulty. The oil is dried with slow stirring at 212°-220° F. so that the water boils off, leaving the oil clear and bright when dry. In every case after this treatment the oil shows traces of acidity notwithstanding the excess of NaOH used.

The Lovibond tintometer test on one of the samples so treated was: red 7.2 on a 35 yellow base, using the standard 5¼" bottle. The untreated oil was so dark that it could not be read on the tintometer, and although subsequent treatments often resulted in a very considerable decolourization, yet most of the colour was always removed by dropping the foots.

Method of Conducting Experiments.

The oil, from which the foots had been dropped in each case, was placed in a beaker and by a small flame brought to the desired temperature (with the stirrer in operation to avoid over-heating). Weighed quantities of the bleaching agents were then added and the stirring continued for the required length of time, the temperature being kept constant. The oil was then filtered.

Series 1	Oil	Fullers' Earth	Temp.	Time	Lovibond reading	
					R.	Y.
(*a*)	250 gms.	3%	100 C.	20 min.	5.9	20
(*b*)	250 "	4%	100 C.	20 "	5.6	20
(*c*)	250 "	5%	100 C.	20 "	5.1	20
(*d*)	250 "	6%	100 C.	20 "	4.9	20
(*e*)	250 "	6%	20 C.	30 "	...	..
(*f*)	250 "	7%	100 C.	20 "	5.1	20
(*g*)	250 "	8%	100 C.	20 "	4.4	20

NOTE.—(*e*) No Lovibond reading made. The result good, but not so good as 6% fuller's earth with hot solution.

As trials (*c*), (*d*) and (*f*) gave very nearly the same colour, a sample of soap was made from (*d*) to determine what effect the bleaching had on the colour of the soap made from the oil in comparison with that made from oil that had the foots dropped, but had not been bleached further. The glycerine was not removed.

Result: The soap from oil 1 (*d*) was of a fine light creamy colour, much lighter than that from the refined but unbleached oil.

Series 2

To determine the effect of adding "Superfiltchar" to the fuller's earth a second series of experiments was run.

Trial	Oil	Fullers' Earth	Superfiltchar	Temp.	Time	Lovibond reading	
Series 2						R.	Y.
(*a*)	250 gms.	3%	¾%	100 C.	20 min.	5.9	20
(*b*)	250 "	4%	¾%	100 C.	20 "	5.4	20
(*c*)	250 "	5%	¾%	100 C.	20 "	5.0	20
(*d*)	250 "	6%	¾%	100 C.	20 "	4.6	20
(*e*)	250 "	6%	1 %	100 C.	20 "	4.0	20

Soap made from oil from trial 2 (*d*) showed only a very slight improvement in colour over that from 1 (*d*).

Series 3 L. A. SALOMON & BRO. PRODUCTS

Trial	Oil	Temp.	Time	Spec. Norite	Oil Norite	Lovibond Reading	
						R.	Y.
(*a*)	200 gms.	100 C.	20 min.	3% Fullers' Earth	...	5.5	20
(*b*)	200 "	100 C.	20 "	5%	2%	4.7	20
(*c*)	200 "	100 C.	20 "	...	2%	5.2	20
(*d*)	200 "	100 C.	20 "	...	3%	4.0	20

Results: "Oil grade Norite" proved more effective than "Special Norite."

Experiment 3 (*d*) using 3% "Oil Norite" is the best result obtained using a vegetable carbon alone. The use of even 3% of carbon of this nature would be very expensive practice, and considering the slight difference that this quantity makes in the colour of the soap, its use would probably not be justified at all unless it is the cheapest method of delaying the appearance of discolouration. More will be said on this point later in the thesis.

Series 4 USING "CARBROX"

Trial	Oil	Temp.	Time	Carbrox	Fullers' Earth	Lovibond Reading	
						R.	Y.
(*a*)	200 gms.	100 C.	20 min.	2%	...	5.3	20
(*b*)	200 "	100 C.	20 "	1%	5%	4.5	20

Comparison of 4 (*a*) with 3 (*c*) shows "Oil Norite" and "Carbrox" to have about equal bleaching effect when used alone.

Series 5. Comparison of "Carbrox", "Darko" and "Oil Norite" at 100° C. for 20 min. In each case 5% fuller's earth was used.

	Lovibond reading	
	R.	Y.
Trial (*a*) 5% Fullers' Earth, 2% Carbrox,	4.0	20
(*b*) 5% " " 2% Darko,	4.4	20
(*c*) 5% " " 2% Oil Norite,	4.0	20

The results of these tests show that "Carbrox" is approximately as efficient as "Oil Norite" when used in conjunction with fullers' earth, and that "Darko" is not quite so efficient as either of the others.

To determine the limit of the bleaching effect of "Oil Norite" a sample of the oil was treated with five successive portions of 5% "Oil Norite", filtering between each treatment. The final product still gave a reading of 3 red on a 20 yellow base.

SUMMARY OF THE WORK ON BLEACHING, USING FULLER'S EARTH AND VEGETABLE CHARS

As this phase of the investigation was merely carried out to enable us to discover if a bleached oil would produce a better keeping soap than an unbleached oil, we did not delve into it thoroughly enough to make any sweeping or positive statement about the absolute merits of the various vegetable chars with regard to their efficiency for bleaching corn oil. The comparisons made are in all cases effective only for the conditions used during the experiments, and may be quite erroneous for different conditions of temperature, acidity, alkalinity, etc. The letters on fyle from the makers of these carbons show that they recognize very clearly the influence of experimental conditions on the efficiency with which their products decolourize various substances. It may be said, however:

(1) That excessive quantities of either fuller's earth or "Oil Norite" would be necessary to decolourize the oil completely, and that we were unable to accomplish that result.

(2) That "Superfiltchar" works better in conjunction with fuller's earth than alone.

(3) That "Oil Norite" and "Carbrox" are the most efficient under the conditions of our experiments of all the carbons we used.

We tried several other decolourizing carbons, as well as Kieselguhr and Catalpo, and we varied the conditions under which we used them. In no case, however, did we obtain as good results as the better ones among those already recorded.

BLEACHING BY BLOWING WITH AIR IN PRESENCE OF A CATALYST[18]

We were able to bleach the oil to a 3.1 red on a 20 yellow base by adding 0.1% manganese oleate and blowing with air for 14 hours at 85-90° C. The iodine number fell from 121 to 103.1, the viscosity rose and the smell resembled that of castor oil. The soap made from this oil was brownish, and we were unable to improve its colour though we tried in several ways to do so. Cobalt oleate gave similar results.

INFLUENCE OF LIGHT, AIR AND MOISTURE

In order to study the effect of these three agents, we made up several batches of soap in the manner described below.

METHOD OF MAKING SOAP FROM CORN OIL IN THE LABORATORY

In the first part of our experimental work on the manufacture of soap from the oil, the method followed was to saponify the oil by boiling for several hours with caustic soda of 15° Bé. in a large evaporating dish. The saponification, once it commenced, was very vigorous for a time. After the velocity of the reaction had decreased the boiling was continued for 5 to 6 hours with an excess of NaOH not sufficient to grain the soap. In our first experiments the glycerine was not separated, and the soaps were dried by direct heat, care being taken not to burn. As the influence of heating in producing an acid condition in the soap was not realized at the outset, and also since the soap was not salted out, the behaviour was probably somewhat different from that of properly finished soap, but nevertheless the samples were well enough made to give the desired information on the influence of light and air.

Samples of soap were made:

1. From oil from which the foots had been dropped.
2. From unrefined oil.

These were put away in desiccators under various conditions.

(*a*) In the light. 1. With $CaCl_2$ in the bottom+air.
2. With water in the bottom+air.
3. Atmosphere of air.

(*b*) In the dark. 1. With $CaCl_2$ in the bottom+air.
2. With water in the bottom+air.
3. Atmosphere of air.

3. From oil bleached with 6% fuller's earth after foots had been dropped.

4. From oil bleached with 5% fuller's earth and ¾% "Superfiltchar".

5. From "foots" filtered, but from which the oil had not been extracted.

After a period of about two months the discolouration of these samples had proceeded sufficiently to enable us to make satisfactory comparisons.

The results may be summed up as follows: All samples placed in the light showed equal discolouration. The samples placed in the dark were coloured slightly less than those in the light, but there was really not much difference between the two series. The samples made from the foots while brown in colour did not show the red blotches.

A sample of commercial soap containing 20% corn oil soap was put away under the same conditions and one sample placed in an atmosphere of nitrogen.

The samples in the air showed gradual discolouration, while the sample in the nitrogen did not show any discolouration over a long period. This was our first demonstration that the oxygen of the air was a factor in the reaction.

To compare the rapidity of discolouration of soaps made from oil bleached with varying quantities of fuller's earth, samples were prepared from oil bleached with 6+6% fuller's earth and 8% fuller's earth and put away under the conditions described above.

The increased bleaching did not apparently delay the development of the discoloured areas.

The samples placed in the moist atmosphere all became gummy and wet on the surface owing to the high solubility of glycerine in water. With this fact in mind it was decided to remove the glycerine from all future samples by graining the soap several times by the use of salt, as carried out in the soap factory. The samples were subsequently dried to about 20% moisture. This salt-washing removed the glycerine, but introduced another variable due to the retention of varying quantities of salt by the soap.

A sample of soap prepared from oil bleached with 3% "Oil Norite" was put away for observation. This sample was of a fine light creamy colour, but it did not resist discolouration in an atmosphere of oxygen.

As results obtained up to this time had indicated that oxygen was a necessary factor in producing the discolouration, a series of samples were prepared using oil which had been bleached with (1) 6+6% fuller's earth, (2) 8% fuller's earth and (3) 3% "Oil Norite", and the samples were put away in desiccators (1) containing oxygen and (2) containing nitrogen.

After 15 days' standing the samples in oxygen were all discoloured. They gave off a disagreeable odour similar to that of the oil blown with catalysts present, and the desiccator top was very hard to remove, which indicated the absorption of oxygen. The samples which had been placed in nitrogen were not discoloured. This series proved definitely that oxygen is necessary for the reaction to proceed at all, and that increasing the concentration of the oxygen increases the rapidity of the deterioration.

On testing all the samples stored in oxygen acidity was found to have developed in every case. This acidity was found wherever discolouration had occurred, and it increased as the discolouration increased.

EXPERIMENTS TO DETERMINE THE EFFECT OF VARYING CONDITIONS ON THE RAPIDITY OF DISCOLOURATION

Samples of neutral soap free from fat were prepared from oil bleached with 5% fuller's earth. Four samples were taken and to them was added 0.0%, 0.05%, 0.2% and 0.5% of NaOH in solution.

No comparative difference in the rapidity of deterioration could be observed after a period of one month. Possibly we should have added higher percentages of NaOH.

To determine the effect of varying the number of salt washes (grainings) samples were prepared after one, three and five salt washes and set aside for observation.

The colour of the freshly prepared samples improved and the discolouration appeared to be retarded by increasing the number of salt washes. As the small quantities of soap used prevented uniform finishing, and as our results indicated that the effect of adding salt in sufficient quantity to bring about graining retarded discolouration slightly, we thought that the results obtained here might be due to salt left in the soap rather than to increasing the number of washings.

As our previous experiments had indicated that oil bleached with 3% "Oil Norite" yielded a soap more resistant to the action of oxygen samples were prepared from oil bleached with 3% "Oil Norite" and oil bleached with fuller's earth that gave the same Lovibond reading.

No difference could be observed in the discolouration of these samples.

Samples of soap prepared from oil bleached with 3, 6 and 9% "Oil Norite" showed approximately the same discolouration and nearly equal development of acidity.

To compare the effect of moisture and dryness two samples were prepared: (1) Finished moist, (2) evaporated further with careful heating.

After a period of two weeks the second sample showed higher acidity than the moist sample. This result was probably due to oxidation during drying. Later experiments supported the hypothesis that drying by heat results in the development of acidity.

A series of experiments was run to determine the effect of adding salt in various quantities to soap placed in oxygen. A large sample was divided into five portions and 0.0%, 0.2%, 2%, 5%, 7.5% salt in solution added, and the samples each dried very carefully to get approximately the same conditions.

The increased additions of salt whitened the samples proportionately, and those containing 5% and 7.5% became granular in texture. Each of these samples was cut into two pieces which were placed in separate desiccators, one in oxygen over H_2SO_4, the other in oxygen saturated with water vapour.

A comparison of the colour of the samples in the wet and dry desiccators, and also of the free fatty acids developed, showed that the deterioration of the soap in the saturated oxygen was much more rapid than that in which the surface of the sample was dried out. The samples with 5% and 7.5% salt developed less colour than those containing smaller quantities of salt, but the result is not conclusive enough for generalization. The dryness of the grained samples containing 5% and 7.5% of salt was probably a favourable condition.

A set of samples was prepared by drying properly finished commercial soap which had 0.3% fat, inoculating the various portions with a trace of:

(1) Salt solution, (2) Corn oil, (3) Oleic acid, and placing the samples in oxygen desiccators. From observation over a period of one month we concluded that inoculation did not in any case have the effect of starting the deterioration, the discolouration and acidity were in all the samples approximately the same after one month.

The following series was prepared by inoculating commercial soap as removed from the kettle after finishing:

(1) No inoculation, (2) Soap heated to 100° C. for 2 hours, (3) Corn oil, (4) Oleic acid, (5) Water.

After one month all the samples showed traces of free fatty acids in the surface layer with slight odour. Free carbonate is still present underneath the surface.

A series of experiments using commercial soap dried carefully over a small flame to about 15% moisture was prepared and inoculated in the same way as the preceding series.

No discolouration took place at the points of inoculation. The acidity was 2% approximately, and the odour and colour development were only slight in a two weeks' period.

A similar series of samples of commercial soap which had been dried in nitrogen over H_2SO_4 were inoculated in the same way. No colour development took place in a period of one month, but a slight odour of rancid soap could be detected. A comparison of these last three series indicates that the effect of heat in drying has a marked influence on the time necessary for deterioration to set in.

A series was prepared for the purpose of comparing the rate of discolouration of (1) neutral soap, (2) neutral soap with 0.5% stearic acid thoroughly incorporated at a temperature above the melting point of the latter.

The addition of 0.5% stearic acid does not appear to retard colour formation.

EXPERIMENTS ON OXYGEN ABSORPTION

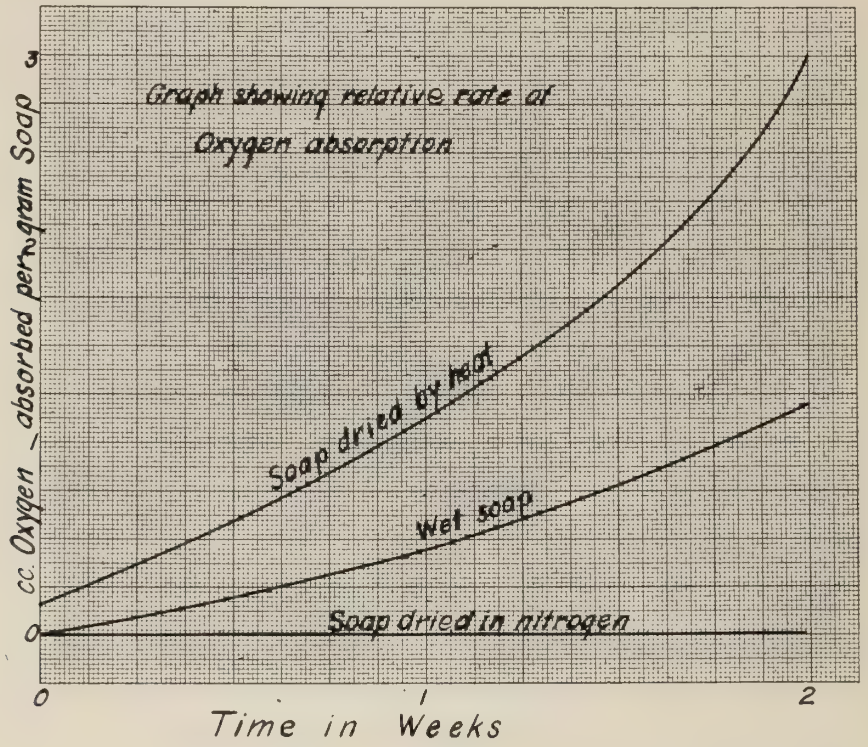
The difficulty of removing the tops from desiccators containing discoloured samples in an oxygen atmosphere, and the fact that oxygen had been shown to be necessary for discolouration to take place, suggested quantitative experiments to determine the absorption of oxygen. Repeated observations by connecting a manometer to a vacuum desiccator containing soap in an oxygen atmosphere showed that oxygen was being absorbed by the samples.

As we had already learned of the development of acidity in samples of soap prepared by drying wet samples (as finished) to commercial moisture content, about 15%, the greatest care was now taken with the evaporation of the water from these wet soaps. A very small flame was used and the soap was stirred continuously to prevent overheating.

To determine the oxygen absorption of corn oil soap prepared on a semi-industrial scale we obtained, through the kindness of the Palmolive Co. of Canada, Limited, a batch of properly made and finished corn oil soap. From this we prepared the following samples: (1) Dried over H_2SO_4 in a nitrogen atmosphere, (2) wet finished soap, (3) dried carefully by direct heat (as described above) to about 15% moisture content.

The samples were placed in three tubes filled with oxygen, connected to oxygen-filled water-jacketed gas-burettes.

The observations on these samples showed (1) no oxygen absorption during a period of one month by the sample dried in nitrogen, (2) medium rate of absorption by the wet sample, (3) rapid rate of absorption in the case of the heated sample. The results of these experiments are illustrated by the accompanying graph.



EXPERIMENTS ON THE DEVELOPMENT OF ACIDITY

During the investigation it was proven that an acid condition was being developed concurrently with the development of colour in the samples.

Determinations of the acidity were accordingly made on samples prepared in various ways. The determinations were made by dissolving the soap in 95% alcohol (neutral), titrating with NaOH solution and calculating the acidity as oleic acid. These results may be summed up as follows:

1. Neutral soaps which were dried with careful heating were shown to have an acid action, always less than 1%.

2. Soap dried at 105° C. for one hour was discoloured and showed 9% acid. This experiment indicates the influence of heating in promoting acidity, and also, that accurate moisture determination for soap containing unsaturated esters ought to be done by evaporation at low temperature, preferably in an inert atmosphere.

In attempting to boil soap in a large beaker contained in a calcium chloride bath at 110° C. a similar discolouration was observed on the sides of the beaker where the soap had an opportunity to dry out and in consequence to become hotter than the wet mass not in contact with the beaker wall.

3. Acidity determinations on soaps prepared from oils bleached with 8% fuller's earth, 6 and 6% fuller's earth and 3% "Oil Norite" after a period of 20 days in oxygen and in nitrogen desiccators showed 6 to 7% acid on samples in oxygen, and less than 1% acid on samples in nitrogen. The slight acidity of the samples in nitrogen is readily attributable to the effect of heating the soap in drying to the proper consistency before placing in the desiccators.

4. Experiments with commercial soap which was discoloured on the outside surface, using samples taken from the outside layer and from the inside of the cake, showed that there was acidity in the outside layer while free carbonate was still present in the inside.

5. Samples cut from discoloured parts of soap prepared in the laboratory and from non-discoloured parts, showed in all cases considerable acidity in the discoloured areas and very little acidity in the non-discoloured portions.

The results of these experiments show that the development of acidity in the soap is simultaneous with the appearance of discolouration and odour, that it is due to the oxygen of the air and not to light, and illustrate the effect of heating in producing initial acidity.

EXTRACTION OF ACIDS FROM CORN OIL SOAP AND SEPARATION OF ACIDS INSOLUBLE IN PETROLEUM ETHER

From a consideration of the acid reaction of discoloured soap a method was worked out to separate this material. The discoloured soap was boiled up with dilute HCl until the fatty acids split off were clear and transparent, thus indicating the complete liberation of the fatty acids. The fatty acid layer which contained the colouring matter was separated from the HCl and salt solution, and washed by repeated boiling with distilled water until free from chlorides. The acids soluble in petroleum ether were then extracted until the ether extract left no residue on evaporation. It may be mentioned that sufficient petroleum ether must be used in the first extraction, or the insoluble material which is very soluble in the colourless fatty acids remains in solution with them and no ethereal layer is formed. After the first extraction a further petroleum ether extract is colourless. The residue that is left after extraction is completed is a red translucent resinous substance of bright appearance. Certain characteristics of this material were determined and are noted as follows:

1. Soluble in ethyl ether, acetone and benzene.
2. Soluble in the colourless fatty acids.
3. Insoluble in hydrochloric acid, carbon disulphide, carbon tetrachloride, petroleum ether and water.
4. Neutralization value 180 (mgs. KOH per gram).
5. Very soluble in NaOH, KOH and ammonia with some frothing.

Time did not permit a further study of the purification and properties of this material. Its identity with those acids produced in the decomposition of the corn oil soap has not been proven as yet. Two ways which suggest themselves for arriving at a decision are the following:

1. A sample of neutral soap prepared from carefully purified oil containing only traces of unsaponifiable matter, which has been put away and discoloured in oxygen should be thoroughly mixed to secure homogeneity. From this, one sample would be taken to determine the neutralizing power to NaOH. From another large weighed sample, the insoluble acids would be extracted quantitatively, and the neutralization value of these acids determined. If these two neutralization values calculated per unit weight of soap are the same the results indicate that the acid extraction yields the same material as is produced in the decomposition of the soap. If, however, the two neutralization values are different it would indicate that the acid reaction of the soap is due to the oxidation of the linolic or other unsaturated acids present in the oil.
2. By extracting the colouring matter present in the soap with an inert solvent and purifying this product carefully, and also the coloured product from petroleum ether extraction, a systematic investigation of these two products would definitely settle the question of identity.

EXPERIMENTS CARRIED OUT TO PROVE THAT THE SURFACE DISCOLOURATION RESULTS FROM THE OXIDATION OF THE SOAPS FORMED FROM THE UNSATURATED GLYCERIDES PRESENT IN CORN OIL

Preparation A—Sodium Stearate

100 gms. stearic acid were weighed into an 8″ evaporating dish and dissolved in as small a quantity of 95% ethyl alcohol as possible. Concentrated NaOH (in water) was added carefully until a drop on the end of a stirring rod turned phenolphthalein on the spot plate red. The sodium stearate was thus slightly alkaline.

The operation was carried out over a gentle flame and on cooling, the sodium stearate appeared as a gelatinous, white, translucent material which redissolved in the alcohol present on further gentle heat and grinding the pieces in the warm evaporating dish.

Preparation B—Sodium Oleate

50 cc. of oleic acid were measured into an 8-inch evaporating dish and made neutral to phenolphthalein with a moderately concentrated

solution of NaOH as described in Preparation A. The creamy, oily liquid was very gently heated and stirred continuously to drive off the moisture. After a time this became thick and viscous, finally assuming the character of a heavy creamy lather.

Preparation C—Sodium Palmitate

Some crude palmitic acid was dissolved in 95% alcohol and a strong aqueous solution of NaOH was added with constant stirring. The amber, oily liquid turned brown and not until the solution was made strongly alkaline did a precipitate form. To a portion of this mass more palmitic acid was added to reduce the alkalinity almost to zero. The sodium palmitate was then salted out (using a solution of NaCl, saturated at room temperature), and filtered through a Büchner funnel.

Preparation D—Sodium Oleate Salted Out

Some sodium oleate was prepared as before, only this time a solution of NaCl was added to salt out the soap. The latter was filtered with difficulty.

The tests to which these four samples were submitted, and their results, appear below:

OVEN TEST

			Colour	
Sample	Temp.	No. hrs.	Before	After
A	120 C.	3	White	White
B	120 C.	3	Creamy	Brown
C	118 C.	14	White	White
D	118 C.	14	White	Brown
D	88 C.	48	White	White
D	100 C.	?	White	Light Brown

Left in Air Six Months

A—Unchanged
B—Light yellow—slight rancidity
C—Unchanged
D—Colour yellow and has rancid odour.

OXYGEN ATMOSPHERE TEST

Sample placed in Oxygen-filled tubes

Sample	Time	Colour Change	Time	Colour Change	Time	Colour Change
A	3 hrs.	None	5 days	None	8 months	None
B	3 hrs.	None	5 days	None	8 months	Slight
C	3 hrs.	None	5 days	None	8 months	None
D	6 hrs.	None	6 weeks	Slight	8 months	Very Brown

REMARKS ON LATHERING PROPERTIES

Sample A—Poor in cold water
" B—Good in cold water
" C—Fair in cold water
" D—Poor in cold water (due to salt).

NITROGEN TEST

All four samples remained uncoloured throughout the test, the duration of which was six months.

As will be seen from the above tables, the soaps made from the unsaturated acids discoloured in the presence of oxygen, but did not in the presence of nitrogen.

The next experiments bring out to what extent (1) the temperature at which saponification was carried out, and (2) the temperature of drying, affect discolouration.

Preparation G—Sodium Oleate

28 grams oleic acid were placed in an 8-inch porcelain dish on a hot water bath and 4 grams NaOH, dissolved in 200 cc. hot water, were carefully added. The sodium oleate formed was heated on the water bath for eight hours to drive off the moisture. It was then put in a desiccator filled with nitrogen and dried by a revolving paper pin wheel, dried nitrogen flowing through the desiccator to carry away the moisture. (Fig. 2).

Preparation H—Sodium Oleate—80°

Made in the same way as G, except that temperature was maintained at 80° C. by an oil bath at 110° C. (Fig. 1).

Preparation J—Sodium Oleate—90°

Made in the same way as G, except that temperature was maintained at 90-100° C. by oil bath at 140° C. An hour was taken to add the NaOH solution. The resulting sodium oleate was clear but dark yellow in colour, similar to the colour of the oxidized material.

Preparation K—Sodium Oleate—Cold.

Prepared in the same way as G, but without heating. To separate the sodium oleate formed, about 50 cc. of NaCl solution were added.

Samples of each of H, J and K were placed in ovens at 100°-90°-80° C.; the progress of colour changes which these samples underwent is recorded in the table on page 223.

Sample J seems to be always more discoloured than H, and in the case of K discolouration seems a little slower in getting under way, but ends up darker than either H or J. It would appear, therefore, that there are no advantages gained by carrying out saponification in the cold. After six months all these were quite rancid.

OVEN TESTS ON SODIUM OLEATE SAMPLES

	Sample	Tuesday 5.00 p.m.	Wednesday 10.00 a.m.	Wednesday 2.30 p.m.	Thursday 10.00 a.m.	Friday 10.30 a.m.
100°	H	Pale yellow	Slightly discoloured	Same	Same	Same
	J	" "	More discoloured than H	Same	Same	Same
	K	" "	Discoloured in spots only	More discoloured	Quite brown	Very dark brown
		Tuesday 3.00 p.m.	Wednesday 9.00 a.m.	Thursday 11.00 a.m.	Friday 10.15 a.m.	Saturday 10.30 a.m.
90°	H	Pale yellow	Slightly discoloured	More discoloured	Still darker	Same
	J	" "	Distinctly discoloured	Same	Slightly darker	Same
	K	White	Slightly discoloured	Quite brown	Very brown	Very dark brown
		Monday 11.30 a.m.	Tuesday 11.00 a.m.	Wednesday 11.00 a.m.	Saturday 11.00 a.m.	
80°	H	Pale yellow	Slightly discoloured	Same	Same	
	J	Slightly darker than H	Distinctly discoloured	Same	Same	
	K	White	Slightly discoloured	Same	Distinctly brown	

QUANTITATIVE STUDY OF THE OXYGEN ABSORBED BY CORN OIL SOAP

The foots were first dropped from the oil in the manner already described before saponification was attempted. The oil refined in this way had the following constants:

Saponification Number	193
Acidity (in mg. KOH per g. of oil)	0.25
Iodine Number	123
Sp. Gr.	0.924
Tintometer reading	15.5 red on 35 yellow base.

Using the saponification value 193 mg. KOH per gm. oil, the amount of alkali necessary was calculated as follows:

1 grm. oil uses 190 mg. KOH = 136 mg. NaOH.

Preparation R—Corn Oil Soap Made in Air

450 cc. (416 grams) of oil to be saponified will require $416 \times .136 = 56.5$ grms. NaOH.

The strength of the NaOH solution is to be 18° Bé, hence as:

14.4 grm. NaOH in 100 cc. give 18° Bé. NaOH,

56.5 grm. NaOH in 400 cc. give 18° Bé. NaOH.

After running the 400 cc. of 18° Bé NaOH solution slowly into the oil a gentle heat was applied for four hours. No definite action to indicate saponification was observed, and at the end of this time the mass had solidified to such an extent that it could not be stirred. To thin the mass and separate the soap from the lye and glycerine a saturated solution of NaCl was added and the whole boiled for a short time. The liquor was then drained off. Heating and stirring was recommenced with the addition of water to wash the stock. The soap was grained a second time, using about 150 grams NaCl, allowed to stand over-night, and was then filtered and pressed in a Büchner funnel.

Preparation S—Corn Oil Soap Made Under Nitrogen

100 cc. corn oil and 196 cc. 18° Bé NaOH were used (Fig. 1). The temperature of the stock was maintained at 90° C. for 2.5 hours, at the end of which time the mass became solid. More water was added and the stock allowed to stand in the apparatus over-night. Next morning the lye and glycerine were decanted off and the stock boiled up with NaCl solution. The soap was grained and washed three times. It was then filtered on a Büchner and finally dried over H_2SO_4 in a nitrogen-filled desiccator.

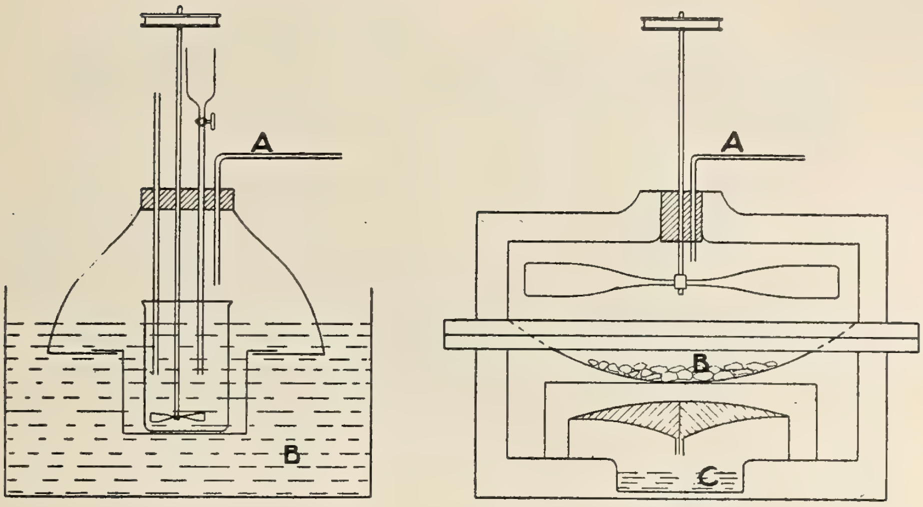


FIG. 1

Preparation T—Corn Oil Soap Carelessly Made

100 cc. corn oil and 196 cc. 18° Bé NaOH were heated together in an open dish over an oil bath till saponification was complete. The soap was washed and salted out twice and dried in a desiccator over H_2SO_4.

These samples of corn oil soap—

S —Moist—Made under nitrogen

S_1—Dry—Made under nitrogen

R—Moist—Made in air—30% H_2O

T—Moist—Made in air carelessly,

were submitted to the following tests:

Known weights of each were placed in tubes filled with oxygen. Each was then attached to a water-jacketed gas burette.

The results of this series of experiments appear in the following tables:

OXYGEN ABSORPTION TESTS RUN ON CORN OIL SOAP SAMPLES

Sample R—5 grams, 30% H_2O			Sample S—5 grams, Moist				Sample S (Continued)		
Date	Hour	c.c. O_2 taken up	Date	Hour	c.c. O_2 taken up		Date	Hour	c.c. O_2 taken up
Dec. 1	2.00 p.m.	0.0	Dec. 12	3.30 p.m.	0.0		Jan. 8	10.00 a.m.	76.0
2	9.00 a.m.	18.1	13	10.00 a.m.	8.8		9	10.00 "	79.0
4	9.00 "	20.0	14	9.00 a.m.	12.0		10	12.00 m.	79.0
4	4.00 p.m.	21.0	15	2.00 p.m.	13.8		11	9.00 a.m.	79.6
5	9.00 a.m.	19.8	16	10.00 a.m.	15.8		12	10.30 "	79.2
6	10.00 "	26.0	18	4.30 p.m.	18.0		13	11.00 "	79.6
6	5.00 p.m.	24.3	19	4.00 "	25.0		15	10.30 "	79.2
7	9.30 a.m.	23.8	20	9.30 a.m.	25.2		16	11.00 "	80.0
8	10.00 "	22.6	21	10.00 "	29.8		17	11.00 "	79.4
8	4.00 p.m.	22.4	21	4.30 p.m.	33.0		22	9.30 "	79.2
9	10.00 a.m.	27.4	22	10.30 a.m.	43.6		23	9.45 "	79.4
11	9.30 "	26.8	23	10.00 "	55.4		24	9.30 "	79.0
12	10.30 "	26.2	26	10.00 "	59.2	(Residual Gas			
13	10.00 "	30.0	27	10.00 "	58.0	(Analyzed			
14	9.15 "	31.4	28	11.00 "	58.8		24	3.00 p.m.	72.0
15	1.45 p.m.	34.0	29	10.30 "	60.6		25	11.00 a.m.	72.2
16	10.00 a.m.	36.6	30	12.00 m.	65.4		26	10.00 "	72.2
18	4.30 p.m.	39.8	Jan. 2	10.00 a.m.	70.4		27	10.30 "	72.0
19	4.00 "	44.4	3	10.00 "	76.0		29	10.00 "	72.2
20	9.30 a.m.	49.0	4	10.00 "	75.6		30	9.30 "	72.2
21	10.00 "	54.8	5	10.30 "	76.0				
22	10.30 "	68.6	6	11.00 "	76.0				

Oxygen Absorption Tests Run on Corn Oil Soap Samples

Sample S_1—5 grams Dry			Sample S_1 (Continued)			Sample T—1 gram Moist		
Date	Hour	c.c. O_2 taken up	Date	Hour	c.c. O_2 taken up	Date	Hour	c.c. O_2 taken up
Dec. 15	3.00 p.m.	0.0	Jan 11	9.30 a.m.	66.8	Jan. 8	11.00 a.m.	0.0
16	10.00 a.m.	3.0	12	10.30 "	66.4	9	10.00 "	−1.5
18	4.30 p.m.	2.8	13	11.00 "	67.2	10	11.45 "	1.2
19	4.00 "	6.0	15	10.30 "	67.0	11	9.30 "	2.0
20	9.30 a.m.	5.8	16	11.00 "	68.0	12	10.30 "	1.2
21	10.00 "	11.4	17	11.00 "	67.4	13	11.00 "	5.6
21	4.30 p.m.	17.4	22	9.30 "	67.2	15	10.30 "	5.8
22	10.30 a.m.	21.2	23	9.45 "	66.6	16	11.00 "	8.6
23	10.00 "	24.0	24	9.30 "	66.4	17	11.00 "	13.0
26	10.00 "	30.4	24	3.00 p.m.	66.0	22	9.30 "	14.6
27	10.00 "	38.2	25	11.00 a.m.	65.8	23	9.45 "	16.6
28	11.00 "	38.2	26	10.00 "	65.8	24	9.30 "	13.8
29	10.30 "	38.2	27	10.30 "	65.4	25	11.00 "	16.6
30	12.00 "	46.0	29	10.00 "	65.4	26	10.00 "	16.0
Jan. 2	10.00 "	53.0	30	9.30 "	64.8	27	10.30 "	19.8
3	10.00 "	59.6				29	10.00 "	25.0
4	10.00 "	60.0				30	9.30 "	30.6
5	10.30 "	59.8						
6	11.00 "	60.0						
8	10.00 "	61.6						
10	12.00 m.	64.8						

It will be seen from these readings that the absorption starts more quickly and proceeds further in the case of the moist than in the dry soap.

The residual gas in the tubes was analyzed and found to contain 1.5% CO_2.

The condition of three of these samples after standing in oxygen six months was:

S —Moist—Completely decomposed, dark brown, slushy and rancid.
S_1—Dry—Badly spotted, but not as bad as S.
R—Moist—Completely decomposed, same as S.
T—Moist—Brown all over.

The condition of the following five samples, after standing in air six months, was as follows:

S —Moist—Badly coloured—decomposition further than S_1.
S_1—Dry—Badly spotted—yellow-brown.
R—Moist—Decomposition bad, especially around edge and bottom where wettest.
T—Moist—Badly coloured—same as S.
T—Dry—Badly spotted—yellow-brown.

In all cases of discolouration the samples appeared to become more moist. Samples from the same series, when placed in nitrogen, showed no signs of discolouration over a period of six months.

The results of these and previously recorded experiments prove conclusively that the discolouration of corn oil soap is due to the oxidation of the unsaturated constituents of the soap, because:

(1) Discolouration does not occur in an atmosphere of nitrogen.

(2) Discolouration is more rapid in pure oxygen than in air.

(3) Soaps made from unsaturated fatty acids discolour; those made from saturated fatty acids do not.

(4) Discolouration starts on the surface.

It is also to be noted that discolouration is more noticeable in moist than dry samples, more in smoothed than in salted samples, and that local overheating such as occurs in careless operation results in a greatly increased rate of deterioration.

EXPERIMENTS AIMED AT PREVENTING THE DEVELOPMENT OF THE BROWN DISCOLOURATION

Having established the cause of discolouring to be the high percentage of unsaturated constituents in corn oil, the next logical step is to find ways and means whereby the percentage of unsaturated oils may be lowered to such a point that the resulting soap is unaffected by the atmosphere, but not so low as to yield a poor lathering soap.

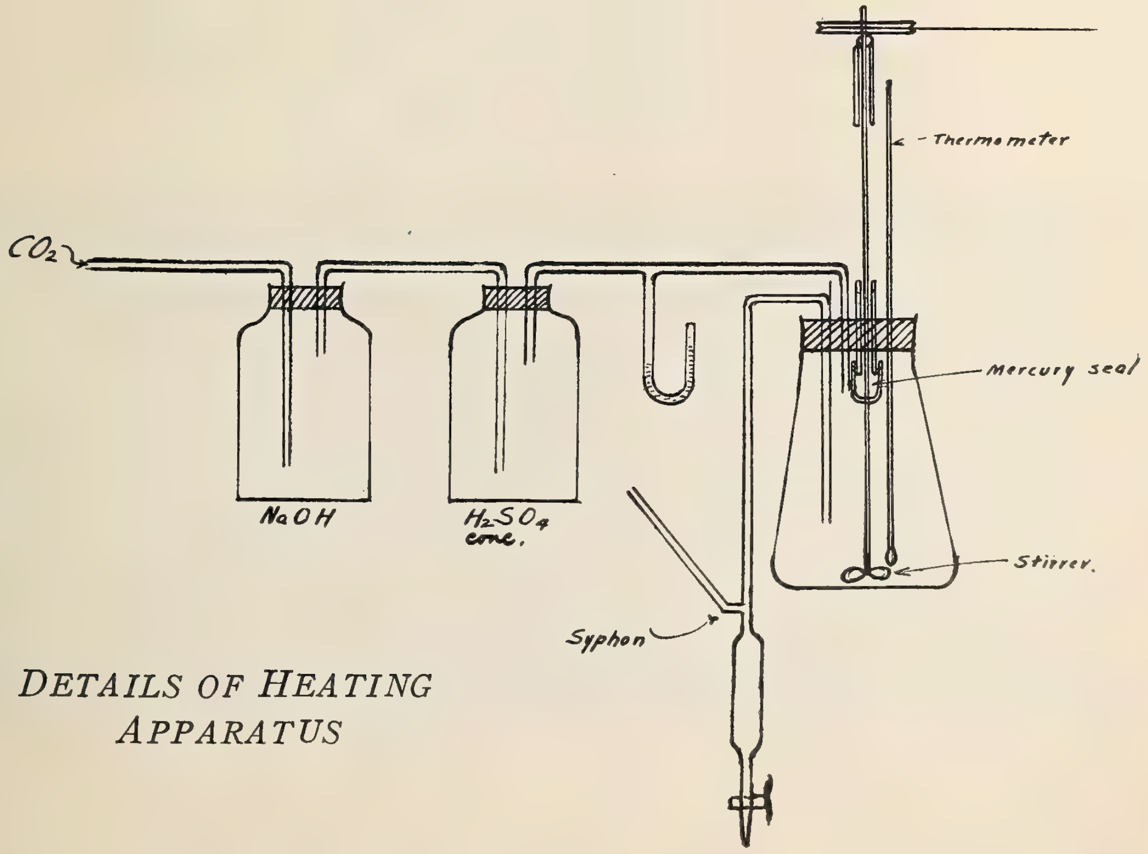
There are two ways of doing this that occur to us; the first is by hydrogenation, the second by polymerization. We might also attain the same practical result by adding something to the soap that will slow down the rate of oxidation.

To-day the selling price of whale oil, cottonseed oil, etc., is in every case too high as compared with the price of palm oil, cocoanut oil, olive oil foots, etc. to render the hydrogenation of the former of economic interest to the soap maker. One of the reasons this method is economical in the foodstuffs industry is that a low priced material is converted into a high priced food product, whereas in the soap industry the cost of the raw material is practically the same as the price of the treated material. Another objection to hydrogenation is, according to Ellis[4], that soaps made from hydrogen-hardened oils have very poor lathering properties.

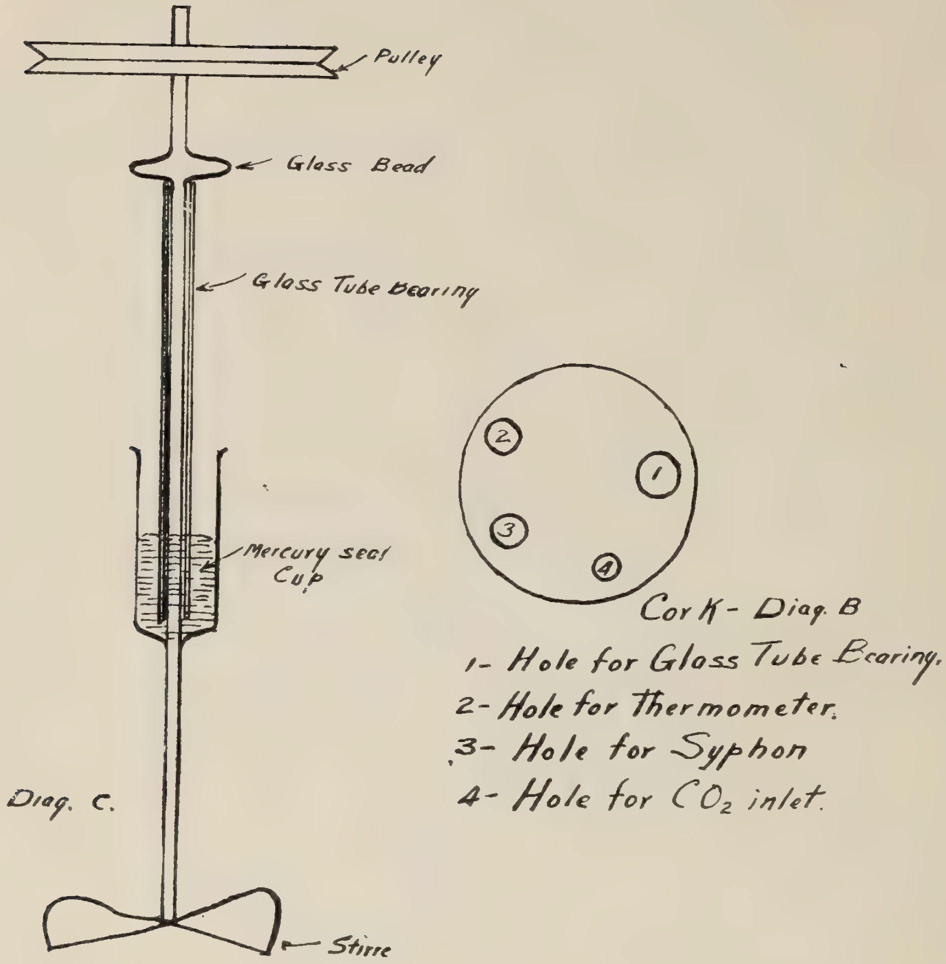
Ellis also refers to soaps made from polymerized oils and states that they lather well. The method Ellis employed to effect polymerization was to heat the oils to relatively high temperatures.

We decided to heat corn oil to see if any polymerization takes place, with a consequent reduction in the extent of its unsaturation. The iodine absorption number[19] of an oil is a measure of the degree of saturation, and this number can be used to trace the progress of polymerization.

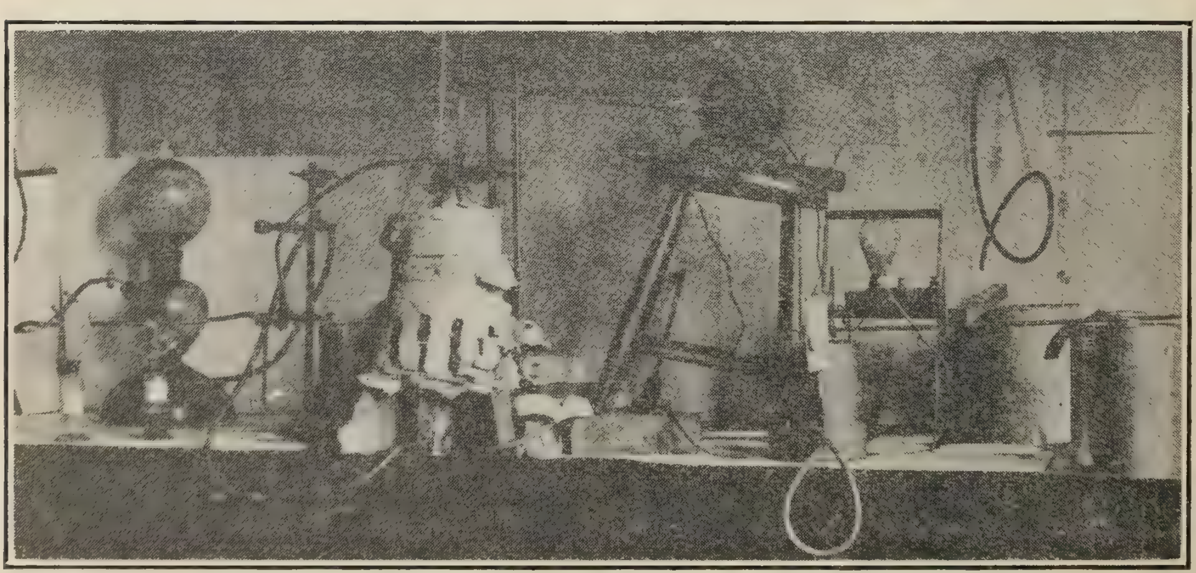
The following is a drawing of the apparatus in which the heating was carried out, and the photograph pictures the apparatus in operation.



DETAILS OF HEATING APPARATUS



DETAILS OF STIRRER



Several batches of oil were heated in an atmosphere of CO_2 in this apparatus at different temperatures for from 20 to 60 hours. The temperatures selected were 180°, 200°, 220°, 240°, 260° and 280° C. Samples were drawn from the flask by means of the syphon as described above, and put through the following tests: Iodine number[19], Lovibond tintometer, saponification value, acidity value and viscosity. In the case of the viscosity, since comparative results only were desired, the time required by 25 cc. of oil at room temperature to run out of a pipette was taken as a measure of the viscosity. This method was much simpler and faster than the usual one involving the use of a standard viscometer.

It was quite difficult to keep the temperature constant, especially at times when the gas pressure increased to such an extent that the regulator was not equal to the task of controlling it.

The values for the untreated oil from which the foots had been dropped are as follows:

Iodine Number	123
Saponification value	193
Acidity (mg. of KOH/10 gms. of oil)	2.8 mg.
Tintometer reading	15.5 red on 35 yellow base
Viscosity (comparative)	100

The following tables show results on heated oils:

TABLE A—AIodine Number (Hanus Method)

Hours of Heating	180° C.	200° C.	220° C.	240° C.	260° C.
3	123	121.2			
4			122.3		
6				123.2	
7	123	121.8			114.3
13	123.3	123.2			114.8
16			123.2		
18				121.4	
22			121.1		
23	123.9	122.7		119.5	
28	123.4	122.5			108
29			118.5		
30				112.4	
33		122.7			110.6
37		122.1			
39					106
42				115.2	
47				115.2	
50				114.1	
53				110.8	
65				101.1	

TABLE B—Tintometer

Hours of Heating	180° C.	200° C.	220° C.	240° C.	260° C.
3	15	12.85			
4			11.55		
6				13.55	
7	15.5	12.6			28.2
13	14.45	12.4			30.3
16			11.82		
18				11.55	
22			15.35		
23	16.6	12.6			15.5
28	16.6	11.3			34.9
29			18.8		
30				15.5	
33	12.35				50/30
37		15			
40					53/30
42				15.75	
47				13.7	
50				17.4	
53				18.75	
65				18.75	

N.B.—All measured on 35 yellow base except the sample at 260° C. for which the yellow base is 30.

TABLE C—Viscosity

Hours of Heating	180° C.	200° C.	220° C.	240° C.	260° C.
3	100	100			
4			104		
6				112	
7	100	110			229
13	100	110			287
16			103		
22			107		
23	100	110		127	
28	100	104			339
29			135		
30				151	
33		110			293
37		112			
40					330
42				190	
47				210	
50				242	
53				390	
65				495	

TABLE D—Acidity

Hours of Heating	180° C.	200° C.	220° C.	240° C.	260° C.
3	2.8	4.0			
4			3.6		
6				5.0	
7	3.6	3.6			31.2
13	4.0	2.4			45.6
16				19.6	
22			5.2		
23	3.8	2.8		18.8	
28	3.6	3.0			48.4
29			24.8		
30				19.2	
33		2.8			61.6
37		2.8			
40					66.4
42				18.0	
47				20.0	
50				18.4	
53				21.6	
65				23.2	

TABLE E—Saponification

Hours of Heating	180° C.	200° C.	220° C.	240° C.	260° C.
3	182	198			
4			196		
6				194	
7	198	178			200
13	178	198			202
16			194		
18				197	
22			194		
23	178	198		196	
28	178	188			196
30			190	202	
33					195
35		181			
37		178			
40					200
42				189	
47				194	
50				186	
53				189	
65				194	

Samples were also heated to 280° C.

1st lot { for 20 hrs.—resulting in a very viscous, turbid and dark coloured oil with an Iodine Number of 101.

2nd lot { for 18 hrs.—good oil, thick like honey; no solids. Iodine Number 107.
for 25 hrs.—solid matter dispersed through the oil. Colour of oil, dark. The temperature rose to 290° on one or two occasions. Iodine Number 100.

3rd lot { for 30 hrs.—good oil, very small amount of dispersed matter.

Iodine Number	98
Acidity (mg. KOH/10grms. oil)	121.6
Saponification Number	189
Viscosity	870
Tintometer	26 red on 50 yellow base

Our problem is to note the effect that the reduction of the iodine number of the oil has upon the colour-keeping qualities of the resulting soap.

In obtaining this range of iodine values it was difficult to duplicate the results shown on Table A. For instance, Table A shows that 65 hours' heating at 240° C. results in an iodine number of 101; on repeating a value of 113 was obtained. As mentioned above, the temperature was difficult to control, and in the former case it probably averaged over 240° C., and in the latter, under 240° C. However, to check this up an iodine number was run on the oil which had previously given a value of 101.1. The result was 106. Both the 101 and 106 determinations were accurate, as every determination was done in duplicate. The cause for this was suggested on consulting Lewkowitsch,[1] who states that when the stearine settles the percentage of unsaturated compounds increases in the supernatant oil with a corresponding rise in the iodine value. A translucent deposit was noted in the bottom of several of the sample flasks; this, no doubt, was the stearine settling out.

The apparatus in which saponification was carried out is shown in the photograph below. A very small flame was all that was required to maintain the temperature of the batch between 70° and 80° C.

The following was the general procedure of saponification:

To 100 or 120 cc. of oil 250 cc. of 13° Bé NaOH solution were added and placed under stirrer, and heated for seven hours at from 70°-80° C. 30 grams of salt were added and stirred for half an hour, then allowed to stand over-night. The lye was drawn off and 100 cc. to 150 cc. of 18° Bé NaOH were added, stirring and heating being then continued for another 6 or 7 hours, or until saponification was completed. 30 grams of salt were added and stirred for half an hour, and then allowed to stand over-night. The lye was drawn off and the soap washed with 200 cc. of water, salted out with 30 grams of salt, washed again (heated to 70° and stirred) and salted again with 30 grams of salt. Time was allowed for a good separation of the lye and salted soap to take place. The lye was then drawn off and 30 to 50 cc. of water added and the soap placed on a water bath to smooth it out. The soap was then dried on a sheet of glass in a fume cupboard maintained at 35° C. As the soap dried it was well worked with a spatula to insure a uniform product. A small sample was placed in a nitrogen filled desiccator, the balance was placed in a stoppered Erlenmeyer flask.

The following soaps were made from:

"foots dropped" oil—Iodine Number	123
"30 hrs.—240°" oil—Iodine Number	122
"65 hrs.—240°" oil—Iodine Number	113
1st batch—turned very brown and saponification very slow. Required an extra wash to improve the colour. 2nd batch—was not so brown and required only 2 washes, as described above in general method.	
"16 hrs.—260°" oil—Iodine Number	121
Always seems to darken up considerably on addition of 18° Bé lye.	
"26 hrs.—260°" oil—Iodine Number	118
"52 hrs.—260°" oil—Iodine Number	110
This oil was very dark coloured, had lost its characteristic odour, and contained much decomposition product dispersed through it. After a short heat with 13° Bé NaOH it became very brown and it was salted out to wash. The lye was then drawn off and a fresh quantity of weak lye added, heated for 4 hrs., and salted and washed again. Saponification was not complete so some more lye was added before finishing in the usual way. This extra work was necessary to obtain a good colour-free soap.	
"18 hrs.—280°" oil—Iodine Number	107

No suspended matter in this oil.

"20 hrs.—280°" oil—Iodine Number.................... 101
Oil very poor and full of dispersed matter similar to the 52 hrs. at 260° oil. It required three washes instead of the regular two, and gave a very poor soap even then.

"25 hrs.—280°" oil—Iodine Number.................... 100
Oil of fair quality with a little suspended matter. Turned brown on addition of 18° Bé NaOH but washed up, giving a cream-coloured soap.

"30 hrs.—280°" oil—Iodine Number.................... 98
Oil had trace of suspended matter and turned brown on addition of 18° Bé NaOH, but whitened up fairly well.

The mix always turned a dark brown upon the addition of the strong caustic. George H. Hurst[3] states that it is unsafe to use lye stronger than 7° Bé on tallow, and so we made a soap with 10° Bé NaOH, starting and finishing on this strength. The soap mix did not darken as on previous occasions, and although the resulting soap was not perfectly white it had a nice creamy colour.

One set of samples of these soaps was placed in an oven at 90° and another set exposed to the air. The results of these tests appear in Tables F, G and H.

The soap samples were subjected to these oven tests to increase the rate of discolouration. The discolouration at room temperature requires several months to develop fully, whereas at 90° C. and 110° C. in the oven it is only a matter of hours.

Wright and Thompson[6] carried out some interesting experiments on soaps for the purpose of determining to what extent hydrolysis occurs. They studied also the effect of salt on this hydrolysis. The following table shows the percentage of NaOH liberated on the hydrolysis of various neutral soaps with water.

Soaps made from	Hydrolysis brought about by 150 molecules of water to 1 mol. anhydrous soap
Stearic Acid	0.7 g. Na_2O per 100 g. Na_2O present as soap
Tallow	1.5
Oleic acid	1.85
Cottonseed	2.25

Hurst[3] also made similar determinations having salt present. One gram of white curd soap dissolved in 50 cc. of water.

TABLE F

AIR EXPOSURE TEST

Sample	Time	Colour	Time	Colour
“Foots dropped oil”—wet	20 days	Unchanged	2 months	Lemon yellow
“ “ “ —½ dry	“ “	1 small spot only	“ “	“ “
“ “ “ —dried	“ “	Unchanged	“ “	Light yellow
“ “ “ —grained	“ “	“	“ “	“ “
“240°—30 hrs.”—48% H_2O	“ “	Light yellow	“ “	Light brown
—18% H_2O	“ “	“ “	“ “	“ “
—dried	“ “	“ “	“ “	“ “
“240°—65 hrs.”—wet	“ “	1 yellow spot only	“ “	Faint yellow
(first batch) —grained	“ “	No change	“ “	No change
—dried	“ “	“ “	“ “	Faint yellow
(second batch)—dried	“ “	Unchanged	“ “	“ “
“260°—16 hrs.”—dried	“ “	“		
“260°—26 hrs.”—dried	“ “	“		
“260°—52 hrs.”—dried	“ “	Light yellow		
“280°—18 hrs.”—dried	“ “	Unchanged		
“280°—20 hrs.”—dried	“ “	Greyish		
“280°—25 hrs.”—dried	“ “	Unchanged		
“280°—30 hrs.”—dried	“ “	“		
“280°—30 hrs.”—dried	“ “	“		

(made from weak lye)

TABLE G

Oven Exposure Test—90°C.

Sample	1 hr.	2 hrs.	4 hrs.	5 hrs.	6 hrs.	12 hrs.	24 hrs.
"foots dropped" 123 (Iodine number)	Colour Started	No further	Same	Brown	Same	Same	Well browned
"30—240°" 122	Colour Started	Brown	Same	Same	Dark brown	Same	Very dark brown
"65—240°" 113	No Change	Same	Same	Same	Same	Small yellow spot	Yellow
"16—260°" 121	No Change	Same	Same	Light brown	Still further brown	Same	Brown
"26—260°" 118	No Change	No Change	No Change	Brown in places	50% brown	Same	Brown
"52—260°" 110	Slight Change	Same	Same	Same	Light brown	Same	Dark brown
"18—280°" 107	No Change	Same	Same	Same	Same	Same	Light yellow
"20—280°" 101	Slight change	Same	Dark yellow	Same	Brown in places	Same	Brown
"25—280°" 100	No Change	Same	Same	Same	Same	Faint yellow	Light brown
"30—280°" 98	No Change	Same	Same	Same	Same	1 spot yellow	Light yellow
Sodium Palmitate	No Change	Same	Same	Same	Same	Same	Faint yellow
Sodium Stearate	Unchanged throughout						
Cheap Castile	No change	Same	Light yellow	Same	Light brown	Same	Same

TABLE H

OVEN EXPOSURE TEST—110° C.

Sample	1 hr.	4 hrs.	5 hrs.	8 hrs.	20 hrs.
"foots dropped"	Dark brown	Same	Same	Black	Same
"30—240"	Black within the hour				
"65—240"	Light yellow	Yellow	Same	Light brown	Same
"16—260"	Light brown	Same	Same	Brown	Same
"26—260"	50% brown	Browner	Same	All brown	Same
"52—260"	Dark brown	Same	Same	Black	Same
"18—280"	Very light yellow	Light yellow	Same	Very light brown	Same
"25—280"	Brown	Same	Same	Dark brown	Same
"30—280"	Light brown	Same	Same	Brown	Dark brown
"30—280" (weak lye)	Very light yellow	Yellow	Same	Same	Light brown
Palmitate	Unchanged	Same	Same	Same	Yellow
Stearate	Unchanged throughout				
Cheap Castile	Unchanged	Same	Light Yellow	Same	Brown
Ivory	Unchanged	Same	Same	Same	Light yellow
Cocoanut soap	Unchanged	Same	Same	Same	Faint yellow

Test Conditions	Total Alkali liberated
Without addition of salt	3.97% of total alkali in the soap
With 1 gram of salt	2.23 " " " "
With 10 grams of salt	0.25 " " " "

We see here that water has a hydrolyzing action on soap and that soap from cottonseed oil, which is somewhat similar to corn oil, hydrolyzes to a greater extent than the other soaps used, also that the addition of salt suppresses this action.

Our observations have shown that the wetter the samples of corn oil soap the sooner the brown colour appears and the more pronounced it becomes. This is probably the reason that salted samples do not discolour as rapidly as smoothed samples, a phenomenon we have often noted.

CONCLUSIONS

1. The discolouration that develops in time on the surface of soap made from corn oil is not due to the colour of the oil nor to sunlight.

2. The discolouration results entirely from atmospheric oxidation of the saponified unsaturated glycerides, *e.g.*, sodium oleate and linolate.

3. It is impossible to make a soap from ordinary corn oil that will remain white for any length of time when exposed to the air. Even when made in an atmosphere of nitrogen the resulting soap will in time turn brown in air. Carelessness in manufacture, *e.g.*, overheating, greatly aggravates the defect.

4. Dry soap, *e.g.*, a well-grained soap or one that has been smoothed and then dried does not become discoloured as quickly as a smoothed or moist soap. The more rapid deterioration of wet samples appears to be due to the NaOH set free by hydrolysis acting as a catalyst towards oxidation. The presence of salt appears to delay the development of colour, probably because it forces back hydrolysis. Even the first class white soaps on the market will turn brown in a few hours when moistened and placed in an oven at 110° C.

5. Heating corn oil in an inert atmosphere to from 240° C. to 280° C. results in a lowering of the iodine number and a thickening of the oil.

6. If the oil has been very carefully heated so that any local heating above 285° C. has been avoided, the resulting oil, if its iodine number has fallen from around 123 to 113 or lower, will give a soap that resists oxidation and hence will remain white much longer than soap made from the unheated oil.

7. It is impossible to produce a satisfactory soap as outlined under (6) unless the heating of the oil has been carried out with the greatest care. The temperature must not be allowed to rise above 285°. At 310° the oil becomes stiff and rubbery and smells much like cheese.

8. Soap made with a weak lye, 10° Bé., retains its whiteness much longer than that made with a stronger lye, 18° Bé.

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CERTAIN USEFUL FORMS FOR THE AZIMUTH FORMULA

By L. B. STEWART, *Professor of Surveying and Geodesy*

The standard method for finding the azimuth of a direction is by observing with a theodolite the horizontal angle between the direction and that of a close circumpolar star, noting the time of pointing to the latter; a repetition of this observation, after reversing the instrument, constitutes a complete observation for azimuth, the result of which is free from the effect of instrumental errors of adjustment. With small instruments the pole star is usually observed, and the angle between the star and the reference point may be measured either by the direction or the repetition method.

The latitude of the place of observation being known, the data to be used in computing the azimuth of the star are:

ϕ, the latitude of the place,
δ, the declination of the star, and
τ, its hour angle,

the last being derived from the observed time.

The formula almost universally used in the reduction is:

$$\tan A = \frac{\sec\phi\cot\delta\sin\tau}{1-\tan\phi\cot\delta\cos\tau} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

and this formula, to be convenient for use, requires either subtraction logarithms, or a special table giving the value of

$$\log\frac{1}{1-a}$$

the argument being log a, where

$$a=\tan\phi\cot\delta\cos\tau;$$

see U.S. Coast and Geodetic Survey Special Publication No. 14, pp. 165 to 173.

The necessity for any special table may be obviated, however, by placing equation (1) in another form, derived as follows: writing it

$$\tan A = \frac{\tan A'}{1-m}$$

in which

$$\tan A'=\sec\phi\cot\delta\sin\tau,$$

and

$$m=\tan\phi\cot\delta\cos\tau,$$

and taking log's. and expanding, it becomes

$$\begin{aligned}\log\tan A &= \log\tan A' - \log(1-m)\\ &= \log\tan A' + \mu m + \tfrac{1}{2}\mu m^2 + \tfrac{1}{3}\mu m^3 +,\end{aligned}$$

μ denoting the modulus of the common system of logarithms. The expression for m may be rendered slightly more convenient by introducing into it the value of $\tan A'$, so that A is given by the equations:

$$\begin{aligned}\tan A' &= \sec\phi\cot\delta\sin\tau\\ m &= \tan A'\sin\phi\cot\tau\\ \log\tan A &= \log\tan A' + \mu m + \tfrac{1}{2}\mu m^2 + \ldots\ldots\ldots\ldots (2)\\ (\log\mu &= \bar{1}.6377843)\end{aligned}$$

This series is very convergent. The example worked out below shows that a sufficient number of terms are retained in equation (2) to give A with the precision necessary to satisfy the exacting requirements of geodetic work, using six-place logarithms; only in extreme cases is an additional term necessary. In reducing observations taken with the engineer's transit, however, where at best the circle is read to the nearest 20″, the term in m^2 may usually be omitted, and five-place logarithms used in the reduction; the resulting error will seldom exceed 1″.

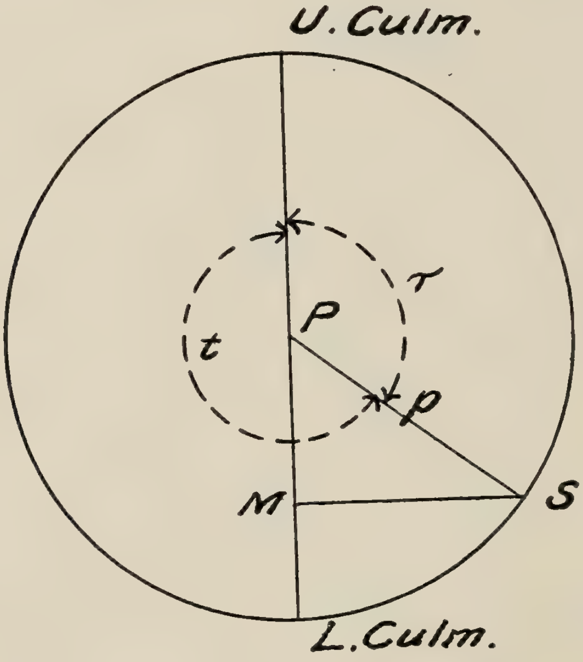

Fig. 1

In azimuth determinations the best time for observing is towards the end of the afternoon, when there is still sufficient daylight to allow the observer to dispense with artificial light, either for illuminating the field of the telescope, or for rendering the reference point visible. Under those conditions a little difficulty may be experienced in finding the star, and the writer has frequently found the following method useful:

Assume the time when observations will begin, and compute the corresponding sidereal time (if not provided with a sidereal timepiece), and thence the hour angle of the star by means of the equation

$$t=\theta-\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which,

t denotes the hour angle reckoned from upper culmination westerly,
θ the sidereal time, and
α the right ascension.

Then draw a circle (Fig. 1) representing the diurnal circle of the star (here assumed to be Polaris), as it would appear from the earth, and lay off t approximately, drawing PS, the declination circle of the star. Also draw SM perpendicular to the meridian PM. Then assuming that the altitude of M is also that of S, the altitude and azimuth of the star are given approximately by the equations:

$$\left.\begin{array}{l} h=\phi-PM=\phi+p\cos\tau \\ A=SM\sec h=p\sin\tau\sec h \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

p being taken to the nearest minute, and $\sin\tau$, etc., to two decimal places. The telescope is then set at the altitude thus found, by means of the vertical circle, the instrument turned in azimuth towards the north, and the star found. The azimuth just computed may then be used to place the instrument approximately in the meridian in order to find the correction of the watch on sidereal time by observing the transit of an equatorial star. The observation for azimuth may then be taken in the usual manner, and the reduction made by equation (1) or (2).

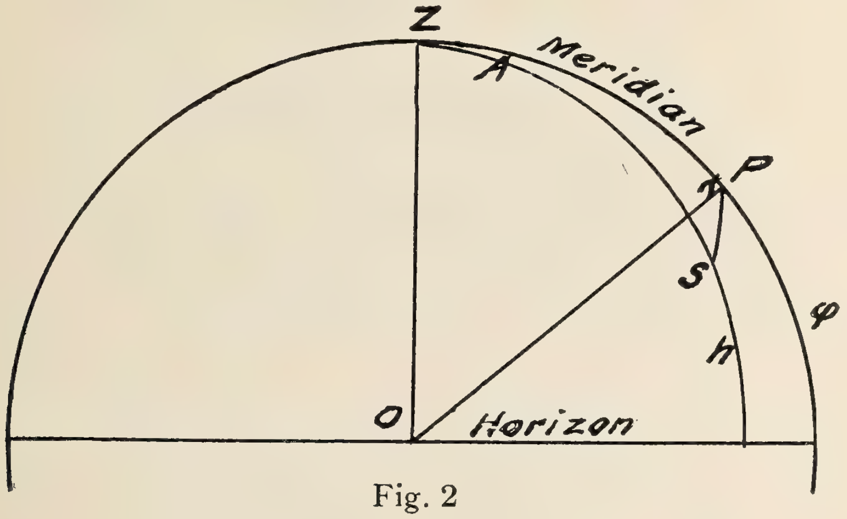

Fig. 2

After carrying out the operations as above outlined the writer always found that the value of A given by the approximate formulæ (4), and using five-place logarithms, agreed very closely with that given by the more precise expression. The following investigation was therefore made to ascertain the degree of precision that might be expected from equations (4), and their adaptability to the final computation of the azimuth.

The rigorous formula from which the second of (4) may be derived is (Fig. 2),

$$\sin A = \sin p \sin \tau \sec h,$$

which may be written,

$$A = \sin^{-1}(\sin p \sin \tau \sec h).$$

Expanding, this becomes

$$A = \sin^{-1}\left\{\left(p - \frac{p^3}{6}\right) \sin \tau \sec h\right\},$$

$$= \left(p - \frac{p^3}{6}\right) \sin \tau \sec h + \frac{p^3}{6} \sin^3 \tau \sec^3 h,$$

(omitting powers of p above the third)

$$= p \sin \tau \sec h + \frac{p^3}{6} \sin^3 \tau \sec^3 h \left(1 - \frac{1}{\sin^2 \tau \sec^2 h}\right),$$

or

$$A = A' + \frac{A'^3}{6}\left(1 - \frac{1}{\sin^2 \tau \sec^2 h}\right)$$

in which

$$A' = p \sin \tau \sec h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This equation may be written in either of the following forms, p, and therefore A' and A, being in seconds:

$$\left.\begin{aligned} A &= A' + \frac{\sin^2 1''}{6} A'^3 \left(1 - \frac{1}{\sin^2 \tau \sec^2 h}\right) \\ A &= A' + \frac{\sin^2 1''}{6} A'^3 \left(1 - \frac{p^2}{A'^2}\right) \end{aligned}\right\}, \ldots\ldots (6)$$

$$\left(\log \frac{\sin^2 1''}{6} = \overline{12}.5929985\right)$$

in which the second term gives the error involved in the use of equation (5); in the example given below it amounts to 0''.307.

This corrective term evidently vanishes when

$$A' = p,$$

or when

$$\sin^2 \tau \sec^2 h = 1,$$

or

$$\sin \tau = \pm \cos h,$$

or

$$h = 90° - \tau, \text{ or } \tau - 90°.$$

Also for values of A' less than p it is evidently negative, and for values greater than p, positive.

The value of A' for which the term is a maximum may be investigated as follows: writing it in the form

$$\frac{\sin^2 1''}{6}(A'^3 - A'p^2),$$

differentiating with respect to A', and equating to zero, we find

$$3A'^2 - p^2 = 0$$

whence
$$A' = \pm \frac{p}{\sqrt{3}},$$
which is the value of A' for which the term in question is a maximum. Substituting in it this value of A', we find
$$\pm \frac{\sin^2 1''}{6} \frac{p^3}{3^{3/2}} (1-3)$$
or
$$\mp \frac{\sin^2 1''}{3^{5/2}} p^3$$
This gives its maximum negative value. With the data of the example given below this is found to be $\mp 0''.099$.

The maximum positive value may be inferred at once by an examination of (6), from which it is clear that the second term increases continually as A' increases from the value p, and becomes a maximum when A' reaches its maximum value at elongation. In this case we have
$$\sin \tau \sec h = \sec \phi,$$
so that
$$A' = p \sec \phi,$$
which, substituted in the second term of (6), gives
$$\frac{\sin^2 1''}{6} p^3 \sec^3 \phi \left(1 - \frac{1}{\sec^2 \phi}\right),$$
or
$$\frac{\sin^2 1''}{6} p^3 \sec^3 \phi \sin^2 \phi,$$
which is the maximum positive value of the second term of (6), and therefore the maximum error in A when computed by means of equation (5). Again using the data of the example below, its value is found to be $0.''564$. It apparently increases rapidly with the latitude, however; in latitude 60° it becomes $1''.550$.

We may therefore sum up as follows the conclusions to be drawn from the above investigation:

For the reduction of observations taken with the engineer's transit, with which the horizontal angle between the star and the reference point is seldom read more closely than to the nearest $20''$, the error arising from the use of equation (5) is unimportant, especially for values of $A < p$. For values of $A > p$, and for latitudes up to 50° the error will seldom exceed $0''.5$. For high latitudes—60° and over—the error may reach values of $1''$ or more.

The form of equation (5) also suggests that the reduction may be materially shortened, if the instrument is provided with a good vertical circle, by observing h in the field, instead of computing it by means of the first of (4). This merely involves reading the vertical circle as well as the horizontal when taking the observation. The question of the amount of error that may thus be introduced into the resulting azimuth is at once raised; and this may be determined as follows:

Taking the rigorous expression

$$\sin A = \frac{\sin \tau \sin p}{\cos h},$$

we find by differentiation

$$\cos A \frac{dA}{dh} = \sin \tau \sin p \frac{\sin h}{\cos^2 h},$$

$$= \frac{\sin \tau \sin p}{\cos h} \tan h,$$

$$= \sin A \tan h;$$

$$\therefore \qquad dA = \tan A \tan h \, dh \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

which gives the error in A due to an error in h. Taking $dh = 20''$, and using the same data as before, $dA = 0''.624$. This error is unimportant when a small instrument is used in the observations. It may be pointed out, moreover, that if the observer is obliged to determine his latitude by observing the meridian altitude of a star, an error nearly equal to that just considered may thus be caused. For, taking the equation

$$\sin \tau \cot A = \cos \phi \cot p - \sin \phi \cos \tau,$$

from which equation (1) is derived, we find by differentiation

$$-\frac{\sin \tau}{\sin^2 A} \cdot \frac{dA}{d\phi} = -\sin \phi \cot p - \cos \phi \cos \tau$$

or

$$\frac{dA}{d\phi} = \frac{\sin \phi \cos p + \cos \phi \sin p \cos \tau}{\sin p} \cdot \frac{\sin^2 A}{\sin \tau}$$

$$= \frac{\sin h}{\sin p} \cdot \frac{\sin^2 \mathrm{A}}{\sin \tau}$$

Then as

$$\frac{\sin A}{\sin \tau} = \frac{\sin p}{\cos h}$$

we have

$$\frac{dA}{d\phi} = \sin A \tan h,$$

or

$$dA = \sin A \tan h \, d\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

As A is small this value of dA is almost identical with that given by (7); in fact, using the same data, the difference is only $0''.0002$. This serves to show that, unless the observer has a fairly accurate knowledge of his latitude, there is little to be gained by using one of the precise formulæ (1) and (2); but that equally good results may be obtained by using the approximate formula (5) and observing the altitude of the star.

The latter procedure is to be preferred to that of observing for latitude, and then using equation (1) or (2) in computing the azimuth.

The following observation, taken in the interior of Anticosti Island on September 3rd, 1920, will serve to illustrate the application of the above formulæ, and to give a comparison of the results obtained by them:

Object obs'd.	*Cir.*	*H.C.R.*	*V.C.R.*	*Watch*
R.P.	R.	185° 22′.5		p.m.
Polaris	R.	116° 14′	(49° 21′)	6h 09m 39s
Polaris	L.	296° 18′	(49° 21′)	6h 13m 11s
R.P.	L.	5° 23′		

The vertical circle was not read in the field, but the altitude was computed from the data, and its value to the nearest minute inserted in the observation to show the use of equation (5). The geographical coordinates of the place of observation were approximately

$$\phi = 49^\circ\ 52'\ 07',$$
$$L = 4^h\ 16^m\ 41^s.$$

The star place was,

$$\begin{cases} \alpha = 1^h\ 33^m\ 16^s \\ \delta = 88^\circ\ 52'\ 40''\ (\therefore p = 4040'') \end{cases}$$

so that $t = 16^h\ 11^m\ 48^s$

and $\tau = 7^h\ 48^m\ 12^s$
$= 117^\circ\ 03'\ 00''$

The azimuth of the star will be computed by means of equations (2), (6), and (5) in turn.

Eq. (2)—

	log.	
sec ϕ	0.1907484	
cot δ	8.2920118	
sin τ	9.9496876	
tan A'	8.4324478	8.4324478
sin ϕ	9.8834164	
cot τ	9.7081022n	
m	8.0239664n	
μ	$\bar{1}.6377843$	
μm	$\bar{3}.6617507n - 0.0045893,\ 45$	
m^2	$\bar{4}.0479328$	
$\frac{1}{2}\mu$	$\bar{1}.3367543$	

$\frac{1}{2}\mu m^2$	$\bar{5}.3846871$	0.0000242, 49
m^3	$\bar{6}.07190n$	
$\frac{1}{3}\mu$	$\bar{1}.16066$	
$\frac{1}{3}\mu m^3$	$\bar{7}.23256n$	$-$0.0000001, 71
tan A		8.4278825

$A = 1° \ 32' \ 03''.366$

Eq. (6)—

h is computed from the data ϕ p and τ, and found to be

49° 20′ 53″.04

		log
p		3.6063814
$\sin\tau$		9.9496876
$\sec h$		0.1861108
A'	$=5523''.061$	3.7421798
A'^3		11.2265394
$\sin^2 1''/6$		$\overline{12}.5929985$
	$0''.6599$	$\bar{1}.8195379$
$\sin^2\tau \sec^2 h$		0.2715968
	0.35314	$\bar{1}.5479411$
	0.30685	
A	$=5523.368$	
	$=1° \ 32' \ 03''.368$	

The close agreement between this value of A and that found by (2) shows that the omitted terms in the expansion (6) have no appreciable influence on the result.

Eq. (5)—

The data are:

$p = 4040''$
$\tau = 117° \ 03'$
$h = 49° \ 21'$

	log
p	3.60638
$\sin\tau$	9.94969
$\sec h$	0.18613

$$
\begin{array}{ll}
A & 3.74220 \\
\hline
A & = 5523''.3 \\
 & = 1^\circ\ 32'\ 03''.3 \\
\hline
\end{array}
$$

The precision of this result is of course partly accidental; in fact, if the more precise value of h given above had been used the effect would have been to diminish A by $0''.2$.

The above investigation shows conclusively that the method of determining azimuth, based upon the simple equation (5), meets all the requirements of an observer provided with the ordinary engineer's transit.